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Recurrence Quantification Analysis

For the investigation of dynamical systems, recurrence based methods have proven its potential even for short and non-stationary data series. A recurrence plot is usually defined as a binary matrix representing the pairwise closeness of the values of a data series:

$$R(i, j) = \Theta(\epsilon - \| \mathbf{x}(i) - \mathbf{x}(j) \|)$$

By quantification of the line structures in a recurrence plot, we are able to characterise the dynamics of the system with measures of complexity. It has been shown that such measures, calculated in moving windows, are able to detect transitions in the dynamics of systems, like chaos-period, chaos-chaos and chaos-SNA transitions (Marwan et al, 2007).

Here we exemplarily use the measures of complexity measuring the fraction of recurrence points forming diagonal (Determinism, DET) and vertical lines (Laminarity, LAM). For both measures, we need the histograms of line lengths $P(l)$. Determinism is the probability that recurrent states will further be recurrent and Laminarity is the probability that (very) slowly changing states remain in similar states. High values of Determinism are typical for deterministic systems and high values of Laminarity are typical for intermittency.

Application to Event Related Potentials (ERP)

In the Oddball experiment, a number of visual or acoustic stimuli of different surprising effect (10% and 90% event probability) is shown to a proband. The averaging of the measured EEG data reveals a P300 component, which is anti-correlated with the event probability. This component

reflects the switching between two modi of cognitive behaviour: During episodes where the frequent stimuli are presented to the subjects, they went into a mode of automatic processing of the events. When suddenly the rare stimulus arises, the brain function is switched to controlled processing.

The investigation of such ERPs on a single trial basis is rather difficult. However, recurrence based methods have the potential to recognize the specific ERP components even on a single trial basis (Marwan and Meinke, 2004; Marwan et al, 2007; Schinkel et al, 2007).

Bootstrap Procedure

As a statistical test for the RQA based transition analysis we propose the following bootstrap procedure:

(1) Merge all local histograms of line lengths $P_i(l)$

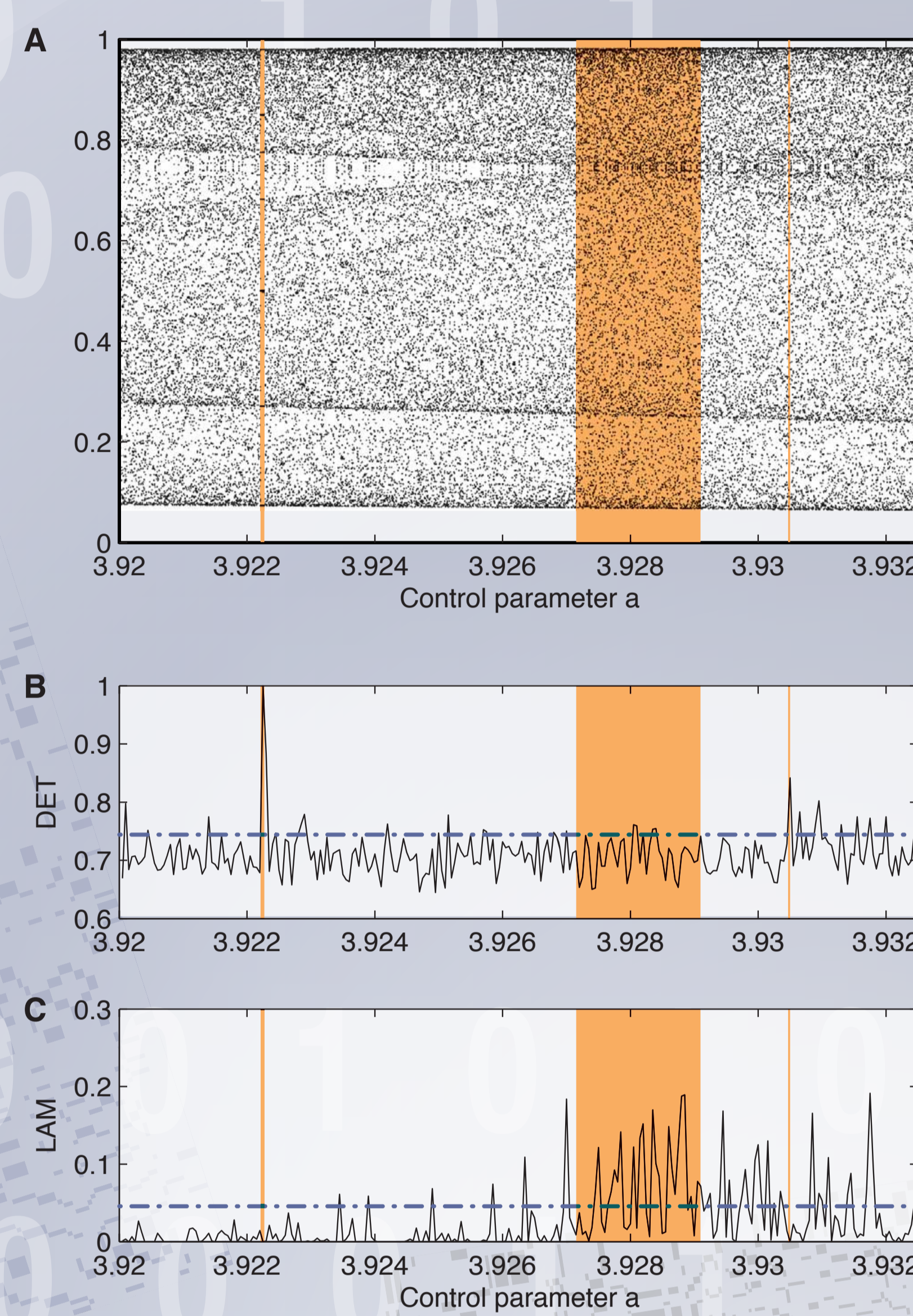
$$Q(l) = \sum_i P_i(l)$$

(2) Now we draw n line structures from $Q(l)$ (n is the mean number of line in a window); we get the empirical distribution of line lengths $P^*(l)$.

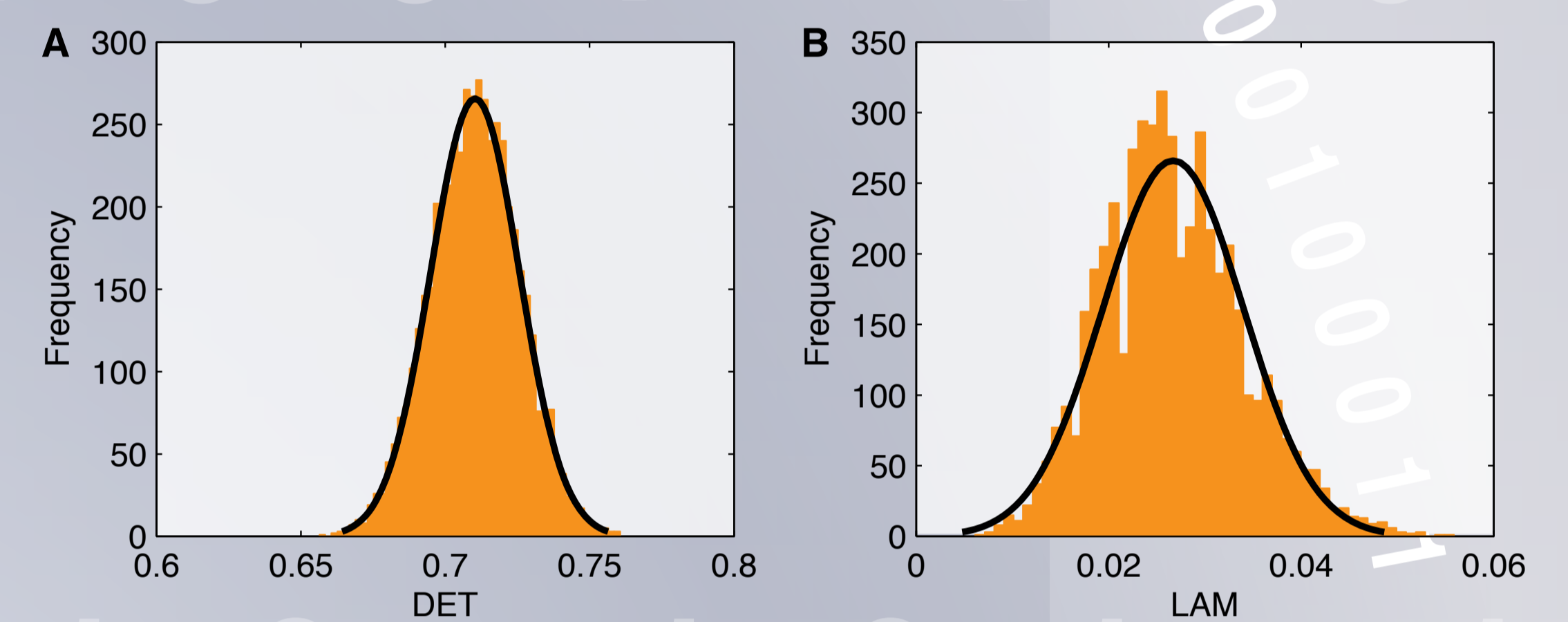
(3) We use $P^*(l)$ for calculating our RQA measure (DET and LAM, resp.).

(4) Repeating steps (2) and (3) we get empirical test distributions for DET and LAM, which we use for statistical test (α -quantiles). In the following examples use 5,000 realisations.

Example: Logistic Map with Transitions

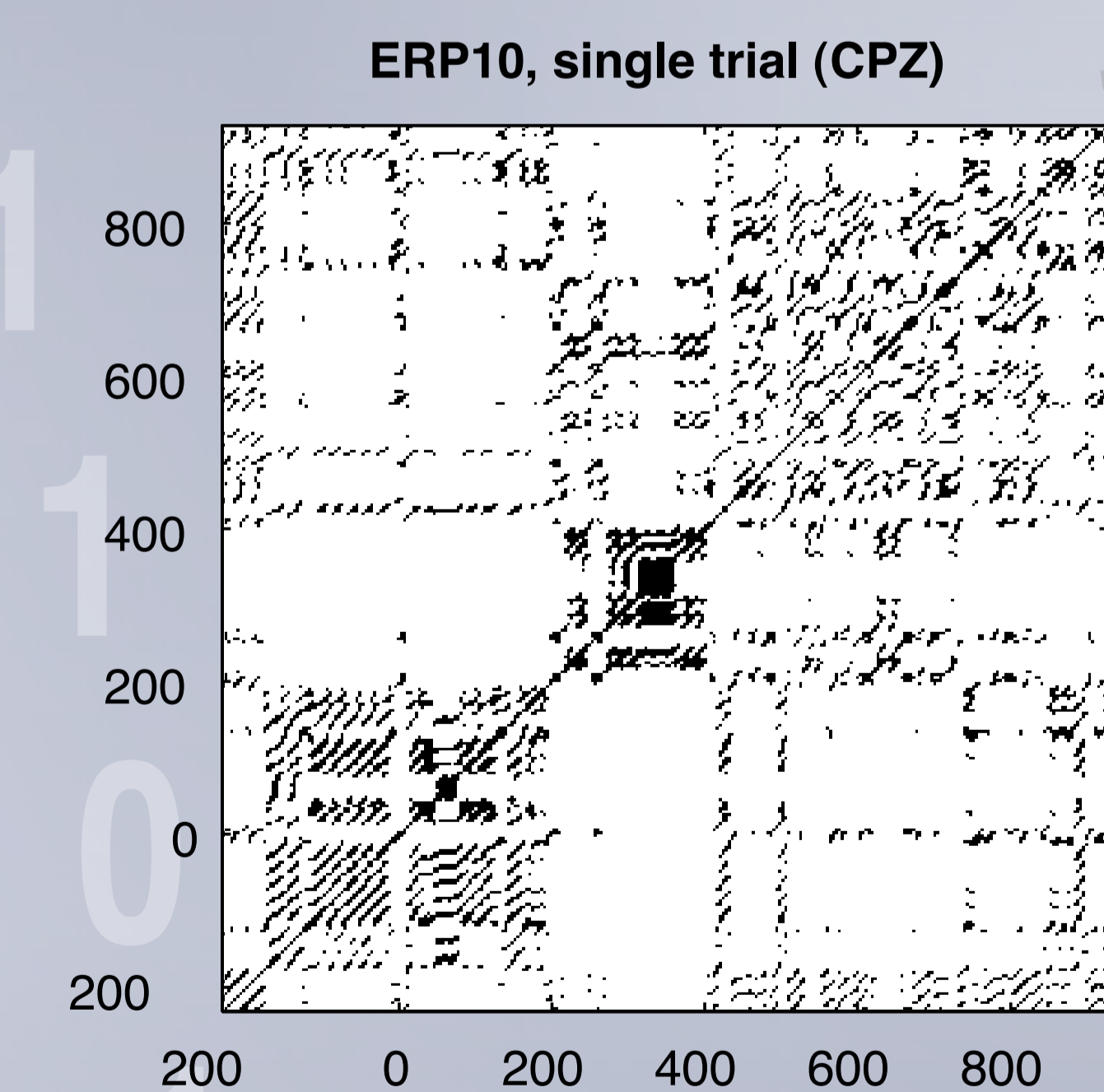
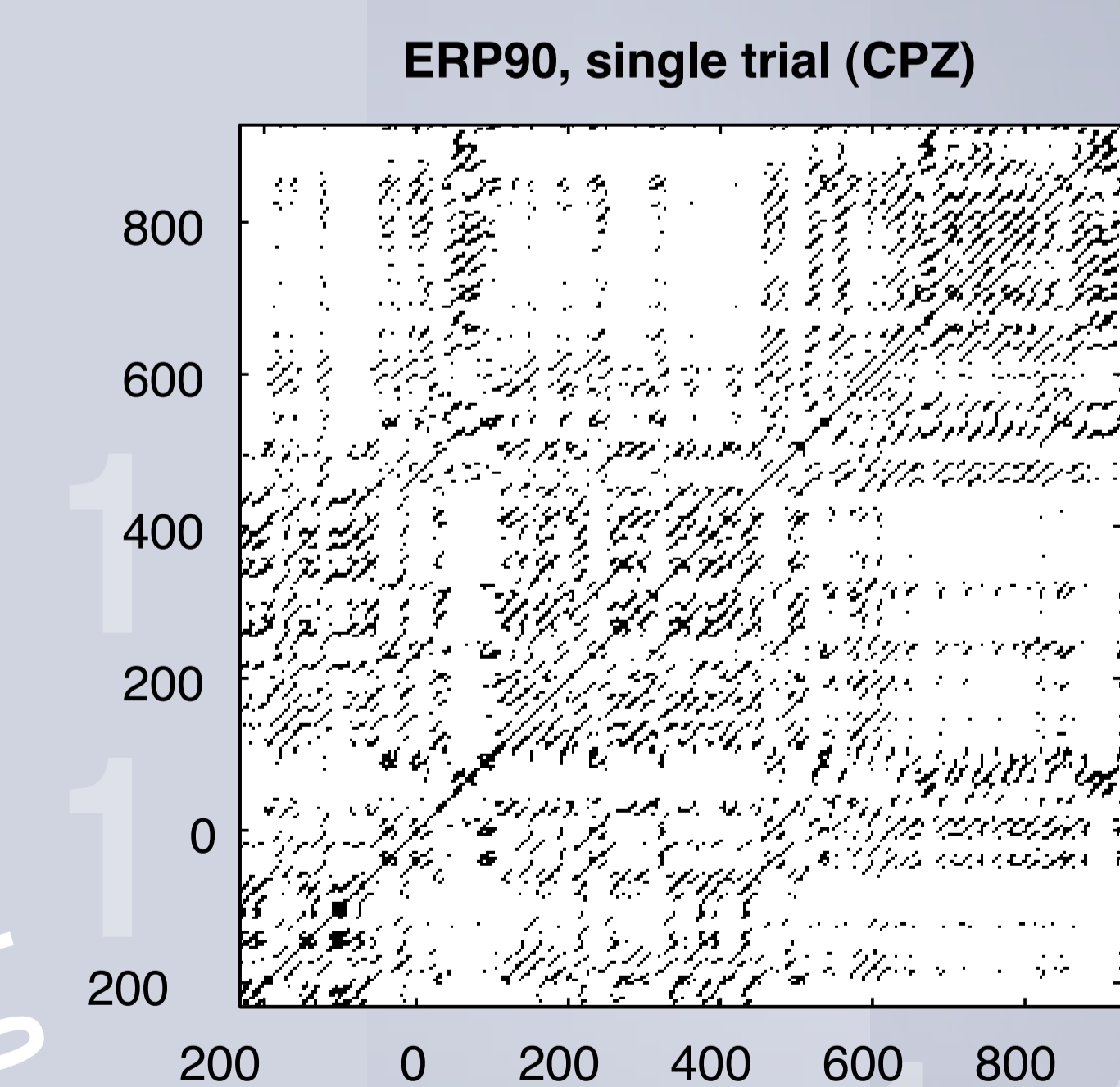
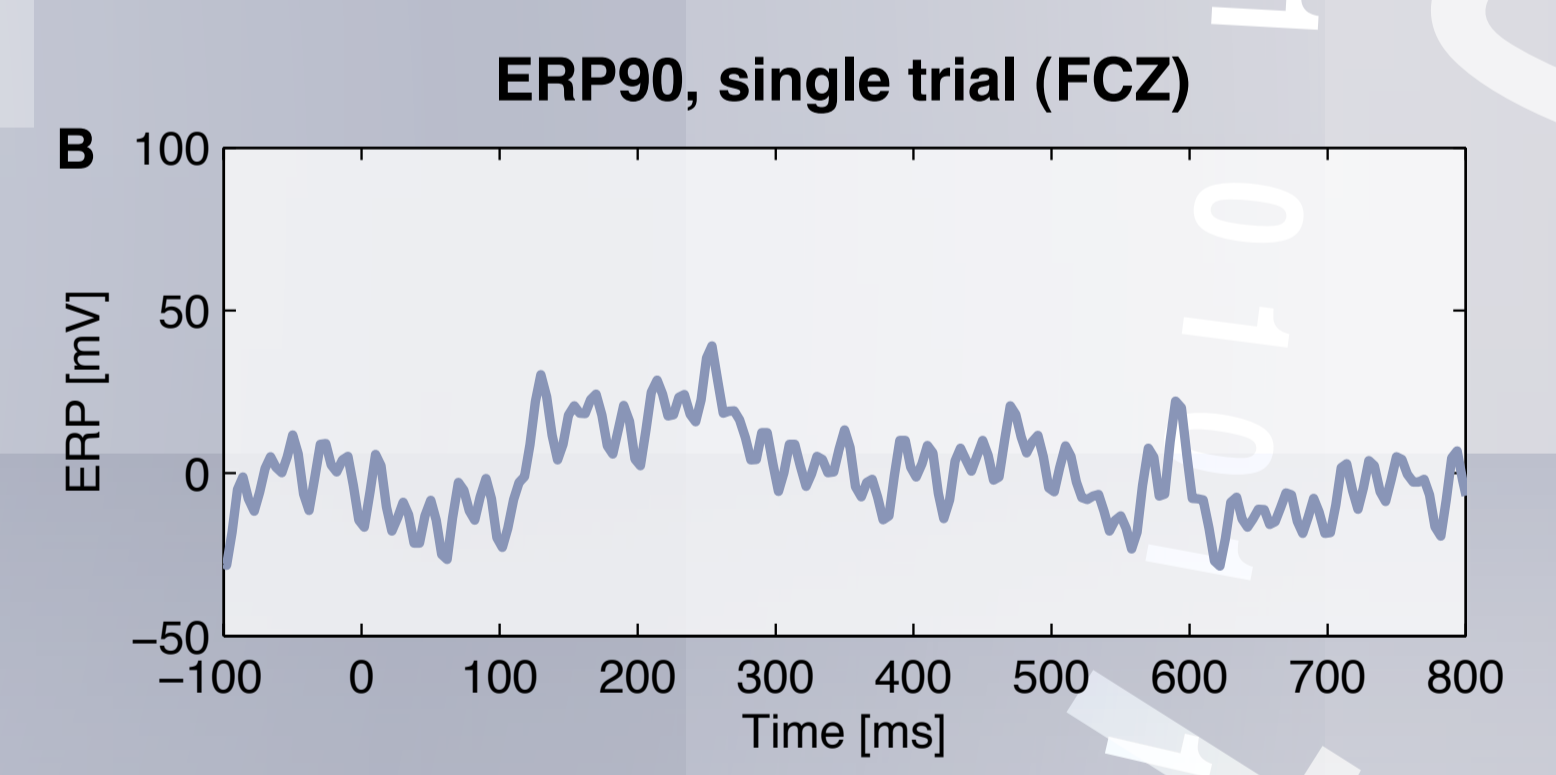
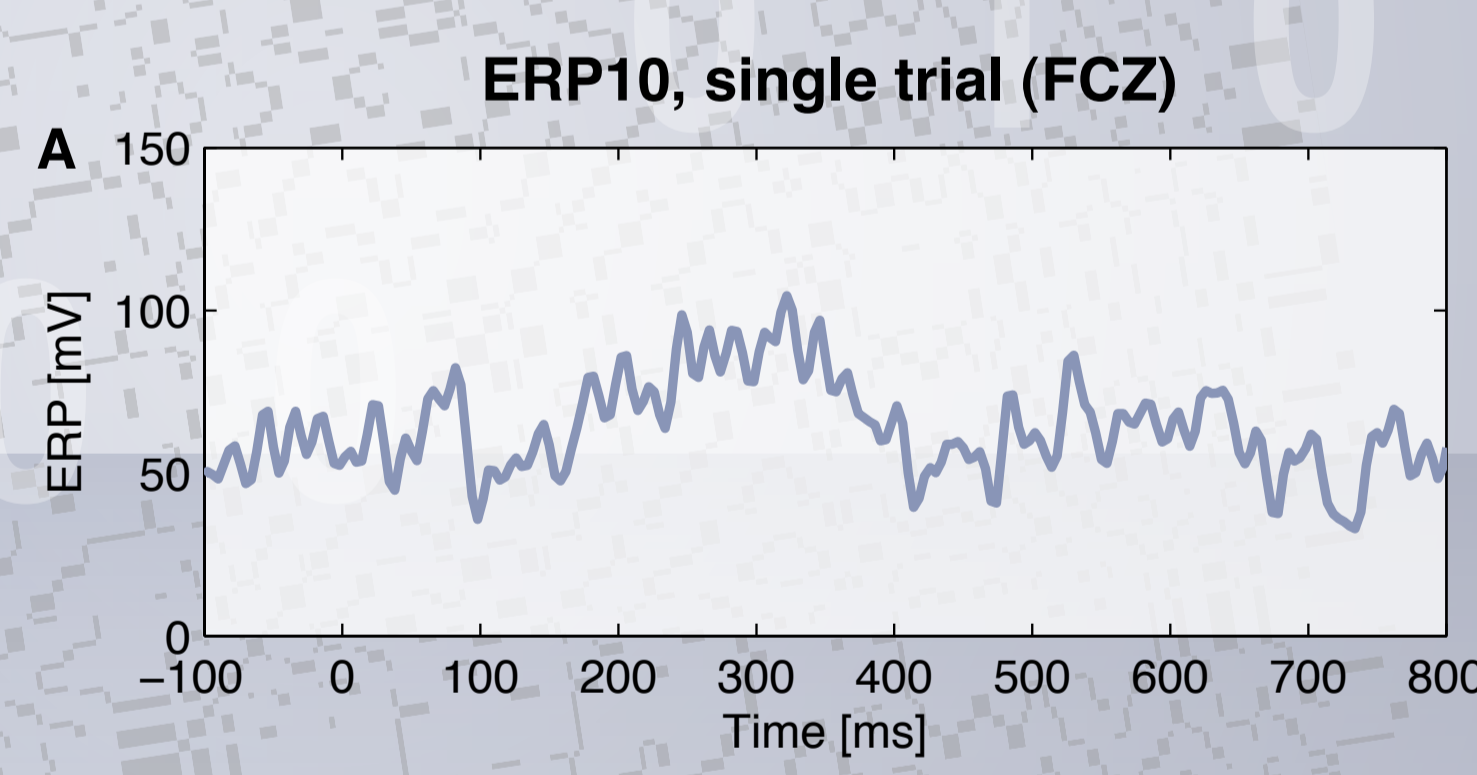
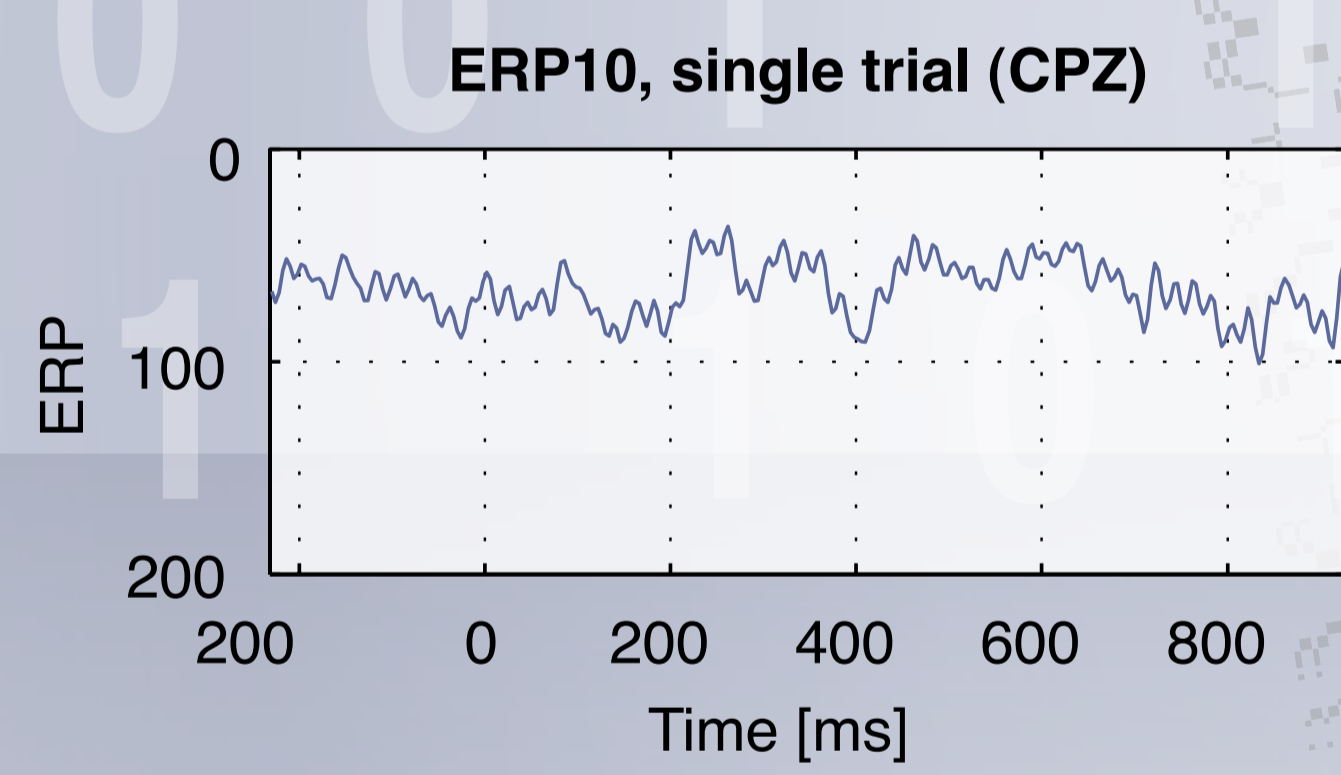
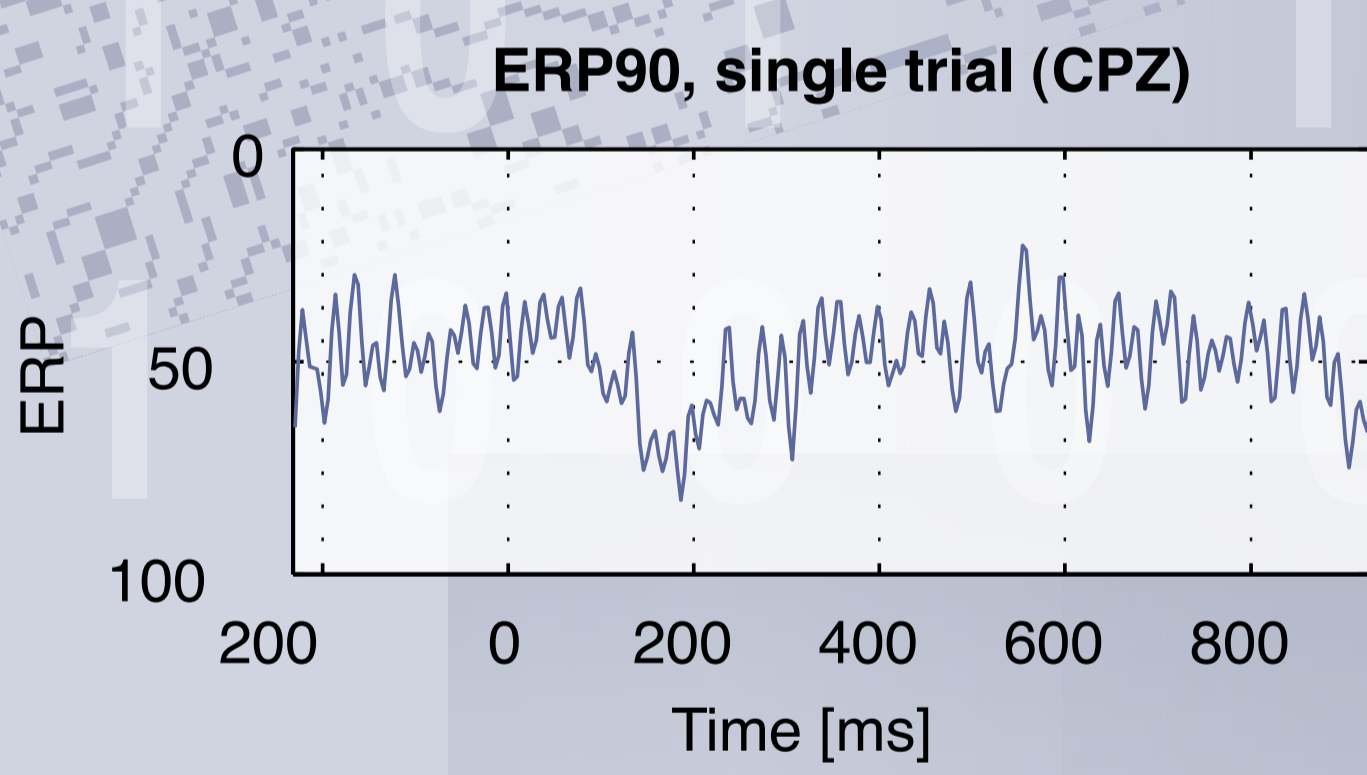


(A) Logistic map with chaos-period and chaos-chaos transitions for control parameter $a = [3.9200 \ 3.9325]$ and corresponding RQA measures (B) DET and (C) LAM. For $a = [3.92221 \ 3.92227]$ we have a period-7 window, for $a = [3.93047 \ 3.93050]$ a period-8 window and at a broad range around $a = 3.928$ intermittency (highlighted with orange bars). 99% confidence bounds are shown as blue dash-dotted lines. RQA settings: no embedding, fixed RR (5%), window = 200 and step = 200.



Empirical distributions for DET and LAM derived from bootstrapping recurrence structures. These distributions follow normal distributions (a fitted normal distribution shown by the black line).

Application: Event Related Potentials



Applying RQA on EEG measurements of an Oddball experiment, we find event related potentials (P300) even in single trials (Marwan and Meinke, 2004; Schinkel et al, 2007). Recurrence plots of EEG signals measured (A) without surprise and (B) with surprise. RQA settings: embedding dimension = 3, delay = 2, fixed RR (5%), window = 50 (200 ms), step = 10 (40 ms).

RQA measures for EEG signals (A, C, E) with surprise and (B, D, F) without surprise. DET and LAM reveal transitions in the signal measured at surprise around 300 ms, corresponding to the ERP P300. By application of the bootstrap test, we find a 99% confidence of these results. In contrast, for the signal without surprise, we cannot detect an ERP with 99% confidence.

References

N. Marwan, A. Groth, J. Kurths: Quantification of Order Patterns Recurrence Plots of Event Related Potentials, *Chaos and Complexity Letters*, 2(2/3), 301-314 (2007).
N. Marwan, M. C. Romano, M. Thiel, J. Kurths: Recurrence Plots for the Analysis of Complex Systems, *Physics Reports*, 438(5-6), 237-329 (2007).

N. Marwan, A. Meinke: Extended recurrence plot analysis and its application to ERP data, *International Journal of Bifurcation and Chaos "Cognition and Complex Brain Dynamics"*, 14(2), 761-771 (2004).
S. Schinkel, N. Marwan, J. Kurths: Order patterns recurrence plots in the analysis of ERP data, *Cognitive Neurodynamics*, 1(4), 317-325 (2007).

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