Independent Component Analysis of Sedimentary Rock Magnetic Data

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One step on the way for the understanding of the variations of the Earth' magnetic field is the study of the past variations. These past variations can be found in various geological archives. One possibility are lake sediments in which the magnetic minerals can store information about the direction and the intensity of the Earth' magnetic field.

However, when we measure the magnetic properties of these sediments, the measurements will also contain a large climatic impact. For example, the concentration of the magnetic minerals depends on the weathering processes which on the other side depend on the climate. Finally, we always measure a mixture of climate and real Earth' magnetic field signals when we measure rock magnetic parameters. Since we are interested only in the information about the Earth' magnetic field, we need a method which can separate these mixtures.

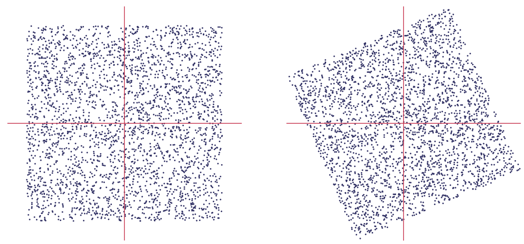
In the following we focus on the independent component analysis which could be successfully applied to rock magnetic data.

Some independent source signals s_i are linearly mixed $x = A\,s$ and only these mixtures x_i can be measured.

One well known method for the separation of the source signals is the principal component analysis which separates the components by using the criterion, which says that these components will be linearly uncorrelated. Another method, which is rather unknown, is the independent component analysis. The difference is that this method will separate components which are not only linearly uncorrelated, but also nonlinearly uncorrelated – with other words, which are independent.

Independent variables are always uncorrelated, but linearly uncorrelated variables are not independent in general. This can be seen on the example of two uniformly distributed random variables (left). If we mix these variables and uncorrelate them by the PCA, we will get the joint distribution shown right, those components are obviously not independent. If we go to the maximal value of the one component, the other component will have some restricted values, which means that it depends on the first. Here the principle of the ICA is obvious: it has to rotate the distribution in that way, that the components become independent.

The ICA uses different approaches, e.g. nonlinear decorrelation by using the mutual information.



Illustrative example

We consider three indepenent oscillations and make a mixture from them. Only the first two components are real mixtures.

$$s_1(t) = \sin\left(\frac{2\pi}{800}t\right) \quad \text{(IID transf.)}$$
$$s_2(t) = \left|\cos\left(\frac{2\pi}{424}t\right)\right|$$
$$s_3(t) = \sin\left(\frac{2\pi}{233}t\right)$$

The mixtures x base on the mixing matrix

$$A = \begin{bmatrix} 0.1 & 0.8 & 0.0 \\ 0.5 & 0.4 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

those ratios between the first two coefficients are

$$\left|\frac{a_{11}}{a_{12}}\right| = 0.125, \quad \left|\frac{a_{21}}{a_{22}}\right| = 1.25$$

The PCA decomposes the observations in three linearly uncorrelated signals, $s = Vx$, e.g. by eigenvalue decomposition of the covariance matrix:

$$C = EDE^T, \quad V = ED^{-1/2}E^T$$

with

$$A = V^{-1} = \begin{bmatrix} 0.38 & 0.93 & 0.00 \\ -0.39 & 0.92 & -0.04 \\ 0.01 & -0.04 & -1.00 \end{bmatrix}$$

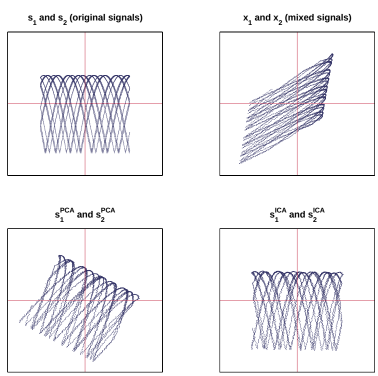
$$\left|\frac{a_{11}}{a_{12}}\right| = 0.41, \quad \left|\frac{a_{21}}{a_{22}}\right| = 0.41$$

The ICA decomposes the observations in three independent signals, $s = Wx$, with

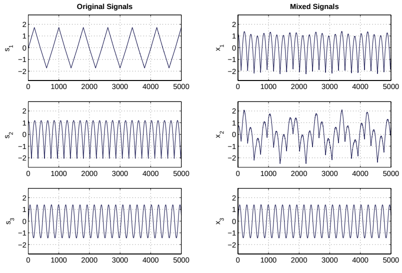
$$A = W^{-1} = \begin{bmatrix} -0.11 & 0.99 & -0.01 \\ -0.78 & 0.63 & 0.01 \\ 0.00 & -0.01 & 1.00 \end{bmatrix}$$

$$\left|\frac{a_{11}}{a_{12}}\right| = 0.11, \quad \left|\frac{a_{21}}{a_{22}}\right| = 1.24$$

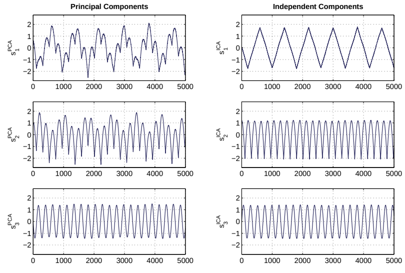
The comparison of the obtained coefficient ratios and mixing matrices of the PCA and the ICA with the original ratios and mixing matrix reveals that the PCA could not successfully find them, whereas the ICA is able to find them rather well (except the absolut values and sign).



The joint distributions of the source, mixing and the separated signals. The mixing distorts and rotates the signals, the PCA rectifies the mixtures, but keeps some rotation, and the ICA rectifies and rotates back the mixtures in that way, that the joint distribution matches rather well with the original joint distribution.



At left the source and at right the mixing signals are shown.



The separated signals gained by using PCA and ICA. The (first two) PCA components do not match with the original signals, whereas the ICA components match rather well (except absolut values/ sign).

The application of the ICA to rock magnetic measurements in order to find a signal of past Earth' magnetic field variations is shown on poster P1203 *The Earth' magnetic field in the last 100 kyr*. Please have a look at it!

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