Cross Recurrence Plot Based Adjustment of Geological Time Series

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Introduction

Thank you Mr. Chairman. Good morning ladies and gentlemen.

- slide: structure

In my presentation, I will speak about a feature of cross recurrence plots and its usage in adjustment of geological data.

I think, this is a very interesting feature which wasn‘t sufficiently respected.

First, I‘ll speak about cross recurrence plots, their construction and properties.

After that, I‘ll show you a possibility of its application in geological data analysis.

As an example, I will present the rescaling of a geological data series to a geological reference time series, which could be borehole data, for instance.

Cross Recurrence Plots

Cross Recurrence Plots

What are cross recurrence plots?

- **slide: Bild und formal RP**

Cross recurrence plots are an extension of recurrence plots.

A recurrence plot visualizes similar states in a system.

This system could be presented by a single measured time series.

Then, we have to reconstruct the phase space trajectory from the time series, for instance by using the time delay method.

The axes of a recurrence plot show the time.

A black point in the recurrence plot means, that the state at a given time is close to a state at another time.

Close means, the Euclidean distance between the trajectories falls below a threshold ε.

One can use another definiton for the closeness.

For instance, one could use a varying threshold or ball in such a way that it contains a fixed number of nearest states.

If the progress of the two states is similar for longer time, this will cause longer diagonal lines in the recurrence plot.

The comparison of the states at the same time means the comparison with itself.

That‘s why, a continuous diagonal line occures in the recurrence plot.

Although, the recurrence plot is mainly a visualization tool, a quantitative analysis of the plot is possible and is already introduced.

But, I will not go into details with recurrence plots.

- **slide: CRP**

We can extend this recurrence analysis of a single phase space trajectory to an analysis of two different phase space trajectories.

This directs us to the cross recurrence plot.

Now, a black point represents a similar state in system one and system two.

Longer diagonal oriented lines reveal epochs of a similar dynamics of the two compared systems.

If the two systems are very similar, the main diagonal line will occur.

What happens, when we introduce some phase delay or differences in the frequencies between the both similar systems?

- **slide: CRP mit Phasendifferenz**

Here, I'll show you two harmonic functions with different frequencies and a phase difference.

The diagonal line is now shifted in vertical direction and has a smaller slope.

The shape of this line reveals the functional relationship between the frequencies of the both systems.

With some simplifications, this relationship is easy to imagine.

- **slide: Formeln 1**

We have two systems, f and g, with different time scales t_1 and t_2 and we compute their cross recurrence plot.

The main diagonal line corresponds with this identity.

This main line will reveal the functional relationship between the both time-scales.

- **slide: Formeln 2 auf den *zweiten* Projektor**

We can consider Fourier series for the functions f and g, which enables us to look up for their frequencies.

If we assume the special case, that the systems have only one frequency, we are able to write an analytical expression for the relation between the time scales and the frequencies and phases.

Finally, the slope of the main line in the cross recurrence plot is the ratio of the frequencies, and a shifting of the diagonal line in vertical direction is caused by a phase difference.

However, I have to note that this will work only if both systems show a similar dynamics.

Example

- slide: sinus & sinus

Here is a further simple example of two sine functions, where in the first sine function the frequency increases with the time squared.

The colours represent the Euclidean distance between the both trajectories.

The main line in the cross recurrence plot is obviously a parabola and reveals the quadratic relation between the frequencies.

Geological Application

Geological Application

Now I would like to illustrate this method with an example from geology.

- **slide: See mit Bohrungen**

Often a large set of data series is gained at various locations and has to be synchronized.

For instance, we have two boreholes in a lake and would like to analyse some parameters, which were determined in these boreholes or from their cores.

Although we consider the same parameters we will get different time series of different length and scale, because the geological conditions are not the same at all locations.

In a lake we would expect various sedimentation rates.

Therefore, the first task would be to synchronize the profiles of the two boreholes.

- **slide: Daten auf den *zweiten* Projektor**

In this example I present you rock magnetic data which were friendly provided by the GeoForschungsZentrum Potsdam.

The synchronization will be done by comparing the parameters of the both profiles.

We can do this visually by comparing each maximum and minimum in the data series by hand.

Instead of this arduous matching, we can apply cross recurrence plots, and we have to fit the diagonal line, which would estimate the transfer-function.

If there is only one parameter, we can reconstruct the phase space by using the time delay embedding method.

Otherwise, we may use each of the parameters for the various dimensions of the embedding – like in this example.

- **slide: CRP von den Daten**

This is the cross recurrence plot of the data, which we have computed from five parameters.

You can see the main line.

Now, we have to find a nonparametric function which would fit to this main line.

This function is the rescaling function between the both scales.

An automatical fitting of the main line isn‘t so easy as it appears.

We are still working on some improvements of the routine, which is finding the rescaling function.

However, we have got a reasonable result, which synchronizes the scale of the one profile, to the scale of the other one.

- **slide: Rescaled Data**

Here, is an example of the original and the rescaled ARM data.

In blue is the first data series and in grey the second, which is to be rescaled to the first data.

Applying the rescaling function we get the red curve.

You can see a good concordance between both curves.

We have estimated a correlation of 0.75 between the reference data and the rescaled data.

Conclusions

Conclusions

Well, let‘s come to the end and summarize the most important statements.

- **slide: Conclusions**

A cross recurrence plot enables us to get an insight into the phase and frequency relation of two systems.

We can use this feature to estimate a rescaling function between the scales of two considered systems.

Perhaps, this feature could also be of interest for the investigation of the phase synchronisation.

I‘ve presented you the application of the cross recurrence plot on geophysical borehole data.

As a result we could synchronize two geophysical profiles.

Furthermore, with this method it is possible to determine the age-depth-function of geological profiles.

Another advantage is the possibility of an automatic adjustment of a large number of data sets.

And, in contrast to a visual and manual adjustment, this method is objectiv.

- **break**

Thank you for your attention.

Cross Recurrence Plot Based Adjustment of Geological Time Series

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The application of Cross Recurrence Plots on geophysical time series allows their easy and automatic adjustment/ ”synchronization“.

The Cross Recurrence Plot (CRP)

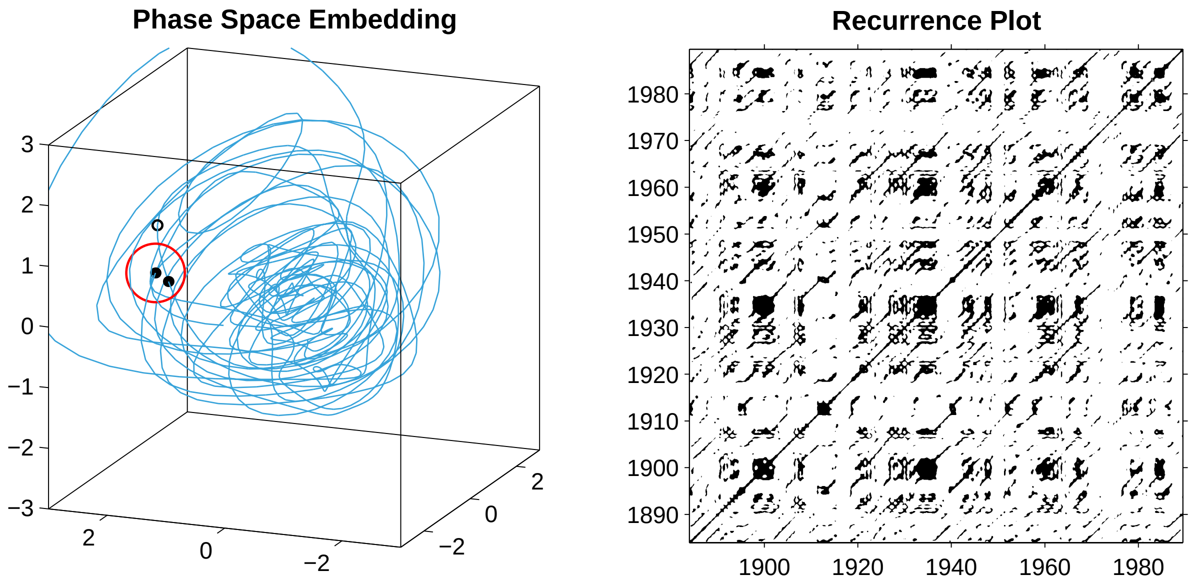
”Synchronization of time series“

Application on adjustment of geophysical profiles

The Recurrence Plot

Recurrence of states in a system x:

$$RP(i,j) = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|)$$



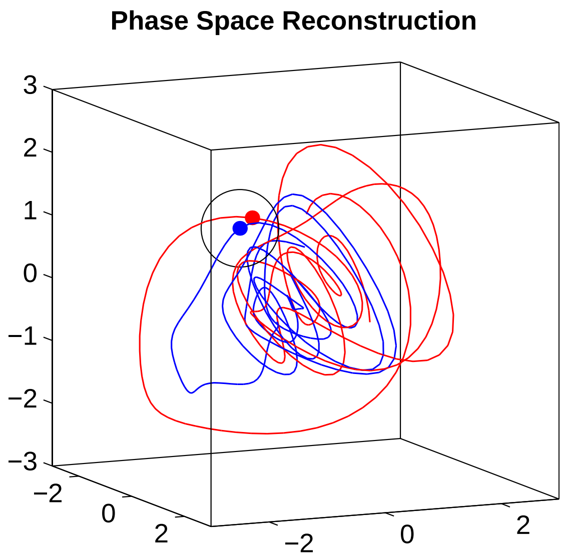
Note:

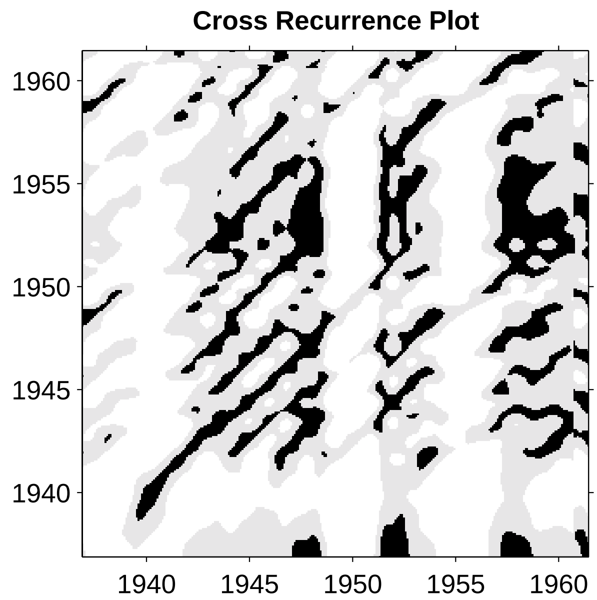
The recurrence of a state can be defined differently: e. g. one may consider the falling of the Euclidean distance below a threshold (above) or a fixed amount of the nearest neighbours.

The Cross Recurrence Plot

Occurrence of “equal” states in two systems x and y:

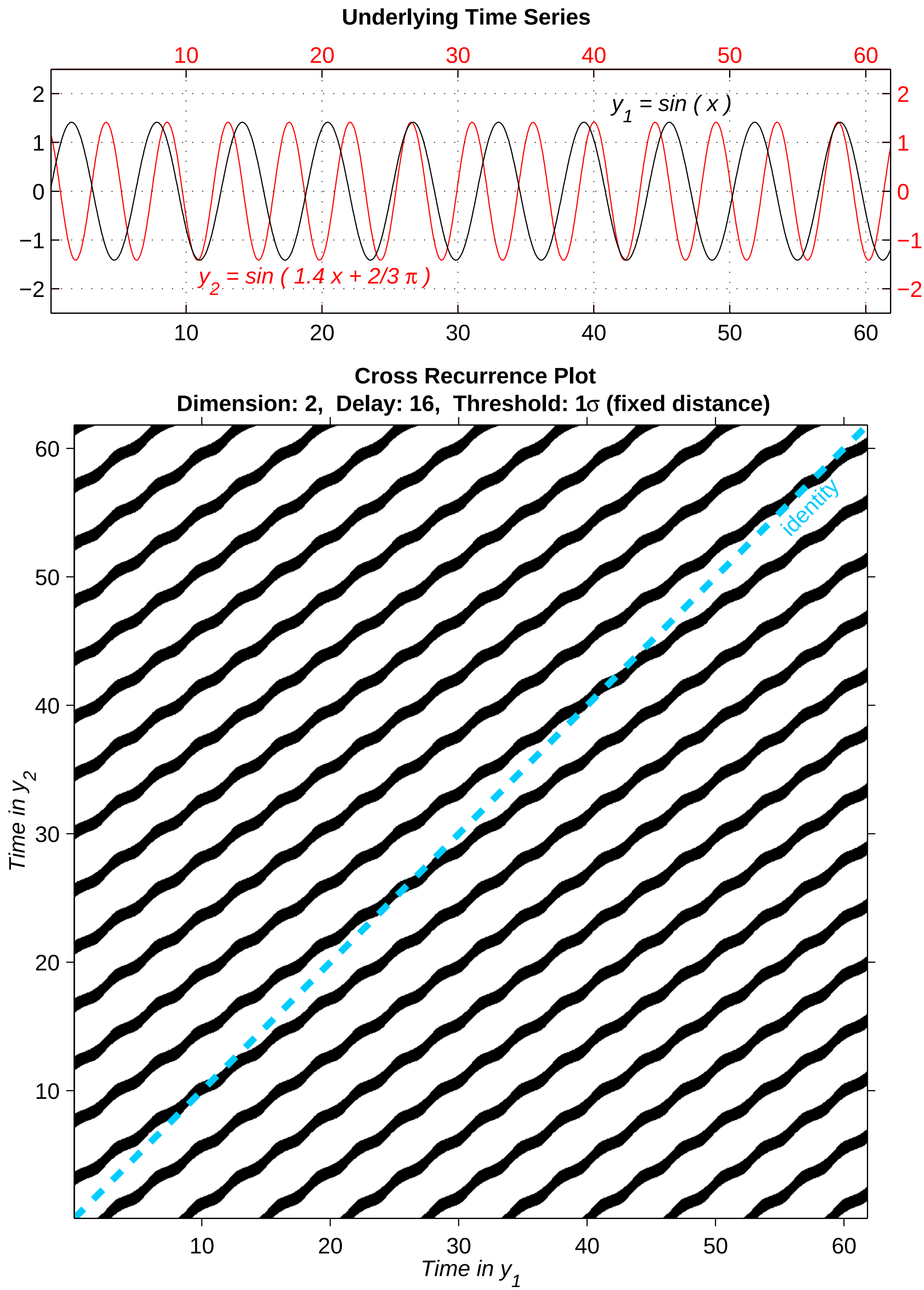
$$CRP(i,j) = \Theta(\varepsilon - \|\vec{x}_i - \vec{y}_j\|)$$





Note:

Diagonal lines correspond to a temporary parallel progress of the states in both systems.


Underlying Time Series
y1 = sin (x)
y2 = sin (1.4 x + 2/3 π)
Cross Recurrence Plot
Dimension: 2, Delay: 16, Threshold: 1σ (fixed distance)
identity
Time in y2
Time in y1

The Main "Diagonal" Line in the Cross Recurrence Plot (CRP)

Consider two measurements taken at different sampling regimes

$$f(t_1) \quad \text{and} \quad g(t_2),$$

there is the CRP (in the one-dimensional case)

$$CRP(t_1, t_2) = \Theta\Big(\varepsilon - \|f(t_1) - g(t_2)\|\Big).$$

Thus, the main "diagonal" line corresponds with the identity

$$f(t_1) \stackrel{!}{=} g(t_2)$$

and reveals the functional relationship between t_1 and t_2

$$t_2 = \varphi(t_1).$$

With Fourier series, the relation between the main line and the frequencies/ phases is obvious:

$$f(t_1) = \sum_i a_i \sin(\phi_i t_1 + \alpha_i)\,, \quad g(t_2) = \sum_j b_j \sin(\psi_j t_2 + \beta_j)$$

where ϕ and ψ may be time dependent frequencies and α and β are the phases. Assuming the special case, the systems have only one frequency ($a_i \approx b_j$):

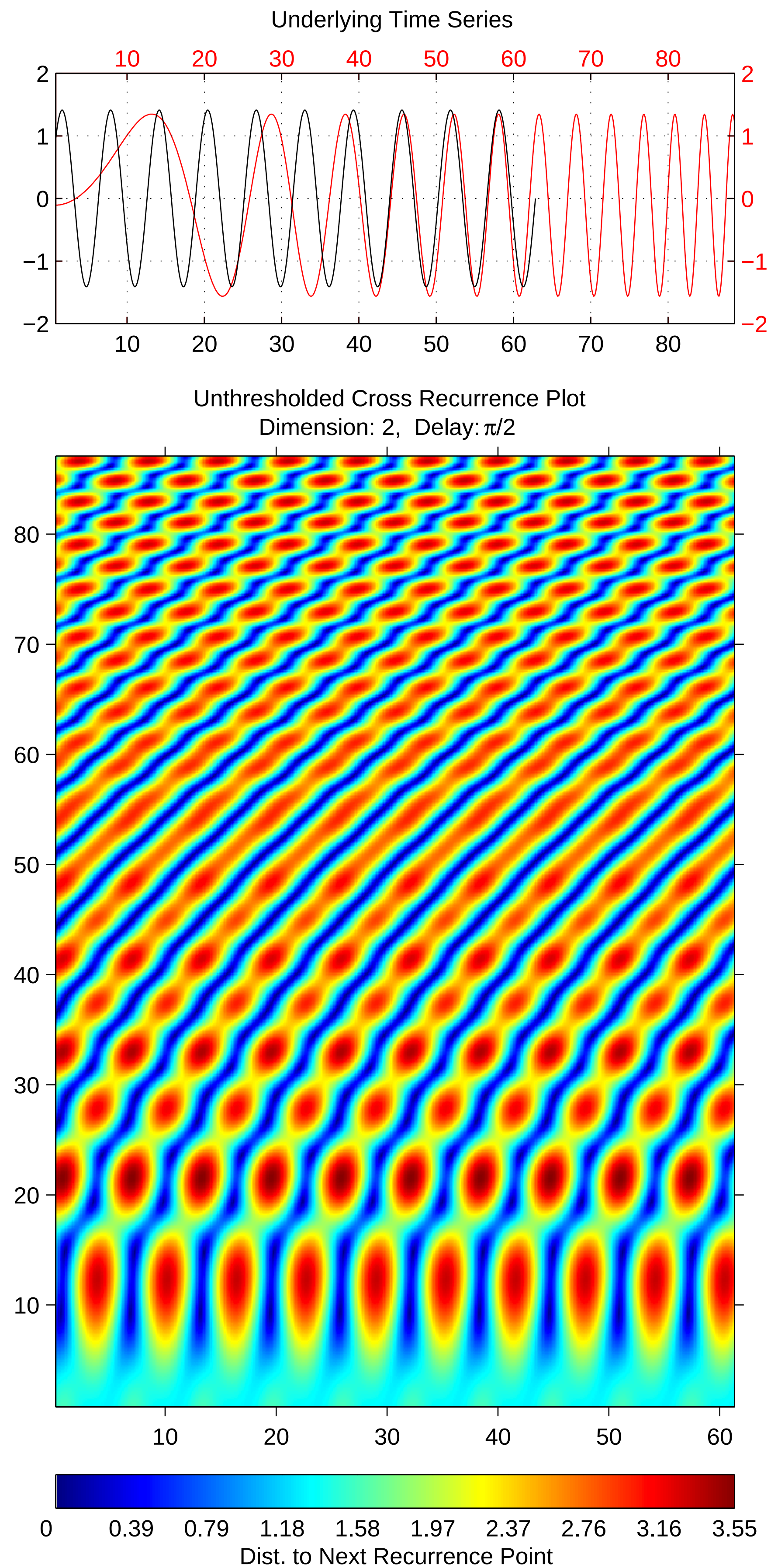
$$f(t_1) - g(t_2) \overset{!}{=} 0$$

$$\sin(\phi\, t_1 + \alpha) - \sin(\psi\, t_2 + \beta) \overset{!}{=} 0$$

$$\Rightarrow \quad t_2 = \left(\frac{\phi}{\psi} t_1 + \gamma\right) = \varphi(t_1)$$

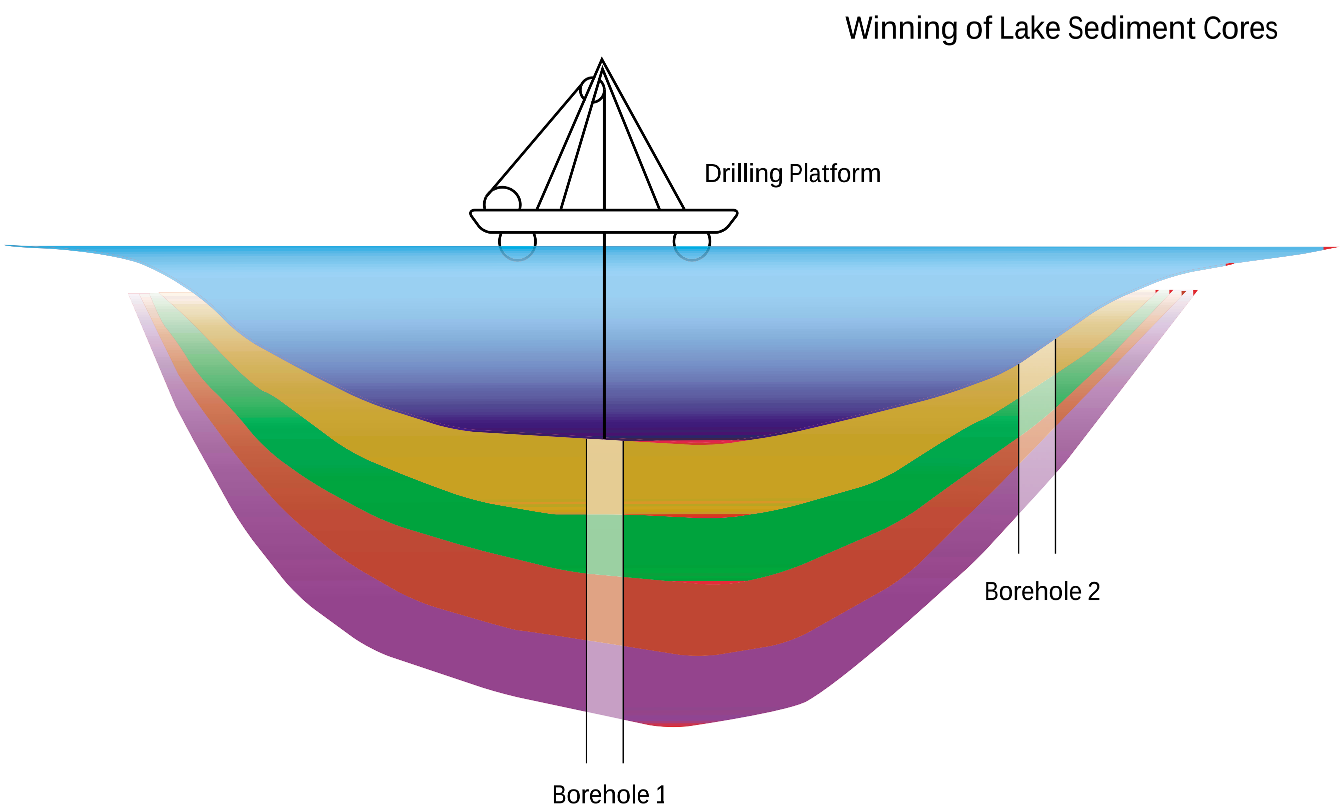
with $\gamma = \frac{\alpha - \beta}{\psi}$.

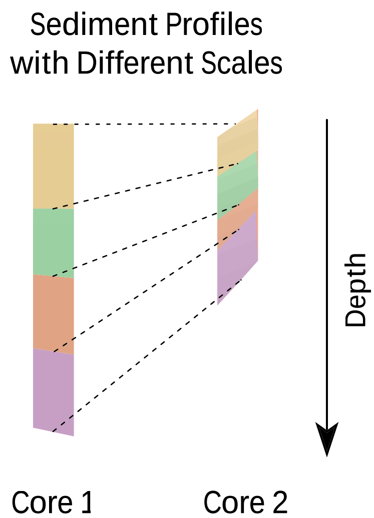
Consequently, the slope of the main line in a cross recurrence plot represents the frequency ratio. The distance of the main line from the identity diagonal line reveals the phase difference.

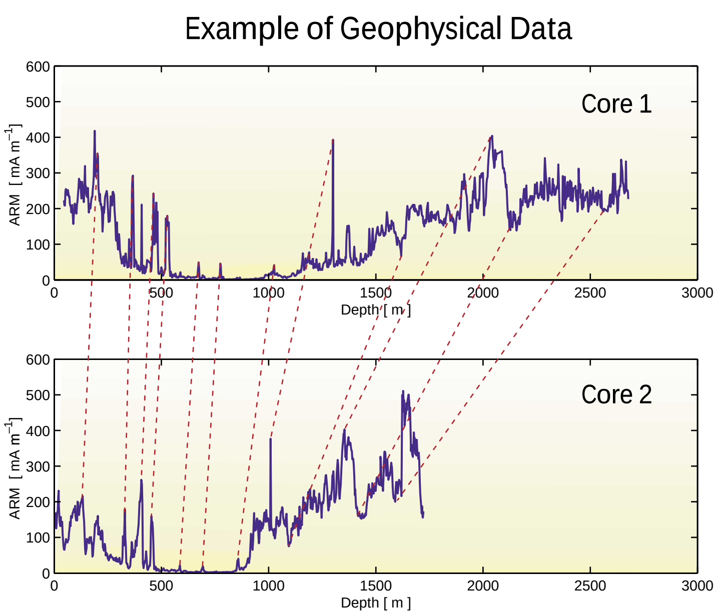


Underlying Time Series
Unthresholded Cross Recurrence Plot
Dimension: 2, Delay: π/2
Dist. to Next Recurrence Point
0
0.39
0.79
1.18
1.58
1.97
2.37
2.76
3.16
3.55

Different Samples of One Geological Content

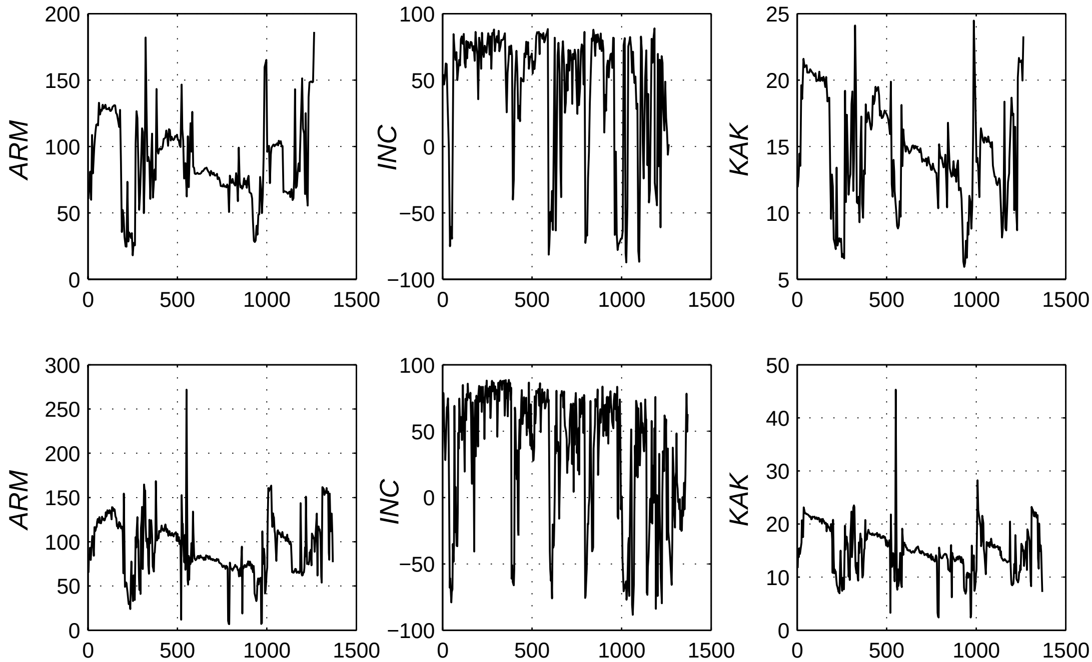






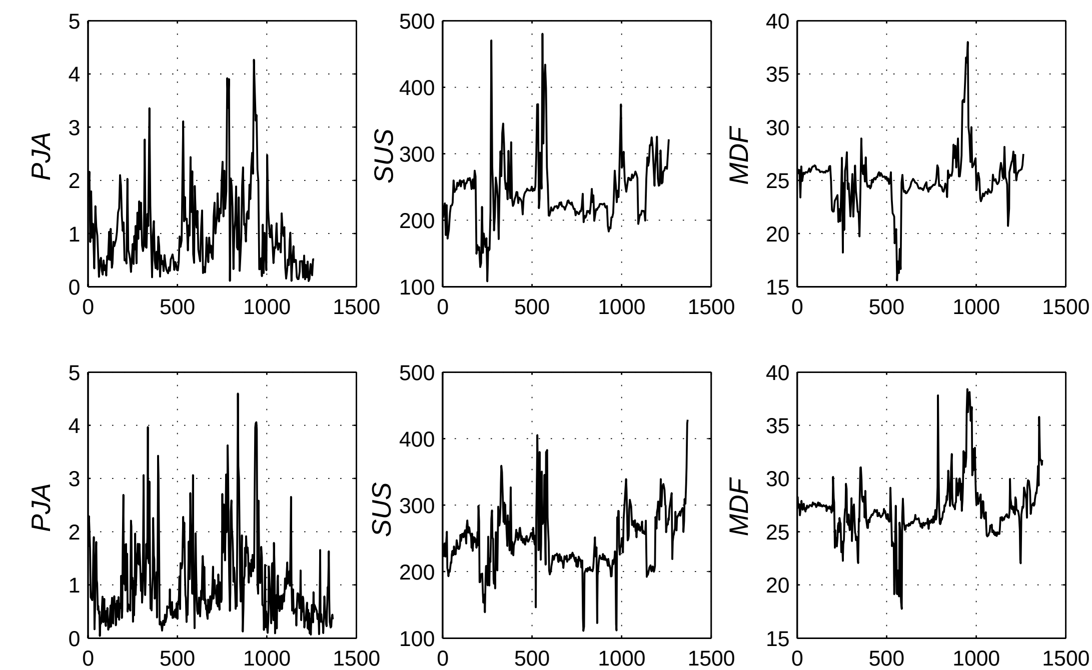
Profile 1

Profile 2

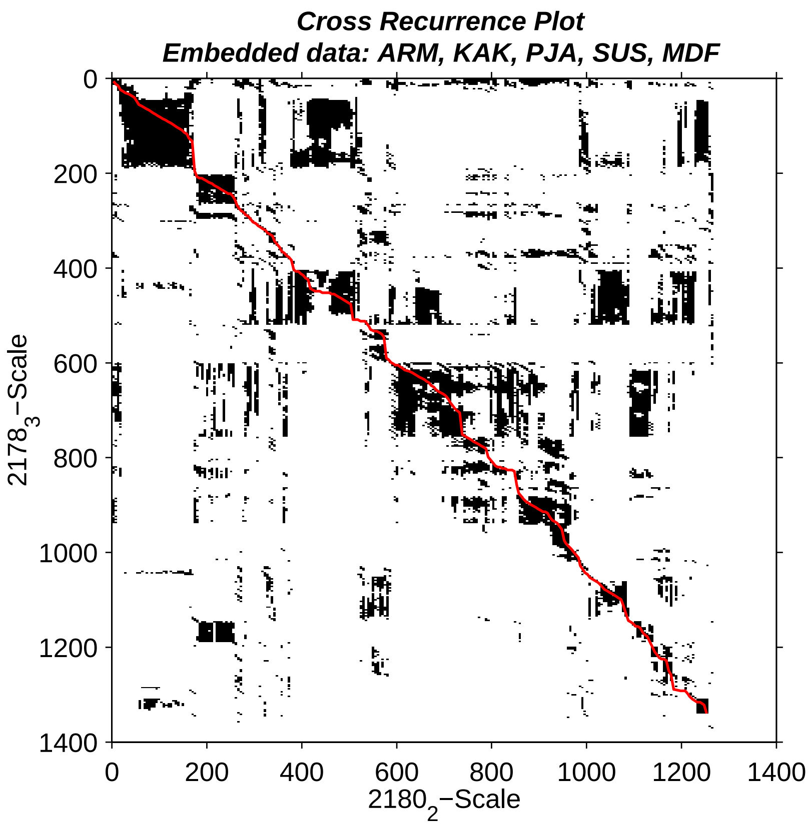


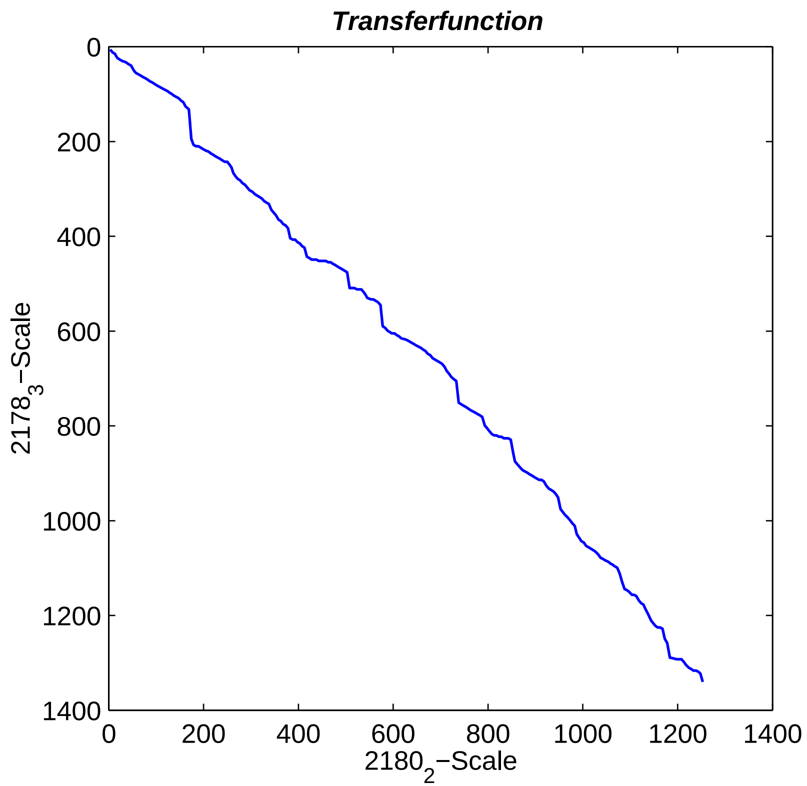
Profile 1

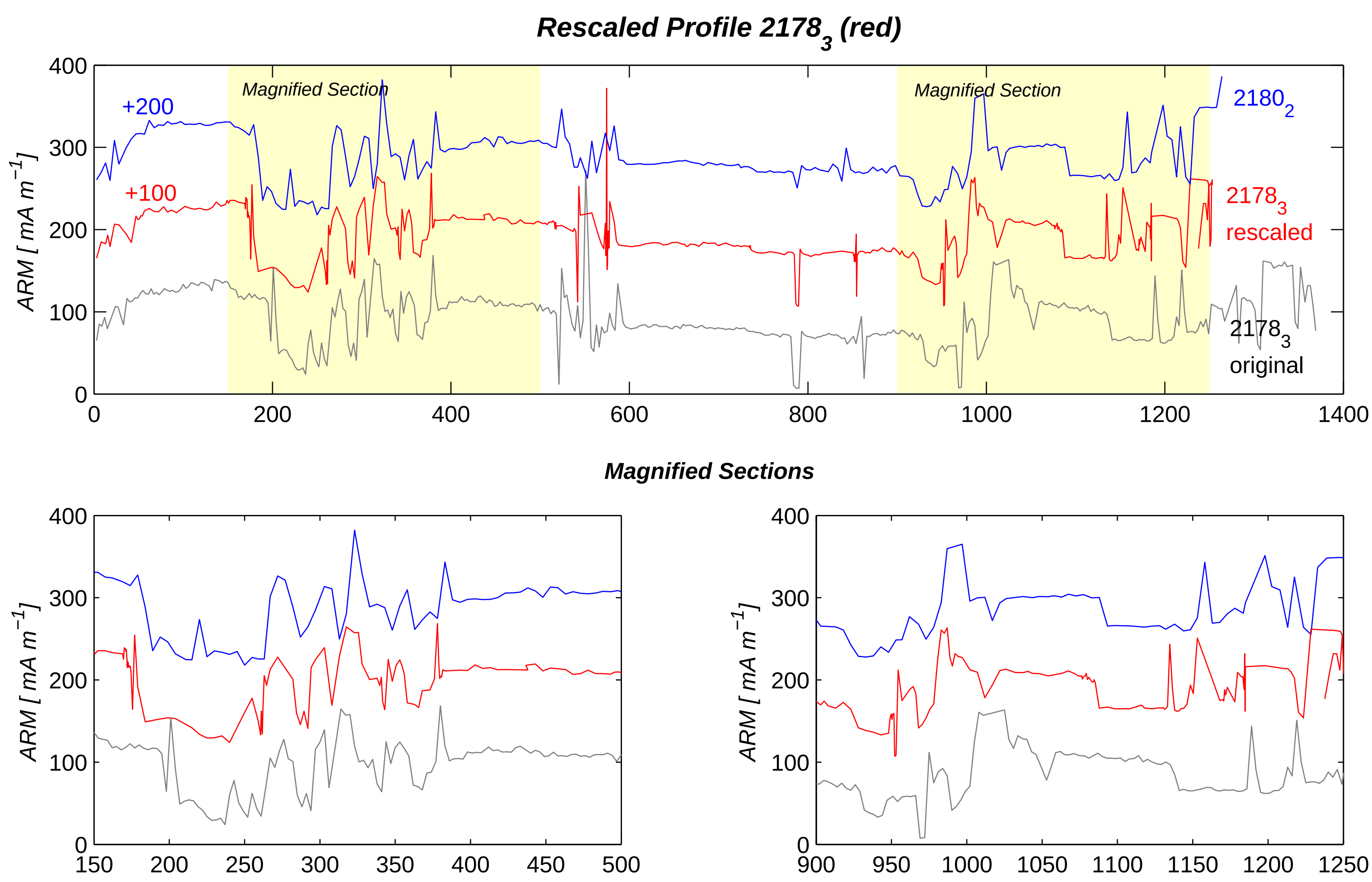
Profile 2



Nowaczyk, GFZ 2000


Cross Recurrence Plot
Embedded data: ARM, KAK, PJA, SUS, MDF
0
200
400
600
800
1000
1200
1400
2178_3−Scale
2180_2−Scale


Transferfunction
0
200
400
600
800
1000
1200
1400
2178_3−Scale
2180_2−Scale



Rescaled Profile 2178$_3$ (red)
Magnified Section
Magnified Section
+200
+100
2180$_2$
2178$_3$ rescaled
2178$_3$ original
ARM [mA m^{-1}]
Magnified Sections
ARM [mA m^{-1}]
ARM [mA m^{-1}]

Cross Recurrence Plot Based Adjustment of Geological Time Series

The application of Cross Recurrence Plots on geophysical time series allows their easy and automatic adjustment/ "synchronization".

Cross Recurrence Plots (CRPs) are related with the phase differences and frequency ratios of the systems

Estimation of the rescaling function

Application of CRPs on "synchronization" of geophysical profiles (age-depth-function)