# Multiplex Recurrence Networks


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Combine recurrence networks with multiplex network approach for multivariate time series analysis.

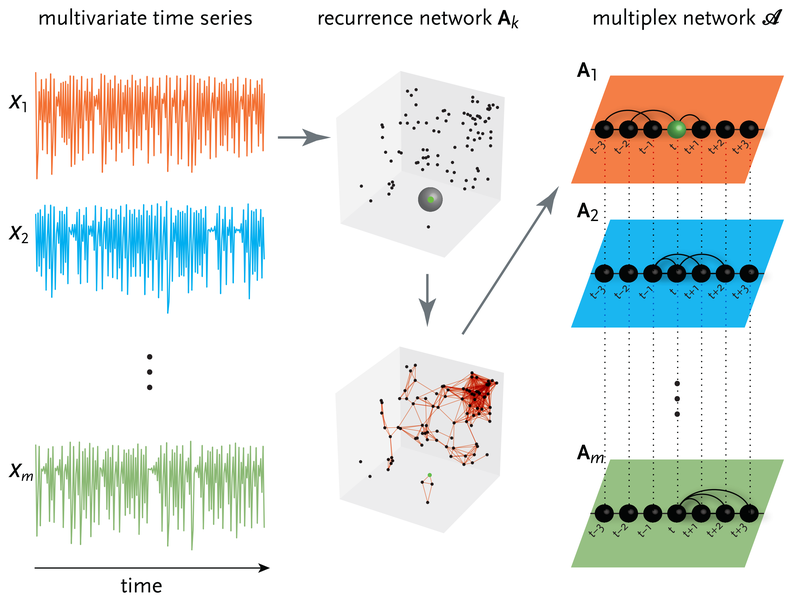
$$R_k(i,j) = \Theta\left(\epsilon - \|\vec{x}_k(i) - \vec{x}_k(j)\|\right)$$

$$\mathbf{A}_k = \mathbf{R}_k - \mathbf{I}_N$$

$$\mathscr{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{I}_N & \dots & \mathbf{I}_N \\ \mathbf{I}_N & \mathbf{A}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{I}_N \\ \mathbf{I}_N & \dots & \mathbf{I}_N & \mathbf{A}_m \end{bmatrix}$$

Similarity of recurrence network in distinct layers:

$$\omega = \frac{\sum_i \sum_{j>i} \sum_k A_k(i,j)}{m \sum_i \sum_{j>i} \left(1 - \delta_{0,\sum_k A_k(i,j)}\right)}$$

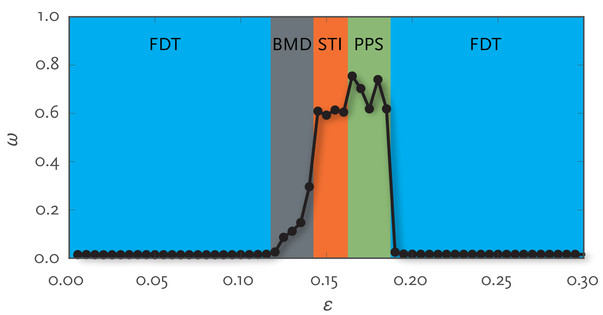



## Prototypical Example: Coupled Map Lattice (CML)

- multi-component dynamical system
- discrete-time model of diffusively coupled oscillators on a ring model of *m* sites
- well-studied dynamical system
- models a variety of nonlinearphenomena

$$x_k(t+1) = (1-\varepsilon) f\left(x_k(t)\right) + \\ + \frac{\varepsilon}{2}\left(f\left(x_{k-1}(t)\right) + f\left(x_{k+1}(t)\right)\right)$$

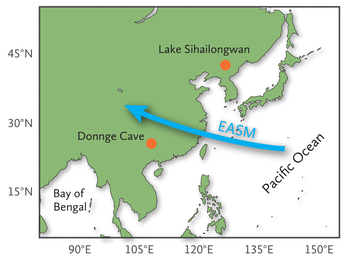
The similarity measure $\omega$ clearly distinguishes the different dynamics depending on $\varepsilon$.




Recurrence network similarity $\omega$ for different CML dynamics (controlled by $\varepsilon$): FDT – Fully Developed Turbulence; BMD – Brownian Motion of Defect; STI – Spatio-Temporal Intermittency; PPS – Periodic Pattern Selection. System size is $m = 200$.

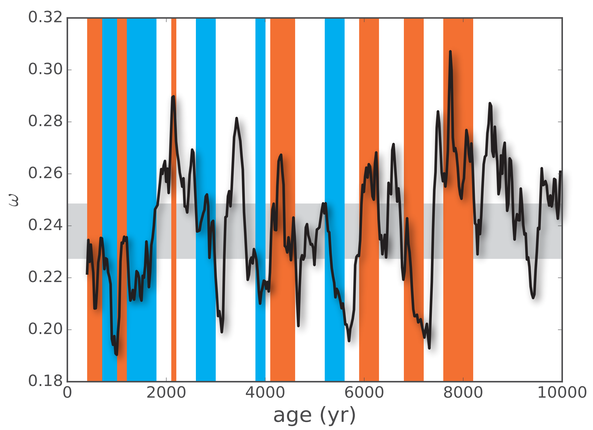
## Real World Application: Historical Vegetation Dynamics

Investigation of historical vegetation dynamics (multivariate pollen records) collected from Lake Sihailongwan.

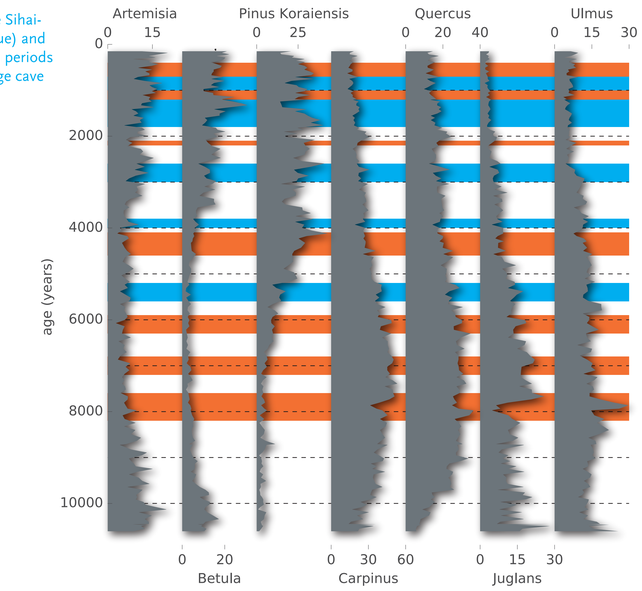


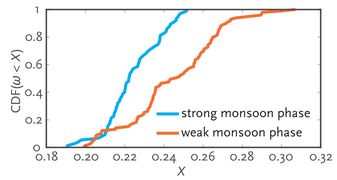

Geographical location of Lake Sihailongwan and Donnge Cave in the East Asian Summer Monsoon (EASM) realm.

Recurrence network similarity $\omega$ for the vegetation around the Lake Sihailongwan during the Holocene. Dynamics of the different vegetation classes is more similar during the weak monsoon phases.




Pollen records from Lake Sihailongwan with strong (blue) and weak (orange) monsoon periods (as indicated by a Donnge cave palaeoclimate archive).




Differences in the recurrence network similarity $\omega$ for strong and weak monsoon periods.

Similarity between the *dynamics* of different vegetation classes changes several times during the Holocene:

weak monsoon periods ▶ higher similarity
strong monsoon periods ▶ less similarity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**References**

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