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A zero-dimensional climate-vegetation model containing global carbon and hydrological cycle

Yuri M. Svirezhev *, Werner von Bloh [1]

Potsdam Institute for Climate Impact Research (PIK), Telegrafenberg, PO Box 601203, D-14412 Potsdam, Germany

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Abstract

A model of a hypothetical zero-dimensional planet containing a global carbon cycle, which describes the fundamental interaction between climate and biosphere, is used as a basis to formulate a new model incorporating a hydrological cycle. Using the conservation law for carbon and water in the system the number of differential equations is thereby reduced by two from five to three. The number and positions of the stable equilibria in the three-dimensional phase space is determined using numerical methods. This numbers is related to the value of the maximum productivity $P_{max.}$, the total amount of carbon A and water B in the system as bifurcation parameters of the system. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Minimal model; Biosphere; Carbon cycle; Hydrological cycle

1. Introduction

So called 'minimal' models (Moiseev and Svirezhev, 1979) are a powerful tool in the description and analysis of the fundamental processes and dependencies acting between geosphere and biosphere. First attempts in this type of vegetation–climate modelling were carried out by Vernadsky (1926) and then by Watson and Lovelock (1983) ('Daisyworld' as a model of some hypothetical planet). Unlike other attempts of modelling the global vegetation (e.g. the Osnabrück biosphere model (Esser, 1991) or the Frankfurt biosphere model (Lüdeke et al., 1995)) the system can be fully understood through analytical and numerical inspections.

In our first publication (Svirezhev and von Bloh, 1996) we introduced such a minimal model describing the climate-biosphere mechanisms of a hypothetical zero-dimensional planet. The model consists mainly of two coupled nonlinear differential equations, for the temperature T of the planet

* Corresponding author. Tel.: + 49 331 2781157; fax: + 49 331 2781204; e-mail: juri@pik-potsdam

[1] Fax: + 49 331 2882600

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and amount of vegetation N. The following publication (Svirezhev and von Bloh, 1997) adds to the proposed model a global carbon cycle increasing the number of equations to three with the amount of carbon in the atmosphere C as the new state variable.

Analytical calculations of the coupled climate-biosphere system indicate that for the T, N system up to two different stable equilibria are possible (the 'dead' planet without any vegetation and the 'living' planet with vegetation). For the system with a carbon cycle the number of stable equilibria is increased to three: the 'living' planet bifurcates into the 'cold' and 'hot' planet with vegetation. Furthermore it was shown that the number of stable states depends on the bifurcation parameters A, the total amount of carbon in the system, and $P_{max.}$, the maximum productivity of the biosphere.

According to the 'virtual biospheres' concept (Svirezhev, 1994), the contemporary Earth biosphere is one of many possible (virtual) biospheres, corresponding to the multiple equilibria of some nonlinear dynamic system 'climate + biosphere'. In the course of planetary history and proper evolution this system has passed through several bifurcation points, when random perturbations determined on which branch of the solution the system would appear. A moving force of this evolution could be the evolution of the 'Earth green cover', which has, in turn, several bifurcation points, for instance, the appearance of terrestrial vegetation and the change of coniferous forest into deciduous forest.

Adding an hydrological cycle on a conceptual level to the model is a further step towards to a full description of planetary evolution as a sequence of bifurcations. Therefore the climatic part of the model is modified to incorporate cloudiness and ice albedo using a two layer approach for the albedo, while the biotic part has to be extended by a water dependent productivity function. The extended model is then analysed numerically in respect to the number of stable equilibria and their sensitivity against external perturbations.

2. Model description and basic equations

The climate of our hypothetical planet is described by one variable, namely, the annual average temperature T of its surface. The planetary atmosphere is an isotropic one with a total amount of carbon in the atmosphere being equal to C and a water vapour content W_a in an atmosphere column over a surface.

The equation for the globally averaged temperature T described by a time dependent energy balance equation is (Petoukhov and Ganopolski, 1994):

$$k\frac{dT}{dt} = S(1-\alpha) - \sigma_{eff}T^4 - k_W W_a n, \quad (1)$$

where k is the surface heat capacity, S is the solar radiation, α is the surface albedo,

$$\sigma_{eff} = \sigma\varphi_C(C)\varphi_W(W_a),$$
$$\varphi_C(0) = \varphi_W(0) = 1, \quad (2)$$

where σ is the Stefan-Boltzmann constant. The 'greenhouse' effect is represented by the functions $\varphi_C(C)$ and $\varphi_W(W_a)$, which are monotonous decreasing functions with saturation levels φ_C^∞ and φ_W^∞ (see Fig. 1 and Fig. 2).

Good approximations for these are Michaelis-Menten hyperbolas. The value k_W is the hidden heat of evaporation. Finally, the cloudiness parameter $n \in [0,1]$ describing the relative cloud cover over the planetary surface can be presented in the form (Smagorinsky, 1960)

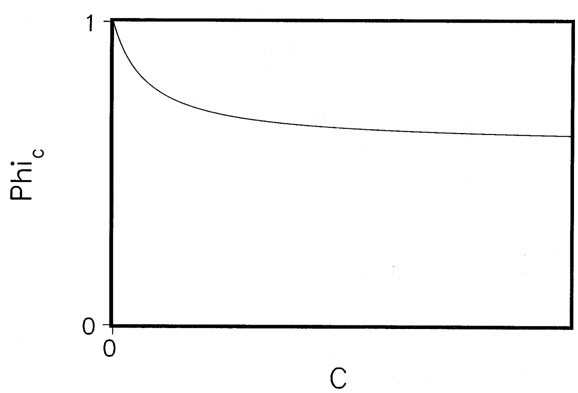

Fig. 1. Function, φ_C describing the greenhouse effect as a function of the amount of carbon in the atmosphere C.

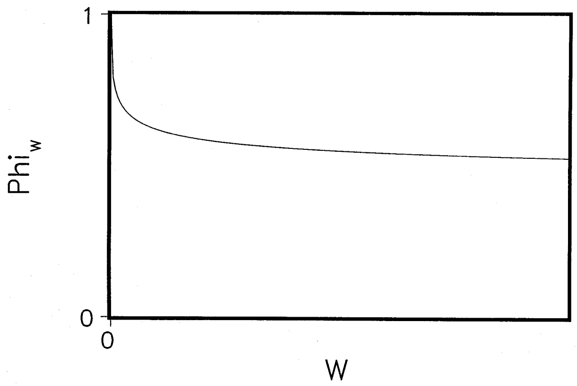

Fig. 2. Function φ_W describing the greenhouse effect as a function of the amount of water in the atmosphere W_a.

$$n = \begin{cases} 2q - 1, & \text{if} \quad 0.5 \le q \le 1; \\ 0, & \text{if} \quad 0 \le q \le 0.5 \end{cases} \tag{3}$$

where q is the relative humidity,

$$q = \frac{W_a}{W_a^*(T)}. \tag{4}$$

$W_a^*(T)$ is the concentration of saturated water vapour depending on temperature according to the Clausius-Clapeyron law (e.g. Fleagle and Businger, 1963) plotted in Fig. 3:

$$W_a^*(T) = e^{(17.638T)/(T + 243.4)} \tag{5}$$

2.1. Albedo

We consider a planet, covered by vegetation, with a 'two-layer' atmosphere. Clouds and the underlying surface (vegetation and ocean, if included in the consideration) reflect a solar radiation, and 'greenhouse' gases transform it. The albedo α in Eq. (1) is a function of

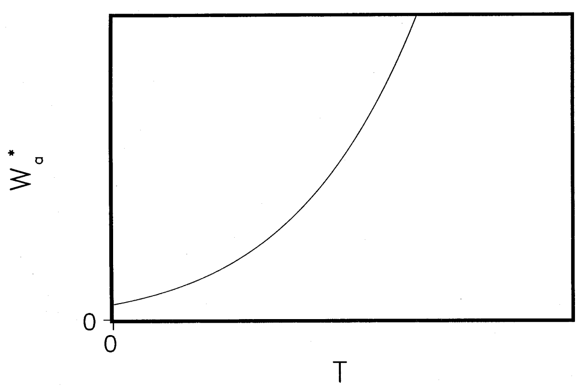

Fig. 3. Concentration of saturated water vapour W_a^* as a function of temperature T.

$$\alpha = \alpha(N, n, T, W_s), \tag{6}$$

where N is the carbon content in vegetation, and W_s is the soil moisture (water in the soil). The albedo depends on the cloudiness n in the following form

$$\alpha = n\alpha_c + (1 - n)\alpha_{nc}, \tag{7}$$

where α_c is the albedo of sky covered by clouds, and α_{nc} is the albedo of a clear sky. Then

$$\alpha_c = \alpha_n + (1 - \alpha_n)^2 \alpha_{surf}. \tag{8}$$

Here the square arises because of a double reflection of clouds and the underlying surface, $\alpha_n \approx 0.5$ is the clouds albedo, α_{surf} is the albedo of the underlying surface. Then

$$\alpha_c = 0.5 + 0.25\alpha_{surf}. \tag{9}$$

In the same way we have

$$\alpha_{nc} = \alpha_a + (1 - \alpha_a)^2 \alpha_{surf}, \tag{10}$$

where α_a is the albedo of the upper atmosphere without clouds ($\alpha_a \approx 0.1$). Then

$$\alpha_{nc} = 0.1 + 0.8\alpha_{surf}. \tag{11}$$

And finally,

$$\alpha = 0.1 + 0.4n + \alpha_{surf}(0.81 - 0.56n). \tag{12}$$

There is the problem concerning how to define the value α_{surf}. Since at a first stage we consider a planet covered by snow and vegetation, without ocean, then

$$\alpha_{surf} = \{\lambda(N)\alpha_{bs} + [1 - \lambda(N)]\alpha_{veg}\}(1 - f_{sn}) + \alpha_{sn} f_{sn}. \tag{13}$$

Here α_{sn} is the albedo of snow ($\alpha_{sn} \approx 0.7$) and α_{bs} is the albedo of bare soil, so that

$$\alpha_{bs} = \alpha_{bs}^0 f_{bs}(W_s / W_s^*), \tag{14}$$

where W_s^* is the moisture of saturated soil. The function f_{sn} describing the fraction of land covered by ice has the following form (see Fig. 4):

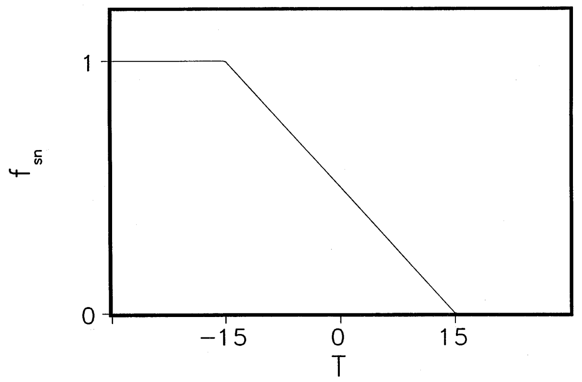

Fig. 4. Function f_{sn} describing the share of land covered by snow as a function of temperature.

$$f_{sn} = \begin{cases} 1 & t < -15 \\ 1 - \frac{t+15}{30} & -15 \leq t < 15. \\ 0 & \text{else} \end{cases} \quad (15)$$

Instead of using a step like function for the albedo as in the Budyko model for glaciation (Budyko, 1969) we use a continuous function. f_{bs} is determined by (Fig. 5).

$$f_{bs} = 1 - W_s \frac{1 - f^*_{bs}}{W^*_s}. \quad (16)$$

The function $\lambda(N) \in [0,1]$ represents the share of land not covered by vegetation as a function of the carbon content in vegetation N. It is a monotonous decreasing function and is shown in Fig. 6.

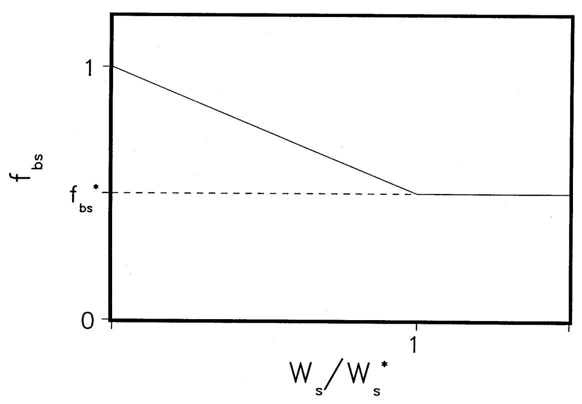

Fig. 5. Function f_{bs} describing the dependence of soil albedo on its relative moisture.

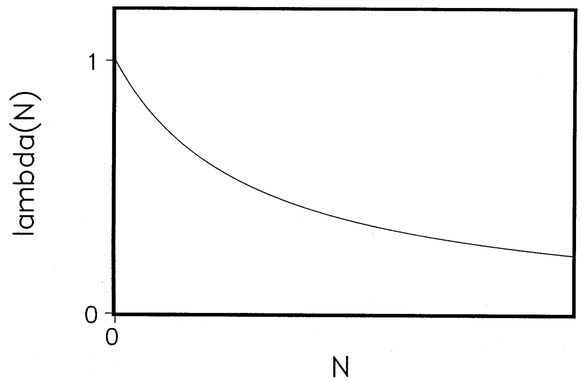

Fig. 6. Function $\lambda(N)$ describing the share of land not covered by vegetation as a function of its amount N.

2.2. Equation of vegetation

The balance equation for carbon content in the vegetation N is very simple:

$$\frac{\partial N}{\partial t} = P - mN, \quad (17)$$

where P is the function describing productivity (annual net production) and $m = 1/\tau_N$, where τ_N is the residence time for living biomass. In accordance with the Liebig principle the productivity (growth) function can be presented in the following multiplicative form as in (Keeling, 1973):

$$P = P_{max} \cdot g_T(T) \cdot g_C(C) \cdot g_N(N) \cdot g_W(W) \quad (18)$$

The factor P_{max} is the maximal value of productivity, when limiting resources are in abundance, and other parameters have optimal values. The possible forms of functions g_T, g_C, g_N, and g_W are shown in Figs. 7–10. Where g_T and g_W are unimodular functions with one maximum which are nonzero for a certain interval, while g_C and g_N are increasing functions with saturation.

2.3. Global carbon cycle

The equations of the global carbon cycle are identical to our previous model (Svirezhev and von Bloh, 1997) with the total amount of carbon in the atmosphere (C) and vegetation (N) as the state variables:

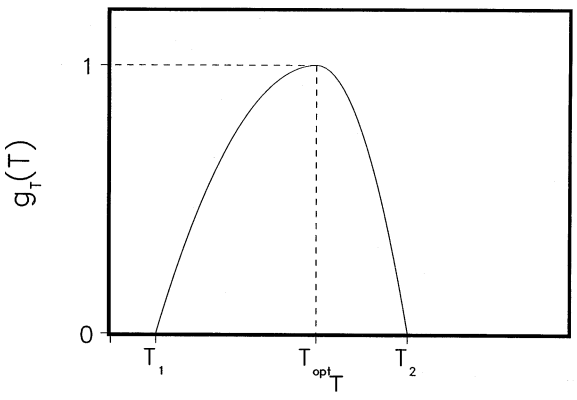

Fig. 7. Qualitative form of the growth function g_T as a function of the temperature T.

$$\frac{\partial C}{\partial t} = -P + mN + e(t),$$

$$\frac{\partial N}{\partial t} = P - mN, \quad (19)$$

where $e(t)$ are the time-dependent (anthropogenic) emissions. If this value is relatively small in comparison with A, then we have the conservation law for carbon in the form:

$$C(t) + N(t) = A = \text{const} \quad (20)$$

Therefore it is possible to replace $C(t)$ by $A - N(t)$ and reduce the number of independent equations by one. For $e(t) > 0$, $e(t) \ll A$ we have some sort of 'adiabatic' process.

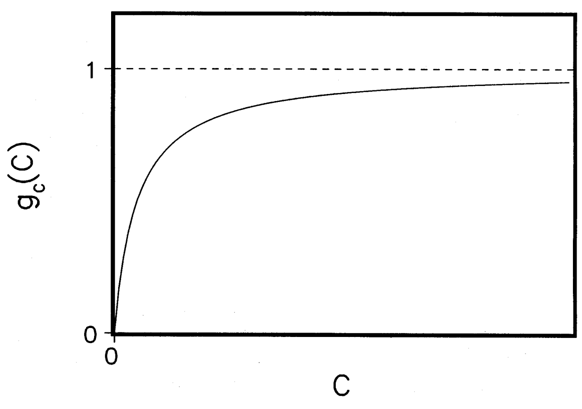

Fig. 8. Qualitative form of the growth function g_C as a function of the amount of carbon in the atmosphere C.

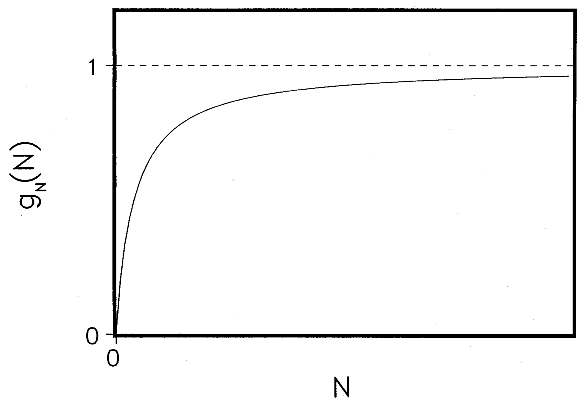

Fig. 9. Qualitative form of the growth function g_N as a function of the amount of vegetation N.

2.4. Water (hydrological) cycle

In the same way we can write the balance equations for water in the atmosphere (W_a) and soil (W_s) (note that we consider the planet to be without ocean). Because the ocean is neglected the change of evaporation of the ocean has to be modelled as an external source for water.

$$\frac{\partial W_a}{\partial t} = -H + E + q_a(t),$$

$$\frac{\partial W_s}{\partial t} = H - E + q_s(t). \quad (21)$$

H is the precipitation and E is the evaporation of soil and evapotranspiration of vegetation cov-

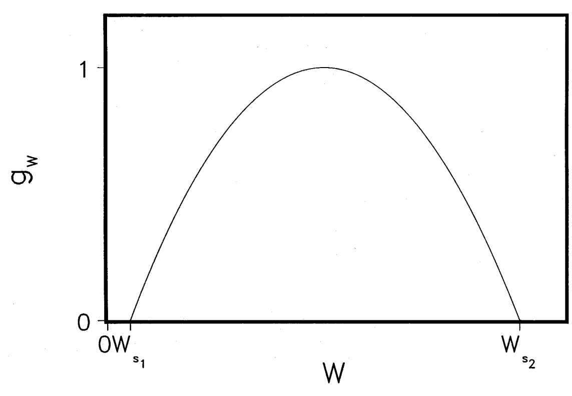

Fig. 10. Qualitative form of the growth function g_W as a function of the amount of water in the soil W_s.

ering the soil. q_a and q_s are external sources and sinks. If q_a and q_s are relatively small and change quasistatically, then we have the conservation law analogous to the carbon cycle for water in a form:

$$W_a(t) + W_s(t) = B = \text{const.} \tag{22}$$

Therefore it is possible to reduce the number of equations by one replacing $W_s(t)$ by $B - W_a(t)$.

The simplest approximation of precipitation H is given by a linear function of the product of the water in the atmosphere W_a and the cloudiness n, so that

$$H = v_a W_a n. \tag{23}$$

The total evaporation E can be presented in the following form:

$$E = \lambda(N)E_{bs} + (1 - \lambda(N)E_{veg}, \tag{24}$$

where E_{bs} is the (physical) evaporation of bare soil,

$$E_{bs} = f_a(T, W_a)W_s/W_s^*,$$

$$f_a(T, W_a) = W_a^*(T) - W_a. \tag{25}$$

Evapotranspiration by vegetation is supposed to be a linear function of the productivity, so that

$$E_{veg} = v_v P. \tag{26}$$

Thus we have the complete closed system of differential equations for the description of 'climate + biosphere' dynamics. Due to the conservation law of carbon and water in the system it consists of the three variables $T(t)$, $N(t)$, and $W_a(t)$.

3. Parametrization

In order to simplify the calculations and allow numerical simulations it is necessary to parametrize the functional behaviour of φ_C, φ_W, λ, g_T, g_C, g_N, and g_W presented in the following:

(1) φ_C and φ_W is parametrized according to Mokhov and Petoukhov (1978). For φ_C we have

$$\varphi_C(C) = 1 - \frac{C(1 - \varphi_C^\infty)}{k_C + C}, \tag{27}$$

φ_W is

$$\varphi_W(W) = \frac{1 + A_W B_W + W^{\beta_W}}{1 + B_W W^{\beta_W}}. \tag{28}$$

The lower bound of $\varphi_W(W)$ is $\varphi_W^\infty = 1/B_w$. The share of land covered by vegetation as function of the total amount of vegetation N is quantitatively described by the equation

$$\lambda(N) = 1 - \frac{n}{k_a + n}. \tag{29}$$

(2) The parametrization of the growth functions is done as described in our previous paper. Let

$$g_T(T) = \begin{cases} 4(T - T_1)(T_2 - T)/\Delta T^2, & \text{if} \quad T \in [T_1, T_2] \\ 0, & \text{if} \quad T \notin [T_1, T_2], \end{cases} \tag{30}$$

where $[T_1, T_2]$ is the tolerance interval for vegetation, i.e. $g_T(T) \geq 0$ for $T \in [T_1, T_2]$, $\Delta T = T_2 - T_1$, i.e. the length of this interval. The function is a unimodular one with one maximum at $T_{opt} = (T_1 + T_2)/2$. The two functions g_C and g_N can be merged to one function $G_N(N) = g_C(A - N) \cdot g_N(N)$. Because $g_C(A - N)$ is a monotonous decreasing function, while $g_N(N)$ is an increasing one, the product is a unimodular function:

$$G_N(N) = \frac{4}{A^2} N(A - N). \tag{31}$$

For the growth function in dependence on the amount of water in the soil we propose a unimodular function similar to that for g_T:

$$g_W(W_s) = \begin{cases} 4(W_s - W_{s1})(W_{s2} - W_s)/\Delta W^2, \\ \end{cases}$$

$$0 \quad \begin{matrix} \text{if} \quad W_s \in [W_{s1}, W_{s2}] \\ \text{if} \quad W_s \notin [W_{s1}, W_{s2}] \end{matrix} \tag{32}$$

where $[W_{s1}, W_{s2}]$ is the water tolerance interval for vegetation and $\Delta W = W_{s2} - W_{s1}$.

A list of all parameter settings is summarized in Table 1.

4. Model results

Despite of the simplicity of the proposed climate–biosphere system, an analytical solution is not possible due to the nonlinearity of the underlying feedback mechanisms acting between the

Table 1
Model parameters and their corresponding units for real Earth scenario

Parameter	Value	Units	Parameter	Value	Units
S	340	W/m^2	k	$3 \cdot 10^7$	J/m^2K
σ	$5.67 \cdot 10^{-8}$	$W/(m^2K^4)$	σ_{sn}	0.4	
α_{veg}	0.1		α^0_{bs}	0.4	
φ^{∞}_C	0.6		k_c	750	Gt
k_a	600	Gt	k_α	600	Gt
A_W	0.01		B_W	2.258	
β_w	0.409		T_1	5	°C
T_2	40	°C	A	1360	Gt
m	0.08	1/year	f^*_{bs}	0.5	
ν_a	70	1/year	ν_{veg}	0.7	$g/(cm^2Gt)$
W^*_s	7.0	g/cm^2	W^1_s	1	g/cm^2
W^2_s	18	g/cm^2	$P_{max.}$	200	Gt/year

climate and biosphere. Therefore numerical solutions of the system are carried out in order to determine possible equilibria of the system.

Due to the different time scales t_W, t_T, t_N for W, T, and N with

$$t_W \ll t_T \ll t_N \quad (33)$$

the system must be solved numerically by an integrator for stiff systems. In order to get a visual impression of the system's behaviour phase portraits can be used. A projection of the three-dimensional phase curves onto the T, N domain eliminating the W variable as the fastest process yields two-dimensional phase diagrams. These two-dimensional phase portraits of the $\{T,N\}$ domain for fixed total amount of water B are plotted for an increasing total amount of carbon A in the system (Fig. 11a–d). Depending on the value of carbon A up to three equilibria are stable. The system bifurcates in dependence on A into different phase space topologies which are characterized in the following:

1. For low values of A in the system (see Fig. 11a) only one equilibrium $N^* \equiv 0$ is stable. Independent of the initial conditions in the $\{N,T\}$ domain vegetation dies out exponentially and the cold desert is reached, ice finally covering the planet.
2. For higher values of A (Fig. 11b) the behaviour slightly changes: one equilibrium is stable as before, but a finite interval in time of occurence of vegetation covering the planet exists. Because the vegetation extracts carbon out of the atmosphere the planet cools down and glaciates completely at the final stage.
3. Above a critical value of A three stable nodes appear (Fig. 11c): one equilibrium (i) with $N^* \equiv 0$ corresponds to the cold desert, while the other two with $N^* > 0$ are stable nodes (ii) (N^*_1, T^*_1) and (iii) (N^*_2, T^*_2) with $N^*_1 > N^*_2$ and $T^*_1 < T^*_2$. Equilibrium (ii) can be identified by the 'cold' planet, and (iii) by the 'hot' planet, respectively. Due to the topology of the basins of attraction a decrease of temperature in equilibrium (ii) forces a glaciation of the planet and the cold desert with $N^* = 0$ is reached. A slight decrease in vegetation, however, is followed by a drastic change in vegetation and equilibrium (iii) is reached. These two possible scenarios can be realized by, e.g. natural disasters, like volcanic eruptions or meteoritic impacts.
4. By increasing A the configuration is again changed. The stable node corresponding to the 'cold' planet disappears and two stable states (ii) and (iii) are obtained. If the carbon in the system is further increased, vegetation becomes completely extinct and the state of a 'hot' desert is reached.

Taking $P_{max.}$, A, and B as the bifurcation parameters a bifurcation diagram can be numeri-

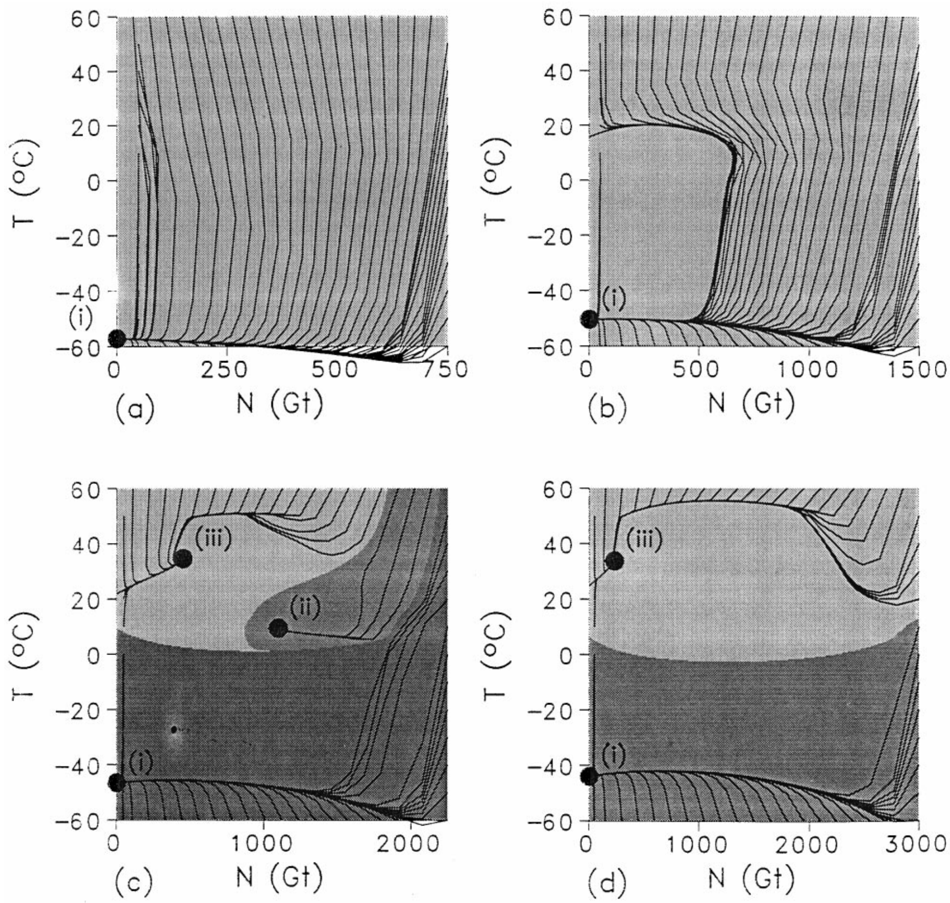

Fig. 11. Phase portrait in the $\{T,N\}$ domain for fixed B ($B = 10$) and different A, (a) $A = 750$, (b) $A = 1500$, (c) $A = 2950$, (d) $A = 3000$. The basins of attraction are marked by different grey shadings.

cally estimated by counting the number of stable equilibria for different settings in the $P_{max.}$, A, and B parameter space. Fig. 12 shows the result for fixed $P_{max.}$ in the $\{A,B\}$ domain.

5. Conclusion

Starting from a very simple global carbon cycle model the proposed climate-biosphere system exhibits some interesting features concerning the stability and sensitivity to external or internal perturbations of the global system. A decrease of temperature or vegetation will force a new equilibrium with significantly lower vegetation or even a glaciation of the whole planet. The number of stable states of the bioplanetary system depends on maximum productivity $P_{max.}$ as a biological factor and the total amount of water and carbon in the system.

Future work will emphasize the determination of transition times between the different equilibria due to random perturbations. Incorporation of a ocean model into the hydrological cycle will improve the validity of the conceptual model.

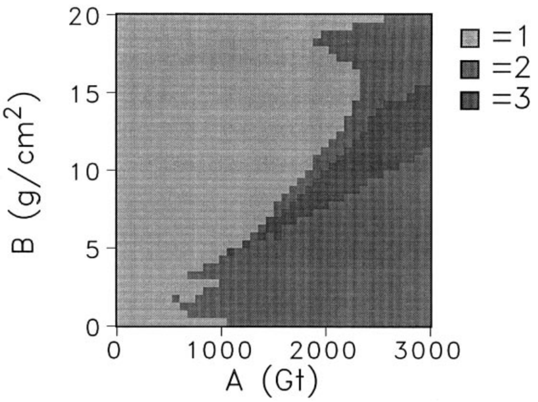

Fig. 12. The number of stable equilibria as a function of total amount of carbon A and water B in the system. P_m is fixed.

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