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Linear response functions to project contributions to future sea level

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Abstract We propose linear response functions to separately estimate the sea-level contributions of thermal expansion and solid ice discharge from Greenland and Antarctica. The response function formalism introduces a time-dependence which allows for future rates of sea-level rise to be influenced by past climate variations. We find that this time-dependence is of the same functional type, $R(t) \sim t^{\alpha}$, for each of the three subsystems considered here. The validity of the approach is assessed by comparing the sea-level estimates obtained via the response functions to projections from comprehensive models. The pure vertical diffusion case in one dimension, corresponding to $\alpha = -0.5$, is a valid approximation for thermal expansion within the ocean up to the middle of the twenty first century for all Representative Concentration Pathways. The approximation is significantly improved for $\alpha = -0.7$. For the solid ice discharge from Greenland we find an optimal value of $\alpha = -0.7$. Different from earlier studies we conclude that solid ice discharge from Greenland due to dynamic thinning is bounded by 0.42 m sea-level equivalent. Ice discharge induced by surface warming on Antarctica is best captured by a positive value of $\alpha = 0.1$ which reflects the fact that ice loss increases with the cumulative amount of heat available for softening the ice in our model.

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1 Introduction

Projections of sea-level rise require a physical understanding of each of the contributing subsystems and their complex dynamics and interactions. Modelling all relevant processes in a fully-coupled Earth system model is a difficult task since some of the key processes are not yet fully understood and various time and spatial resolutions need to be comprised. So-called semi-empirical models provide an alternative to estimate future sea-level rise from either the global mean temperature increase (Vermeer and Rahmstorf 2009) or changes in radiative forcing (Jevrejeva et al. 2010). In principle the associated equations could be applied not only to global mean values but for specific subsystems, if sea-level contribution time series for these were available. In comprehensive model simulations, the sea-level response of a single component to a given climate forcing often exhibits a general functional form (Gillett et al. 2011; Huybrechts et al. 2011; Mikolajewicz et al. 2007; Solomon et al. 2009; Swingedouw et al. 2008; Vizcaino et al. 2010; Winguth et al. 2005; Winkelmann et al. 2012) which can be captured by means of a response function. Within the range of applicability, as discussed below, knowing the response of the subsystem to one forcing thus enables the construction of response functions $R^{(i)}$ with which the response to any other forcing can be predicted for a limited time into the future. Thus following general response theory, sea-level rise $\dot{S}$ can be described as a convolution of the applied climate forcing F, for example temperature increase, and system-specific response functions $R^{(i)}$

$$\dot{S}(t) = \int_0^t dt' R^{(1)}(t,t') \cdot F(t') + \int_0^t dt' \int_0^{t'} dt'' R^{(2)}(t,t',t'') \cdot F(t')F(t'') + \cdots \quad (1)$$

This approach has previously been used for the interpretation of comprehensive climate model results in order to translate perturbations in global mean temperature into the response of a climate variable (e.g., Good et al. 2011; e.g., Hooss et al. 2001). For the contributions to sea-level rise considered here, we will restrict the approach to linear response functions, neglecting all but the first order in Eq. (1), with $R^{(1)}(t,t') = R(t - t')$ such that the sea-level response is represented by

$$\dot{S}(t) = \int_0^t dt' R(t-t') f(t'). \quad (2)$$

This means that the response will be linearly dependent on the magnitude of the forcing. It does not mean that the temporal evolution of the response has to be a linear function of time, but merely that if the same forcing time-series is scaled by, for example, a factor of two then the response will double as well. Linear response theory requires the response to be (1) linear, (2) causal and (3) stationary. For the sea-level contributions considered here, linearity is assumed and shown to yield a good approximation within the multi-centennial timeframe of the future projections we are interested in. Causality and stationarity are not necessarily given, since observed sea-level rise cannot be attributed entirely to global warming but depends for instance also on the history of the ice sheets. We therefore need to restrict the approach to sea-level contributions caused by global warming.

There are different ways to determine the response function of a given system. One method is to perturb the system with an abrupt forcing that is short-lived on the response-time scale of the system. In this case, the forcing term $F(t)$ in Eq. (2) can be considered a delta-function in time and the response of the system to this specific perturbation yields the response function $R(t)$. The response function can also be obtained by perturbing the system with a step-functional perturbation. Equation (2) reveals that the response of the system to a step function yields the integral of the response function as the system response. Differentiation yields the linear response function $R(t)$. In principle, forcing with a random perturbation signal can also be used to determine the response function, but due to the necessity of long-time series this method is less practical for systems with long-time scales such as the oceanic thermal expansion and the response of the ice sheets. Here we will restrict ourselves to the first two approaches.

Rahmstorf (2007) proposed that the rate of global sea-level change is proportional to the global mean temperature change above the preindustrial value, $\dot{S} = a \cdot \Delta T_g$. This relation was tested with observed temperature and sea-level data for the time-period between 1880 and 2001, yielding a significant correlation between the global time-series. Rahmstorf (2007) also showed that this approach holds for the thermal expansion component until the end of the twenty first century. It is noteworthy that the future projections obtained by this approach strongly depend on the functional form considered. As seen in Fig. 1a, similar approaches $\dot{S} = a \cdot \Delta T_g^{\beta}$ with $\beta \in [1;2]$ yield a similarly good correlation with past sea-level between 1950 and 2000 when trained for the period 1880–1950 while values below 1 cannot capture the sea-level response as well. However, while the past may be similarly well represented for various $\beta \in [1;2]$, these approaches give very different projections for the next century for an idealized scenario with a linear temperature increase by 4 K (see Fig. 1b). The physical motivation of the specific formula applied is thus essential. An approach as used by Rahmstorf (2007) and Vermeer and Rahmstorf (2009) can be motivated from the linear response approach: In the response function formalism, this approach can be understood as the sea-level response captured by a delta-function

$$R_{\delta}(t) = \delta(t) \quad (3)$$

such that sea-level rise occurs as an instantaneous reaction to the current climate forcing. Throughout this paper, an approach with a response function of type R_{δ} is referred to as *instantaneous approach*. Allowing for an explicit time-dependence of R as proposed in this study can be regarded as a generalization of the instantaneous approach which takes the history of climate change into account.

Here, we propose time-dependent response functions for the solid ice discharge from Greenland (Sect. 2) and Antarctica (Sect. 3) and for thermal expansion (Sect. 4) to regional climate forcing. Note that due to their time-dependence, the relations presented here are only valid when considering temperature changes compared to an equilibrium state of the respective subsystem. For each of the components, we will apply the proposed response function to estimate the sea-level response to a given forcing and compare this estimate to projections from a comprehensive model.

2 Greenland—solid ice discharge

Price et al. (2011) have shown in a higher-order ice-sheet model that the solid ice discharge from Greenland glaciers in response to a reduction in mechanical frontal stress

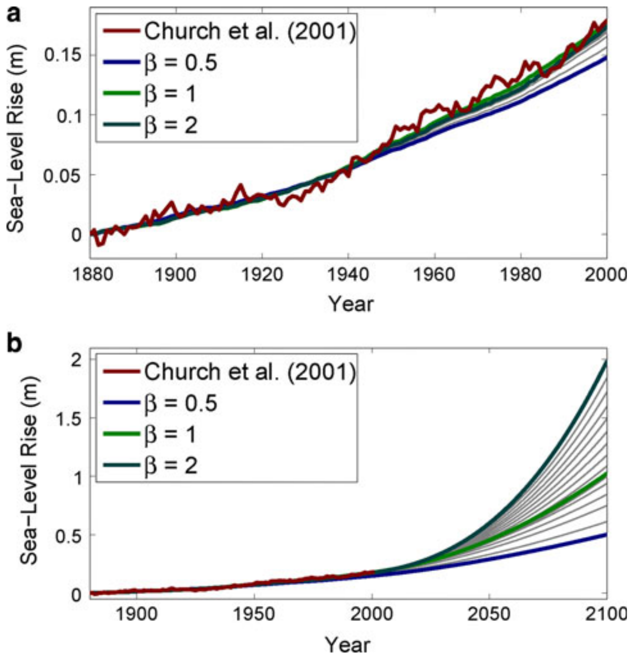

Fig. 1 Comparison of different power-law approaches. **a** Observed sea-level rise (*red*) and sea-level rise computed from $\dot{S} = a \cdot \Delta T_g^{\beta}$ where the exponent β is varied from 0.5 to 2 in steps of 0.1. The coefficient a is learned from the temperature (Hansen et al. 2001) and sea-level data (Church et al. 2001) between years 1880 and 1950. **b** Sea-level projections for a linear increase in temperature by 4 K until 2100, with the same coefficients a and β as in **a**

shows a universal functional form. They simulate the response of Jakobshavn Isbræ, Helheim Glacier and Kangerdlugssuaq Glacier to observationally constrained perturbations at the glaciers' termini and then use a scaling approach to compute the sea-level change for the whole Greenland Ice Sheet. The resulting sea-level response of the Greenland Ice Sheet from (Price et al. 2011, c.f. Fig. 4B) can be described by a function

$$S_{\mathrm{GIS}}(t) \sim \left(\frac{t}{t_0}\right)^{\alpha_{\mathrm{GIS}}+1} \tag{4}$$

with $t_0 = 1$ s and $\alpha_{\mathrm{GIS}} + 1 = 0.3$ (Fig. 2). Since the response time of the ice sheet is much larger than the perturbation time of a few years it is justified to represent the perturbation mathematically as a delta function. As can be seen from Eq. (2), this is consistent with a response of the system to the release of frontal stress with the linear response function

$$R_{\mathrm{GIS}}(t) = \Gamma_{\mathrm{GIS}} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{\mathrm{GIS}}} \tag{5}$$

with $\alpha_{\mathrm{GIS}} = -0.7 < 0$ and $\Gamma_{\mathrm{GIS}} = 0.55$ mm/year. (The coefficients for the response functions presented here are summarized in Table 1).

Solid ice discharge from the Greenland Ice Sheet can hence be expressed as

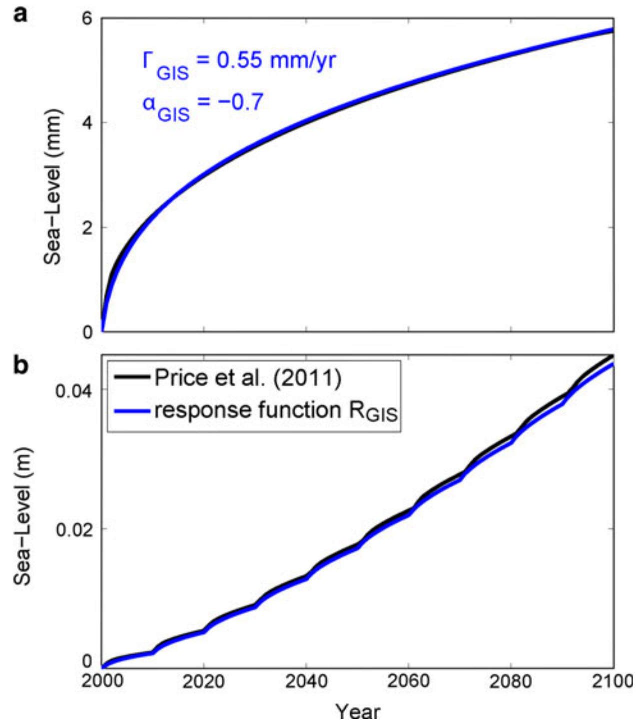

Fig. 2 Dynamic sea-level contribution of the Greenland Ice Sheet. **a** *Black curve* repeats Fig. 4B from Price et al. (2011), showing the cumulative sea-level rise from the Greenland Ice Sheet. *Blue* shows the result obtained via response function R_{GIS} [see Eq. (5)]. **b** *Black curve* repeats Fig. S6 from Price et al. (2011), showing the cumulative sea-level rise from Greenland until 2100 assuming a perturbation recurrence interval of 10 years. *Blue* gives the result obtained via response function R_{GIS} when using Γ_{GIS} and α_{GIS} obtained from the sea-level response depicted in **a**

$$\dot{S}_{GIS}(t) = \int_0^t dt' R_{\mathrm{GIS}}(t-t') f(t') = \Gamma_{\mathrm{GIS}} \cdot \int_0^t dt' \left(\frac{t-t'}{t_0}\right)^{\alpha_{\mathrm{GIS}}} f(t'). \tag{6}$$

where f(t) represents the rate of change of the non-dimensionalized forcing which in Price et al. (2011) is a reduction in mechanical frontal stress. Figure 2a shows a comparison between the committed sea-level rise from Greenland simulated with the higher-order ice-sheet model by Price et al. (2011) and the result obtained via response function R_{GIS} with a root mean square error of 0.05 mm. In order to verify that such a linear response representation of the dynamic sea-level rise from the Greenland Ice Sheet is valid we apply it to multiple delta function perturbations which can be interpreted as a release in backstress recurring every 10 years, as discussed by Price et al. (2011). The resulting sea-level curve compares well with the simulation from Price et al. (2011) with a root mean square error of 0.6 mm until 2100 (see Fig. 2b). As can be seen from the figure, the deviation of the response-function

Table 1 Coefficients for the response functions R_{GIS}, R_{AIS} and R_{TE}

Component	Γ	α
Greenland Ice Sheet: ice discharge from frontal stress release	0.55 mm/year	−0.7
Antarctic Ice Sheet: ice discharge from surface warming	0.05 mm/year/K	0.1
Thermal expansion	0.013 mm/year/K	−0.7

result from the model result grows with time which is consistent with the linear approach chosen here.

Price et al. (2011) stress that their estimate is minimal in the sense that their study does not account for changes in surface mass balance or possible future dynamical changes. Under the assumption that the dynamic response to a release in frontal stress will qualitatively stay the same in the future, the authors give 45 mm as an upper bound for dynamic sea-level rise from the Greenland Ice Sheet within the twenty first century, resulting from a release in frontal stress occurring every 10 years. Under the same assumption, we can use R_{GIS} to project the sea-level response of the Greenland Ice Sheet to higher perturbation frequencies, which can be interpreted as a result of a change in ocean dynamics, causing warm ocean waters to reach the glacier termini at shorter time-intervals. Figure 3 shows the dynamic sea-level change resulting from perturbations with a frequency of $\tau \in \{1, 2, 3, 5, 10\}$ years, i.e.,

$$\dot{S}_{GIS}(t) = \int_0^t dt' R_{GIS}(t - t') \sum_{n=1}^{\infty} \delta(t' - n\tau). \qquad (7)$$

The maximal sea-level response obtained is 42 cm, for an annual incursion of warm ocean water. The sea-level change increases non-linearly with the perturbation frequency. It has to be noted that the upper limit for the dynamic sea-level contribution from Greenland is limited by the total amount of ice that is susceptible to changes in frontal stress under climate change which might be significantly smaller than the 42 cm obtained here.

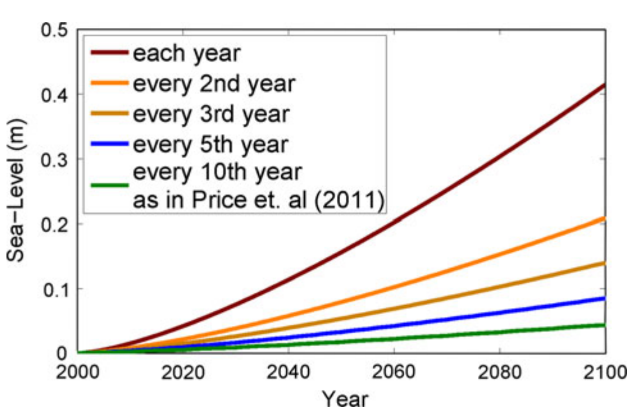

Fig. 3 Sea-level contribution of the Greenland Ice Sheet obtained via response function R_{GIS} for a perturbation frequency of 1 to 10 years

Price et al. (2011) compare their results to other published sea-level estimates for the cumulative sea-level rise from Greenland by 2100 (Meier et al. 2007; Pfeffer et al. 2008; Rahmstorf 2007; Jevrejeva et al. 2010). To this end, they downscale the sea-level contributions projected by these models in order to compute the part which is solely due to Greenland ice dynamics, as repeated in Table 2. Overall, the minimum dynamic sea-level contribution of 6 mm from Price et al. (2011) lies within 2–26 % of the other studies. Table 2 shows the twenty first century estimates based on the other studies (ranging from 23 to 240 mm) along with the deduced perturbation frequency which would have to be applied in the (Price et al. 2011)-model to attain the same sea-level change by 2100. This translates into a required release in backstress every 2nd–20th year.

3 Antarctica—solid ice discharge through surface warming

Here we derive a linear response function for the solid ice discharge as computed with the Potsdam Parallel Ice Sheet Model PISM-PIK (Winkelmann et al. 2011) under surface warming scenarios. PISM-PIK is a continental-scale ice-sheet model based on the Parallel Ice Sheet Model (PISM, Bueler and Brown 2009). For the model's performance under present-day boundary conditions confer Martin et al. (2011). The sea-level response of the Antarctic Ice Sheet to surface warming has been investigated in a 81-member ensemble of equilibrium states derived for present-day boundary conditions. The surface temperature was subjected to changes according to the Extended Concentration Pathway ECP-8.5 (Meinshausen et al. 2011). The ECP-8.5 scenario is the extension of the Representative Concentration Pathway RCP-8.5 (Riahi et al. 2007) with overall rising greenhouse gas concentrations until 2500. Of the four ECPs, it leads to the strongest warming globally and results in an increase of Antarctic surface temperatures of about 11 K until 2500 depending on the climate sensitivity and the polar amplification. For details on the projections see Winkelmann et al. (2012). Each simulation starts from one of the 81 equilibrium states of the Antarctic Ice Sheet which all closely resemble present-day Antarctica with respect to their geometry but differ in the basal friction between ice and bedrock and in the ice softness. In order to

Table 2 Dynamic sea-level rise from the Greenland Ice Sheet, as derived by Price et al. (2011) via a scaling-approach from the results by Meier et al. (2007), Pfeffer et al. (2008), Rahmstorf (2007) and Jevrejeva et al. (2010)

Study		Dyn. SLR from GIS by 2100 (mm)	Required frequency τ (years)
Meier et al. (2007)	No acceleration	23	20
	Acc. at current rate	123	3
Rahmstorf (2007)	Lower bound	45–68	6–10
	Upper bound	127–191	2–3
Pfeffer et al. (2008)	Doubled flow rates	93	5
Jevrejeva et al. (2010)	Lower bound	60–90	5–7
	Upper bound	160–240	2–3

By means of response function R_{GIS}, the corresponding perturbation frequency τ is deduced

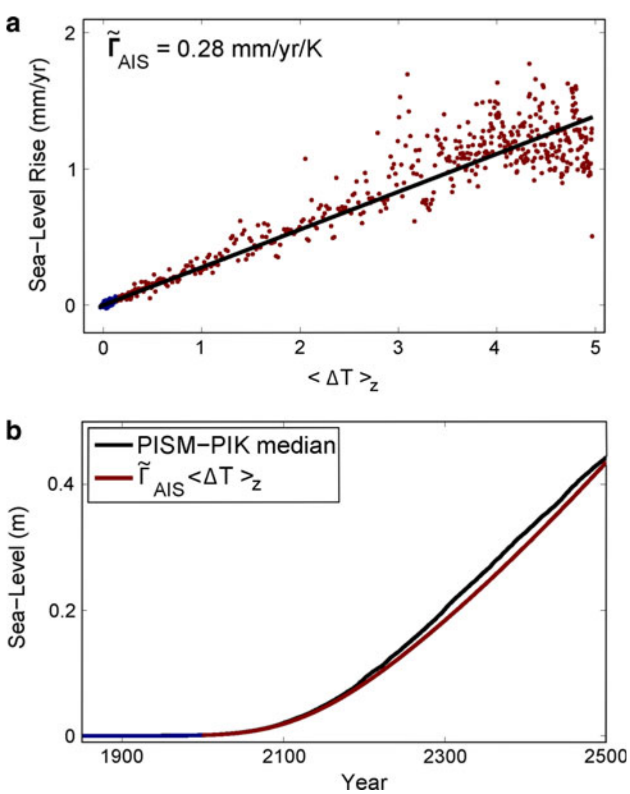

Fig. 4 Linear relation between sea-level rise and vertically-averaged temperature anomalies of the Antarctic Ice Sheet. **a** Sea-level rise shown against the vertically-averaged temperature anomaly for the ECP-8.5 scenario as modelled with PISM-PIK. **b** Comparison of cumulative sea-level rise modelled with PISM-PIK (*black*) and computed from the temperature via Eq. (9). Here, the coefficient $\tilde{\Gamma}_{\mathrm{AIS}}$ is found from the modelled sea-level rise and temperature until year 2000 (*blue*) and then used to predict the sea-level change until 2500 (*red*)

separate the dynamic effect of surface warming from other processes, surface melting, ocean warming and precipitation increase are not included in these simulations. The median of sea-level change due to surface warming reaches a maximum of 44 cm in 2500. Here we use this ensemble mean of the ECP-8.5 projections in order to derive the functional form of the corresponding response function (Fig. 4b).

The major influence of surface warming on the ice discharge is through softening of the ice. The propagation of the surface temperature changes into the ice body leads to its softening which increases the overall of ice flux across the grounding line. Consistently, we find a quasi-linear relation between the averaged temperature increase within the ice and the ice discharge rate (Fig. 4a). Based on this observation we assume that sea-level rise can be approximated to be proportional to the total amount of accumulated heat

$$\dot{S}_{\mathrm{AIS}} \sim \int dt W, \tag{8}$$

where W is the heat flux into the ice (or into the ocean, see Sect. 4), and can be expressed in terms of the depth-averaged temperature anomaly of the ice-sheet

$$\dot{S}_{\mathrm{AIS}}(t) = \tilde{\Gamma}_{\mathrm{AIS}} \cdot \frac{1}{H} \int_0^H dz \Delta T(z,t) \equiv \tilde{\Gamma}_{\mathrm{AIS}} \cdot \langle \Delta T \rangle_z, \tag{9}$$

where H denotes the ice thickness. Figure 4b shows the projection of sea-level based on Eq. (9) with $\tilde{\Gamma}_{\mathrm{AIS}} = 0.28$ mm/year/K in comparison to the simulation with PISM-PIK.

In order to relate the vertically-averaged temperature anomaly to the surface warming, we assume that the propagation of surface temperature changes ΔT_s into the ice body is dominated by vertical diffusion. Thus the ice temperature change $\Delta T(z,t)$ at depth z and time t is governed by

$$\frac{\partial \Delta T(z,t)}{\partial t} = \kappa \frac{\partial^2 \Delta T(z,t)}{\partial z^2}, \tag{10}$$

with vertical diffusivity κ and the boundary condition

$$\Delta T(H,t) = \Delta T_s(t). \tag{11}$$

The solution of Eq. (10) in response to a step change of 1 K in surface temperature is given by

$$\Delta T_{\wedge}(z,t) = \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right), \quad (12)$$

where erfc denotes the complementary error function, i.e. one minus error function [see for example (Marčelja 2010)]. This is easily verified by insertion into Eq. (10). For a given temporal evolution $\Delta T_s(t)$, the ice temperature change $\Delta T(z,t)$ can be derived by convolution with the step-change solution Eq. (12), i.e.,

$$\Delta T(z,t) = \int_0^t dt' \frac{d\Delta T_s(t')}{dt'} \cdot \Delta T_{\wedge}(z, t-t'). \quad (13)$$

Using this solution in Eq. (9), we find

$$\begin{aligned}\dot{S}_{\mathrm{AIS}}(t) &= \tilde{\Gamma}_{\mathrm{AIS}} \cdot \frac{1}{H}\int_0^H dz \int_0^t dt' \left(\frac{d\Delta T_s(t')}{dt'} \cdot \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa(t-t')}}\right)\right)\\ &= \tilde{\Gamma}_{\mathrm{AIS}} \cdot \frac{1}{H}\int_0^t dt' \left(\frac{d\Delta T_s(t')}{dt'} \cdot 2\sqrt{\kappa(t-t')}\int_0^H dz' \mathrm{erfc}(z')\right)\\ &= \tilde{\Gamma}_{\mathrm{AIS}} \cdot \frac{1}{H}\int_0^t dt' \frac{d\Delta T_s(t')}{dt'} \cdot 2\sqrt{\kappa(t-t')} \cdot c_e\\ &= \Gamma_{\mathrm{AIS}} \cdot \int_0^t dt' \frac{d\Delta T_s(t')}{dt'} \cdot \sqrt{t-t'},\end{aligned} \quad (14)$$

where we made use of the substitution $z' = \frac{z}{2\sqrt{\kappa(t-t')}}$. The constant c_e depends on the ice thickness, but assuming that the surface temperature changes do not reach the ice-sheet base, we can use the approximation $\int_0^\infty dx \,\mathrm{erfc}(x) = 1/\sqrt{\pi}$. We therefore propose the following response function

$$R_{\mathrm{AIS},1/2}(t) = \Gamma_{\mathrm{AIS}} \cdot \sqrt{t}, \quad (15)$$

with constant Γ_{AIS}, to compute the sea-level contribution of Antarctica in response to surface warming.

The form of response function $R_{\mathrm{AIS},1/2}$ with positive exponent $\alpha = 0.5$ implies that past variations in surface temperature dominate current sea-level rise. This means that, although the heat uptake of the ice-sheet decreases with time, sea-level keeps rising because the amount of heat already stored in the ice-sheet causes a decrease in viscosity which in turn enhances the ice flux. This relation must fail on longer time-scales where the Antarctic Ice Sheet loses part of the accumulated heat, so the applicability of response function $R_{\mathrm{AIS},1/2}$ is clearly limited. It is also important to note that $R_{\mathrm{AIS},1/2}$ can only be used for temperature projections in comparison to an equilibrium state of the Antarctic Ice Sheet, i.e., $\Delta T_s(0) = 0$. We can test response function $R_{\mathrm{AIS},1/2}$ using the surface temperature changes from the ECP-8.5 scenario underlying in the model simulations with PISM-PIK. The solid red line in Fig. 5 shows the sea-level projected via the response function in comparison to the median of the PISM-PIK simulations, with $\Gamma_{\mathrm{AIS}} = 0.008$ mm/year/K. The coefficient Γ_{AIS} is found by minimizing the error between the modelled sea-level rise and the projection via response function $R_{\mathrm{AIS},1/2}$ until year 2500. Although the sea-level curve derived via $R_{\mathrm{AIS},1/2}$, based on the assumption that the heat-transport in the ice-sheet is solely governed by vertical diffusion, is close to the PISM-PIK median, we cannot find the appropriate coefficient Γ_{AIS} only based on the knowledge of past surface temperatures and sea-level rise data. This becomes evident when constricting the learning process until year 2000 (dashed line in Fig. 5).

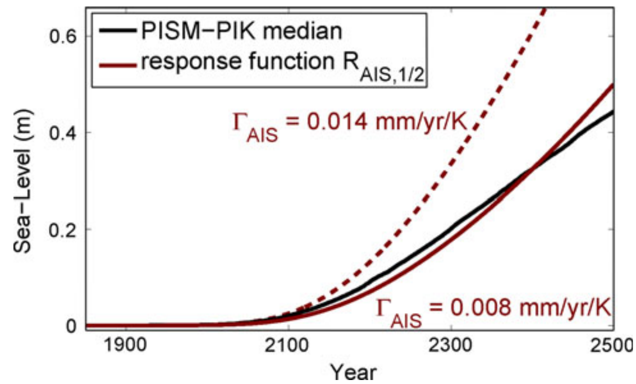

Fig. 5 Comparison of cumulative sea-level rise obtained via response function $R_{\mathrm{AIS},1/2}$ with learning periods until 2000 (*dashed red line*) and 2500 (*solid red line*) to the result from PISM-PIK (*black line*)

However, the importance of taking past temperature changes into account becomes clear when we compare this approach to an instantaneous approach, where we assume that sea-level rise directly follows Antarctic surface warming

$$\dot{S}_{\mathrm{AIS},\delta}(t) = a_{\mathrm{AIS}} \cdot \Delta T_s. \quad (16)$$

Training this relation with the temperature and sea-level data until the year 2000, we find $a_{\mathrm{AIS}} = 0.06$ mm/year/K. While the instantaneous approach holds well within the twenty first century with a root mean square deviation of 0.17 mm (Fig. 6a), the projected sea-level differs from the model results with a root mean square of 62 mm until year 2500 (green curve in Fig. 6b). We therefore conclude that the history of surface temperature changes is of increasing importance for the future sea-level contribution of Antarctica. The results also suggest that the heat transport is not solely governed by downward diffusion but other processes such as temperature advection play a significant role. Motivated by the fact that the Antarctic sea-level contribution depends on past temperature changes, but the functional form of this dependence is a-priori not clear, we generalize the response-function approach, similar to the response function for solid

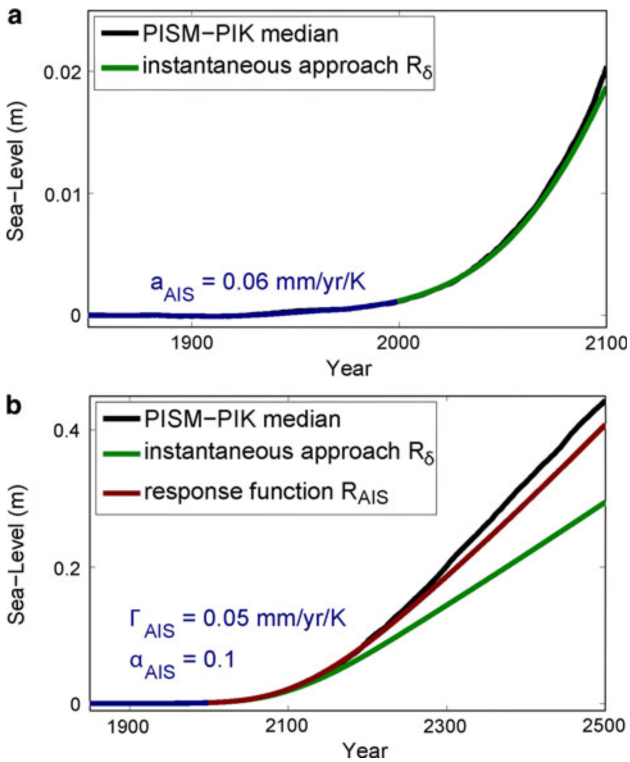

Fig. 6 **a** Antarctic sea-level contribution within the twenty first century simulated with PISM-PIK and estimated assuming an instantaneous approach for the relation between sea-level rise and surface warming [Eq. (16)]. The coefficient a_{AIS} is found from the temperature and sea-level data until year 2000 and used to project the future sea-level change. **b** Comparison of the instantaneous approach [*green*, see Eq. (16)] and the generalized response function [*red*, see Eq. (17)] to the dynamic sea-level response of the Antarctic Ice Sheet to surface warming as modelled with PISM-PIK

ice discharge from the Greenland Ice Sheet given in Sect. 2, by allowing for other exponents α_{AIS}

$$R_{AIS}(t) = \Gamma_{AIS} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{AIS}}. \tag{17}$$

with $\alpha_{AIS} > 0$. From the known temperature anomalies and sea-level rise for the time-period between 1850 and 2000, we find those parameters Γ_{AIS} and α_{AIS} which maximize the correlation between the sea-level response computed via R_{AIS} and the modelled sea-level change, yielding $\Gamma_{AIS} =$ 0.05 mm/year/K and $\alpha_{AIS} = 0.1$. Figure 6b shows the sea-level change until the year 2500, projected with the generalized response function R_{AIS} based on these parameters.

4 Thermal expansion

In contrast to the ice loss from Antarctic surface warming, as described in the previous section, thermosteric sea-level rise is to first order proportional to the heat flux W into the ocean (e.g., Church et al. 2011)

$$\dot{S}_{TE}(t) \sim W. \tag{18}$$

When ocean heat uptake is dominated by vertical diffusion of surface temperature anomalies, we can make use of Eqs. (12) and (13) to compute the resulting change in sea-level from the amount of heat accumulated in the ocean:

$$S_{TE}(t) = \tilde{\Gamma}_{TE} \cdot \int_0^\infty dz \int_0^t dt' \left(\frac{d\Delta T_s(t')}{dt'} \cdot \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa(t-t')}}\right)\right) \tag{19}$$

$$= \tilde{\Gamma}_{TE} \cdot 2\sqrt{\frac{\kappa}{\pi}} \cdot \int_0^t dt' \frac{d\Delta T_s(t')}{dt'} \cdot \sqrt{t-t'}. \tag{20}$$

where κ denotes here the vertical eddy diffusivity. The resulting sea-level rise is then given by

$$\dot{S}_{TE}(t) = \tilde{\Gamma}_{TE} \cdot \sqrt{\frac{\kappa}{\pi}} \int_0^t dt' \frac{d\Delta T_s(t')}{dt'} \cdot \frac{1}{\sqrt{t-t'}} \tag{21}$$

(see also Allen et al. 2009, supplement). This can be understood as the response of thermosteric sea-level rise to changes in sea-surface temperature, governed by the response function

$$R_{TE,-1/2}(t) = \frac{\Gamma_{TE}}{\sqrt{t}}. \tag{22}$$

Since thermosteric sea-level rise is directly proportional to the heat uptake, it is dominated by present changes in sea-surface temperature and the influence of past changes is dampened which is reflected in the time-component introduced by the response function $R_{TE,-1/2}$. That is in contrast to R_{AIS} with a positive exponent $\alpha_{AIS} > 0$ which emphasizes past temperature changes. As a first-order approximation it is therefore valid to understand sea-level rise as an immediate response to a change in sea-surface temperature. Rahmstorf (2007) showed that future thermosteric sea-level rise can be well-approximated within the twenty first century using an instantaneous approach, i.e.

$$\dot{S}_{TE,\delta}(t) = a_{TE} \cdot \Delta T_s \tag{23}$$

where ΔT_s is the difference in sea-surface temperature compared to pre-industrial levels. From model results for the A1F1 scenario with the coupled climate model CLIMBER-3α (Montoya et al. 2005), Rahmstorf (2007) found $a_{TE} = 1.6$ mm/year/K. We confirm this result for different climate forcings given by the ECP scenarios using projections of sea-surface temperature and thermosteric sea-level rise conducted with CLIMBER-3α (Schewe et al. 2011). From the model results for years 1765–2000 of the

ECP-8.5 scenario, we find the coefficients a_{TE} for the instantaneous approach and Γ_{TE} for the response function $R_{TE,-1/2}$. We then test these approaches by comparing the projected sea-level rise for the twenty first and twenty second centuries with the results obtained with CLIMBER-3α. Figure 7b shows the resulting change in sea-level in comparison to the modelled sea-level change from Schewe et al. (2011). Both approaches hold well until 2100 with root mean square errors of 20 mm for the instantaneous approach and 9.6 mm for the response function. Thereafter, the sea-level projections deviate more strongly from the CLIMBER-3α-result, suggesting that past temperature changes need to be taken into account on longer time-scales, but this dependence may be of a different form. The results indicate that the assumption of a purely diffusive ocean, on which the response function $R_{TE,-1/2}$ is based, only holds on relatively short time-scales and other processes such as convection, and the evolution of ocean currents need to be taken into account.

In a next step, we test the different approaches for the extended RCP-2.6 scenario (van Vuuren et al. 2006, 2007). The ECP-2.6 scenario differs qualitatively from ECP-8.5 in that the radiative forcing reaches its maximum within the twenty first century and declines thereafter. The global mean temperature changes also peak in the 21st, slightly above 1.5 K. With the same coefficients Γ_{TE} and a_{TE} found for the ECP-8.5 scenario, we compute the sea-level response to the changes in surface temperature obtained for

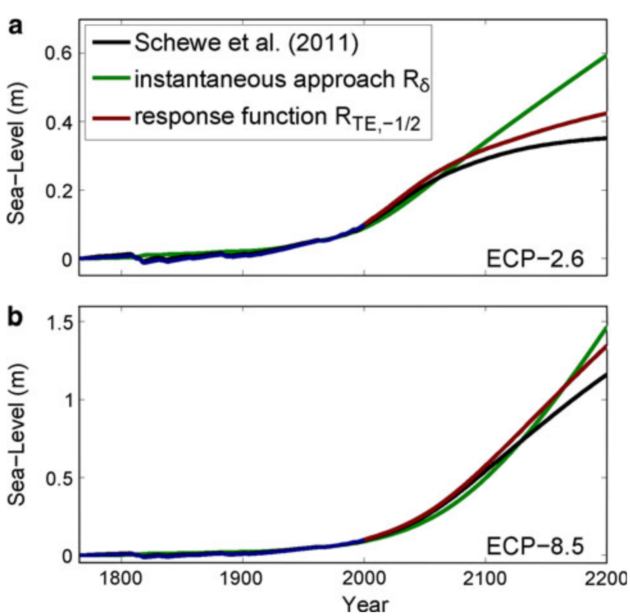

Fig. 7 Thermosteric sea-level change for scenarios ECP-2.6 (**a**) and ECP-8.5 (**b**). *Black curves* repeat results from Schewe et al. (2011, Fig. 6) as modelled with the coupled climate model CLIMBER-3α. The results obtained using the instantaneous approach R_δ and response function $R_{TE,-1/2}$ [Eq. (22)] are given in *green* and *red*, respectively

ECP-2.6 and compare the results with the thermosteric sea-level response from CLIMBER-3α. Figure 7a shows that both approaches can in principle be used for projecting near-future sea-level changes to very different climate forcing. While both begin to fail after the twenty first century, the response function captures modelled deceleration more closely. Aiming at a generalized response function, we allow for different forms of time-dependence as we have done for the sea-level contributions of Greenland (Sect. 2) and Antarctica (Sect. 3):

$$R_{TE}(t) = \Gamma_{TE} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{TE}} \tag{24}$$

with $\alpha_{TE} < 0$. With this general response function, we test how much information on future sea-level rise is gained by extending the learning period: we assume we had knowledge about the surface temperature and sea-level evolution until year 2000, 2100 and 2200, only for scenario ECP-8.5. While this results in $\Gamma_{TE} = 0.02$ mm/year/K and $\alpha_{TE} = -0.9$ for a learning period until year 2000, the approach yields $\Gamma_{TE} = 0.01$ mm/year/K and $\alpha_{TE} = -0.6$ when extending the learning period to the year 2100 and $\Gamma_{TE} = 0.013$ mm/year/K and $\alpha_{TE} = -0.7$ when extending the learning period to the year 2200. In each case, the

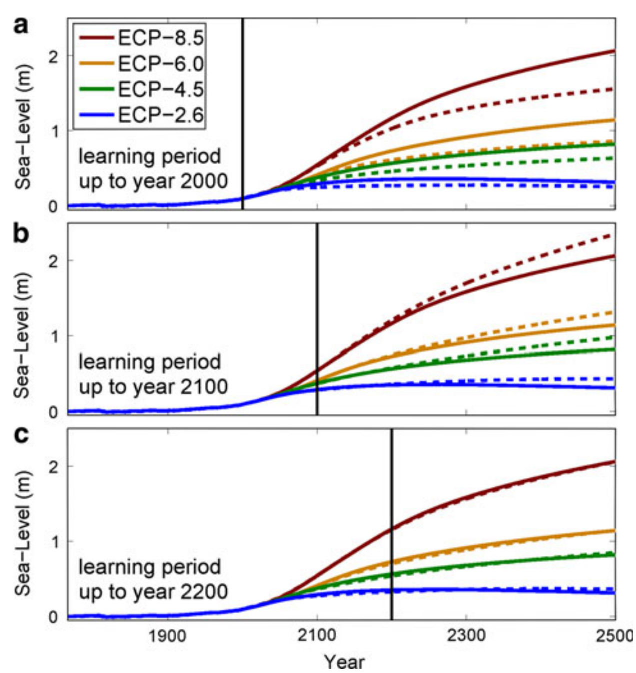

Fig. 8 Thermosteric sea-level change for scenarios ECP-2.6 (*blue*), ECP-4.5 (*green*), ECP-6.0 (*yellow*) and ECP-8.5 (*red*). *Solid lines* depict model results from Schewe et al. (2011, Fig. 6). *Dashed lines* are estimates using response function R_{TE} [Eq. (24)]. The parameters Γ_{TE} and α_{TE} are found from the sea-surface temperatures and sea-level rise until year 2000 (**a**), 2100 (**b**) and 2200 (**c**) of scenario ECP-8.5 and used to predict the sea-level changes until 2500 and for the other scenarios

exponent α_{TE} is smaller than -0.5, suggesting that the assumption of a purely diffusive ocean is not entirely appropriate and faster processes such as downwelling, leading to a more immediate response of the ocean, cannot be neglected. From the values of Γ_{TE} and α_{TE} obtained within the learning period for the ECP-8.5 scenario, we compute the expected sea-level change until the end of the twenty fifth century. The results are given in Fig. 8 (dashed lines) in comparison to the modelled sea-level from CLIMBER-3α (solid lines). While the average error of the response-function estimate until year 2500 is about 133 mm given only past sea-level information, it is reduced by an order of magnitude to about 14 mm when we presuppose sea-level data until 2200, independent of the choice of scenario for the learning process.

5 Conclusions and discussion

Linear response functions provide a useful method to estimate individual sea-level contributions, provided that the underlying dynamic mechanism does not undergo a qualitative change. We have found response functions capturing thermosteric sea-level rise as well as the dynamic sea-level contributions of Greenland (in response to a reduction in frontal stress) and Antarctica (in response to surface warming) on a centennial time-scale. Linear response functions as discussed in this paper can be understood as a generalization of an approach which assumes an instantaneous sea-level response to a change in forcing such that $R = \delta$. Examples for such a relation could be the precipitation changes on Greenland or Antarctica, following directly from atmospheric warming due to its enhanced moisture-carrying capacity.

The response functions for the three components of sea-level rise considered here introduce a time-component which accounts for the fact that past climate change will continue to influence sea-level rise in the future. They have a universal functional form

$$R(t) = c \cdot \left(\frac{t}{t_0}\right)^{\alpha}. \quad (25)$$

The time-dependence on the external forcing is determined by the coefficient α and the sea-level contribution is then computed according to

$$\dot{S}(t) = \int_0^t dt' R(t - t') f(t') \quad (26)$$

with the corresponding forcing time series f(t) which is related to the global temperature perturbation. The exponent α is found to be negative both for the dynamic sea-level contribution from Greenland and for the thermosteric oceanic contribution, such that an influence of past climate changes on current sea-level rise exists but is dampened over time. For the dynamic sea-level contribution from Antarctica, however, α is positive. This implies an inherent limitation of this particular response function R_{AIS} since it is based on the assumption that heat keeps being accumulated in the ice-sheet and past warming contributes to further softening of the ice. The response function must necessarily overestimate sea-level rise when heat is lost from the ice-body, which is however not significant for the time-periods considered here.

How can this approach be generalized? Global temperature changes and regional climate changes show a close linear relation in many of the comprehensive climate models of the last Assessment Report of the IPCC as shown by Frieler et al. (2011) for Greenland temperature and precipitation changes as well as for Antarctic surface and ocean temperature changes (Winkelmann et al. 2012). The expression for the Antarctic sea-level contribution (Sect. 3) can therefore easily be reformulated in terms of global mean temperature changes by introducing the scaling coefficient $c_{AIS} = \Delta T_{s,AIS}/\Delta T_g$ where ΔT_g denotes the global mean temperature change and $\Delta T_{s,AIS}$ the surface warming averaged over Antarctica. For Greenland, Price et al. (2011) state that most of the dynamic sea-level response during the last decade was caused by intrusion of warm ocean waters. This regional warming can be due to an increase in global mean temperature. If it was translated into a reduction in frontal stress, a second coefficient c_{GIS} could be used to express the sea-level contribution of Greenland in terms of global warming. This would allow merging the response functions found for the individual components into a single response function

$$R_g(t) = \Gamma_{TE} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{TE}} + c_{GIS}\Gamma_{GIS} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{GIS}} + c_{AIS}\Gamma_{AIS} \cdot \left(\frac{t}{t_0}\right)^{\alpha_{AIS}} + \cdots \quad (27)$$

such that

$$\dot{S}_g(t) = \int_0^t dt' \frac{\partial \Delta T_g(t')}{\partial t'} \cdot R_g\left(\frac{t - t'}{t_0}\right) \quad (28)$$

estimates the global mean sea-level rise for a given global mean temperature anomaly. It needs to be noted that while we have restricted the response function formalism to linear response functions in this paper, some sea-level contributions cannot be approximated in this way. An example is the surface-elevation feedback which introduces strong non-linearities into the response of the Greenland Ice Sheet to surface warming (e.g., Ridley et al. 2010).

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