The Probabilistic Tolerable Windows Approach

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Abstract. The integrated assessment of climate change aims to consider the entire chain of cause and effect of climate change. If one looks at this chain in more detail, considerable uncertainty has to be acknowledged. This uncertainty needs to be taken into account. Therefore we develop a probabilistic extension to the Tolerable Windows Approach (TWA) in this paper. In the TWA, the aim is to determine the complete set of emission strategies that are compatible with so-called guardrails. Guardrails are limits to impacts of climate change, to climate change itself, or to the impacts of climate mitigation strategies. Therefore, the TWA determines the "maneuvering space" humanity has, if certain impacts of climate change are to be avoided. Due to uncertainty it is not possible to definitely exclude the impacts of climate change considered, but there will always be a certain probability of violating a guardrail. Therefore the TWA is extended to a probabilistic TWA that is able to consider "probabilistic uncertainty", i. e. uncertainty that can be expressed as a probability distribution of uncertain parameters or uncertainty that arises through natural variability.
As a first application, temperature guardrails are imposed, and the dependence of emission reduction strategies on probability distributions for climate sensitivities is investigated. The analysis suggests that it will be difficult to observe a temperature guardrail of 2°C with high probabilities of actually meeting the target.

1. Introduction

The *tolerable windows approach* (TWA) (Petschel-Held et al., 1999; Bruckner et al., 1999; Bruckner et al., 2003; Toth, 2003; Toth et al., 2003a; Toth et al., 2003b), also called the *guardrail approach*, is an approach to the integrated assessment of climate change (IA).

In the integrated assessment of climate change an attempt is made to evaluate the entire chain of cause and effect of climate change, ranging from the anthropogeneous emissions of greenhouse gases, over the changes in climate these emissions cause, to the impacts the induced

climate change will have. Prototypically, this is done in a comprehensive and coordinated analysis. Since this mainly involves the *future* changes in climate, a strong emphasis is placed on models as tools for IA.

With regard to the methodology employed, three paradigms of IA can be distinguished that differ with respect to how they consider the control problem of IA. Formally, IA is a control problem with the basic formulation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t; \mathbf{u}). \tag{1}$$

In this equation $\mathbf{x} \in \mathbb{R}^n$ is the state vector of the system, and $\mathbf{u} \in \mathbb{U}$ is a vector of control variables that can be freely chosen in $\mathbb{U}$. In the climate change problem, $\mathbf{x}$ would be the state vector of the coupled system of socio-economy, climate system, and impacts of climate change, while $\mathbf{u}$ could be the reductions in CO_2 emissions, or the emissions themselves. With the help of this basic equation, three basic approaches to IA can be identified (adapted from Weyant et al. (1996)):

- Policy evaluation modelling: in policy evaluation modelling the physical, ecologic economic and social consequences of *predefined* climate protection strategies are evaluated. Formally, a single control function $\mathbf{u}(\cdot)$ is specified as an input, and the solutions $\mathbf{x}(\cdot)$ are sought.
  A representative of this approach is the IMAGE family of models (Rotmans et al., 1989; Alcamo et al., 1998).

- Policy optimisation modelling: in policy optimisation modelling it is attempted to determine control vectors in such a way that a predefined goal function is maximised. This function may be determined by costs and benefits of climate protection strategies in a single metric, i. e. US \$, but other definitions are possible as well. After defining a goal function $J(t) = \int_0^t c(\mathbf{x}, t')dt'$, solutions $\tilde{\mathbf{u}}(\cdot)$ are sought, such that

  $$\tilde{\mathbf{u}}(\cdot) = \arg \max_u(t) \int_0^t c(\mathbf{x}, t')dt' \text{ with } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t; \mathbf{u}). \tag{2}$$

  Policy optimisation modelling usually takes place either as *cost-benefit analysis* or, in cases where additional constraints are acknowledged, as *cost-effectiveness analysis*. Typical representatives of this approach are the models DICE (Nordhaus, 1994), RICE (Nordhaus and Yang, 1996), MERGE (Manne et al., 1995), and SIAM (Hasselmann et al., 1997).

- Policy guidance modelling: policy guidance modelling aims to determine the *complete set* of climate protection strategies $\mathbf{u}(\cdot)$ that

are compatible with normative constraints, formally defined as $\mathbf{h}(\mathbf{x}, t; \mathbf{u}) \leq \mathbf{0}$. These constraints may be set in order to limit the impacts of climate change, but they may also limit the costs of emission reduction or any other element that is represented in the coupled assessment model. This problem can formally be represented as a differential inclusion (Aubin and Cellina, 1984; Deimling, 1992)

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}, t) \quad \text{with} \quad \mathbb{F} := \{\mathbf{f}(\mathbf{x}, t; \mathbf{u}) | \mathbf{u} \in \mathbb{U}\} \tag{3}$$

under the condition

$$\mathbf{h}(\mathbf{x}, t; \mathbf{u}) \leq 0. \tag{4}$$

Representatives of this approach are the *safe landing analysis* (Swart et al., 1998), which partly fulfils the abovementioned characteristics, and the *tolerable windows approach* (TWA) (Petschel-Held et al., 1999; Bruckner et al., 1999; Toth, 2003; Bruckner et al., 2003).

The *tolerable windows approach* (TWA) was originally proposed by the German Advisory Council on Global Change (WBGU, 1995) during the preparations for the 1st Conference of the Parties (COP) to the United Nations Framework Convention on Climate Change (UNFCCC) in Berlin. Its main objective is to support climate change decision-making by separating scientific analysis from the normative setting of climate protection targets (Petschel-Held et al., 1999; Bruckner et al., 2003).

A major motivation for the TWA stems from Article 2 of the UNFCCC. This article calls for the stabilisation of greenhouse gas concentrations at levels that prevent dangerous anthropogenic interference with the climate system (United Nations, 1995). The TWA is an approach that enables an operationalisation of Article 2, since it aims to determine the maneuvering space humanity has, if it wants to avoid "dangerous anthropogenic interference". In the TWA the integrated assessment process starts by assessing which impacts of climate change, or mitigation measures, are undesirable. These impacts are then excluded by setting normative constraints, "guardrails" in the language of the TWA. In a subsequent step, the TWA then determines sets of emission reduction strategies that are compatible with the predefined guardrails.

Since guardrails will often be formulated with respect to impacts of climate change, a new representation of impacts was developed for the TWA. In the TWA, impacts can be represented as climate impact response functions (CIRF) (Bruckner et al., 1999; Füssel et al., 2003). CIRFs are derived from climate impact assessments and indicate how

the system under consideration reacts to climate change. Therefore impacts of climate change need not be expressed in monetary terms, as in cost-benefit modelling, but they can rather be expressed in a metric that is suitable to the impact under consideration.

If one considers the chain of cause and effect of climate change in more detail, one has to acknowledge that considerable uncertainty is present in every element of this chain. In IA, uncertainty has so far mostly been considered for *parameter uncertainty*, i. e. uncertainty about climatic processes that can be represented as uncertain parameters in models. Tol (1999), for example, has investigated probability distributions for uncertain parameters in a policy optimisation model, which has also been done by Nordhaus (1994) and Plambeck et al. (1997). Dowlatabadi (2000) and van Asselt and Rotmans (1996) have also investigated parameter uncertainties, with the latter not assuming probability distributions for parameters, but instead investigating the consequences of different cultural perspectives by different actors on the choice of parameters. Finally, Lempert et al. (2000) have investigated the influence of climate *variability* in a policy evaluation model.

With respect to policy guidance modelling, the consideration of uncertainty has been limited so far. Toth et al. (2003b) have presented emission corridors that arise, if parameters in the model or guardrail settings are varied. Similarly Zickfeld and Bruckner (2003) have determined emission corridors for various probabilities of a collapse of the thermohaline circulation, as well as for different climate sensitivities, while Kriegler and Bruckner (2004) have investigated the sensitivity of emission corridors to changes in various parameters. In all of these cases, it was just possible to test certain parameter settings and to derive the different emission corridors arising out of variations of single parameters, but a more comprehensive treatment of uncertainty remains desirable.

While these studies have performed the first steps in considering uncertainty in the TWA, the present study aims at modifying the conceptual framework in order to enable a more comprehensive treatment of uncertainty within an extended TWA formulated in terms of probabilities. The extension of the deterministic TWA to a probabilistic TWA has two advantages. First, the natural variability of climate and impacts can be considered explicitly. Second, in cases of parameter uncertainty, where probability distributions for uncertain parameters are known, the information about these uncertainties can be utilised fully. This allows a further improvement in policy advice applications, since the uncertainty can be considered explicitly, which facilitates its communication.

In Section 2 the deterministic TWA will be reviewed, and the consequences of uncertainty for the TWA will be explored. In Section 3 the

conceptual framework of a probabilistic TWA will be developed, while a first application will be shown in Section 4. The paper will end with a summary and some conclusions in Section 5.

2. The TWA under probabilistic uncertainty

2.1. A simple climate model

The consideration of uncertainty in the TWA has a number of consequences for the original deterministic TWA. In this section we will explore these consequences. We will consider two sources of uncertainty. On the one hand, we will employ probability distributions for climate sensitivity, and on the other hand we will investigate the natural variability in global mean temperature.

For this purpose a simple climate model will be used, which has to be adapted to the question investigated. The climate model employed was originally published in Petschel-Held et al. (1999) and described in more detail by Kriegler and Bruckner (2004). It is a reduced-form climate model with very low requirements with regard to computational resources. These low requirements allow extensive ensemble experiments in order to explore the consequences of probabilistic uncertainty for the TWA. The model describes the climate response to anthropogenic forcing, with CO_2 emissions considered as the only greenhouse gas.

The model consists of a very simple carbon cycle coupled to a temperature equation. The carbon cycle approximates a pulse-response model that has been calibrated against carbon cycle and GCM experiments (Maier-Reimer and Hasselmann, 1987; Hasselmann et al., 1997).

The model is made up of the three differential equations

$$\dot{F} = E \tag{5}$$

$$\dot{C} = \beta E + BF - \gamma C \tag{6}$$

$$\dot{T} = \mu \ln\left(\frac{C}{C_{pi}}\right) - \alpha\left(T - T_{pi}\right) \tag{7}$$

for the cumulative emissions F, the CO_2 concentration C and the global mean temperature T. Inputs and parameters to the climate model are the CO_2 emissions E in GtC, the atmospheric retention factor $B/(\beta\gamma)$, with the CO_2 emission to concentration conversion factor β, and the carbon cycle response parameter γ in Equation 6. In Equation 7, there are the parameters μ and α, and the preindustrial CO_2 concentration

C_{pi} and temperature T_{pi}. The parameters μ and α can be identified as

$$\mu = \frac{Q_{2xCO_2}}{c_{oc} \times \ln 2}, \qquad \alpha = \frac{Q_{2xCO_2}}{c_{oc} \times T_{2xCO_2}} \tag{8}$$

with Q_{2xCO_2} the radiative forcing at a doubling of CO_2, and c_{oc} the effective oceanic heat capacity (Kriegler and Bruckner, 2004). The parameter values used are summarised in Table I, as well as initial (1990) and preindustrial conditions.

Table I. Model parameter values, initial conditions and preindustrial values used in the climate model. Values are set following Kriegler and Bruckner with the exception of c_{oc}, which was changed to reflect IPCC TAR. All values except T_{2xCO_2} are held constant in the ensemble experiments.

Parameter	value	Initial condition	value
β	$0.47\frac{\text{ppm}}{\text{GtC}}$	E_0	$7.9\frac{\text{GtC}}{\text{a}}$
B	$1.51 \times 10^{-3}\frac{\text{ppm}}{\text{GtC a}}$	F_0	426GtC
γ	0.0215a^{-1}	C_0	360ppm
c_{oc}	$43.6\frac{\text{Wa}}{\text{m}^2\text{K}}$	C_{pi}	280ppm
T_{2xCO_2}	3K	T_{pi}	14.6°C
Q_{2xCO_2}	$3.7\frac{\text{W}}{\text{m}^2}$		

In order to be able to consider the sources of uncertainty under investigation, the deterministic model presented above has to be modified to a stochastic formulation in order to simulate the natural variability in global mean temperature. If one considers the global mean temperature T_{GM}, as it is simulated by large 3D GCMs, it becomes apparent that this quantity displays a certain variability. Collins et al. (2001) report that the global mean temperature in the GCM HadCM 3 has a standard deviation of 0.12K, whereas T_{GM} in HadCM 2 had a standard deviation of 0.13K. A stochastic extension to the climate model has therefore been implemented. This extension reproduces the natural variability in global mean temperature T_{GM} shown by HadCM 3.

In order to correctly simulate the natural variability in T_{GM}, Eq. 7 has to be modified to a stochastic formulation

$$\dot{T} = \mu \ln\left(\frac{C}{C_{pi}}\right) - \alpha\left(T - T_{pi}\right) + \sigma\xi. \tag{9}$$

In this equation, the stochastic extension is the term $\sigma\xi$. It consists of a white noise process ξ with standard deviation σ. By using this extension to the original model, the variance of T_{GM} can be reproduced.

The second uncertain element that will be investigated is the uncertainty in climate sensitivity. Considering Eq. 8 it is obvious that a

probability distribution for T_{2xCO_2} results in a probability distribution for α.

2.2. Probabilistic uncertainty

There have been numerous attempts at classifying the uncertainties inherent in the coupled system of humanity and climate. Some of these attempts are based on more general theories and concepts. In control theory, for example, one distinguishes *aleatory* and *epistemic* uncertainty (Paté-Cornell, 1996). This classification can be found in the integrated assessment of climate change, e. g. in publications by Rotmans and van Asselt (2001), who distinguish between internal variability of the system on the one hand, and unknowns on the other hand. On the basis of these coarse categories, one can distinguish different causes of uncertainty, e. g. random chance inherent in natural processes, or the diversity of human values and behaviour.

In another typology, Toth et al. (2003b) distinguish between uncertainties in processes, uncertainty about the predictions of future development, and uncertainty about values and political decisions. This classification of uncertainties is based on the distinction between the different relations to the decision-making process for climate protection strategies.

For the purposes of this study, three types of uncertainty in the integrated assessment of climate change can be distinguished:

1. uncertainty that is caused by natural variability,
2. uncertainty caused by insufficient knowledge, and
3. uncertainty caused by the unpredictability of human society.

The latter uncertainty is in part anticipated by the TWA, since the TWA doesn't try to predict the future development of human society. By determining the set of emission reduction strategies that is compatible with the predefined guardrails it maps the "maneuvering space" humanity has, if certain impacts are to be avoided. Therefore the uncertainty about the future development of human society is considered by not making predictions about it.

The other two causes of uncertainty can be considered in a TWA that is extended to a probabilistic framework. Here the employment of a probabilistic framework may improve on the deterministic TWA.

Uncertainty caused by insufficient knowledge is impossible to consider comprehensively, since unknown factors cannot be represented in models. What can be considered in a practical application is uncertainty

that can be expressed as uncertainty in model parameters. If all that is known about an uncertain parameter is a possible range of values, then a probabilistic approach will not help much in considering it, but if a probability distribution of model parameter values is known, then a probabilistic TWA can help in considering the uncertainty.

This case of uncertainty through insufficient knowledge, as well as uncertainty that arises from natural variability, can also be classified as *probabilistic uncertainty*. Probabilistic uncertainty is the term we are using for uncertainty that either arises through the consideration of natural variability, which leads to a probability distribution for the outcomes of an ensemble of experiments, or uncertainty that can be represented by the consideration of probability distributions for uncertain parameters. While the underlying causes of these two sources of uncertainty may be different, the consideration of them leads to similar experiments and results. Both types of uncertainty can be considered in Monte-Carlo experiments (Press et al., 1997) – in the first case sampling from different realisations of the stochastic process, and in the second case sampling from the probability distribution of uncertain parameters –, and both types of uncertainty lead to similar results for experiments. Experiments do not return a single solution, but a probability distribution of experiment outcomes.

This kind of uncertainty is the domain of the probabilistic TWA. The application of the probabilistic TWA will be demonstrated by considering uncertainty in the climate sensitivity, and by considering uncertainty arising through the natural variability of global mean temperature.

2.3. Uncertainty in climate sensitivity

One of the key uncertain factors in the assessment of changes in climate is the *equilibrium climate sensitivity* T_{2xCO_2}. The equilibrium climate sensitivity, also simply called climate sensitivity, is the change in global mean temperature that results when the climate system, or a climate model, attains a new equilibrium after a forcing change resulting from a doubling of the atmospheric CO_2 concentration (Cubasch et al., 2001). There are various estimates for T_{2xCO_2}.

The estimate of climate sensitivity published by the IPCC is the range $T_{2xCO_2} \in [1.5°\mathrm{C}, 4.5°\mathrm{C}]$ (Cubasch et al., 2001), without any further specification of probability distribution or most probable value. However, a few estimates of a probability distribution for T_{2xCO_2} exist, which were derived by various means, e. g. the estimates by Morgan and Keith (1995), Webster and Sokolov (2000), Andronova and Schlesinger (2001), Gregory et al. (2002), and Forest et al. (2002). Of these dis-

tributions, the ones by Andronova and Schlesinger (2001) and Forest et al. (2002) are considered here.

Andronova and Schlesinger (2001) used a simple climate/ocean model, the observed near-surface temperature record, and a bootstrap technique to objectively estimate a probability density function (pdf) for T_{2xCO_2}. Their climate model was able to consider five different mechanisms for radiative forcing. These were the radiative forcing by all greenhouse gases other than tropospheric ozone, the forcing by tropospheric ozone, the direct and indirect forcing by sulfate aerosols, the forcing by volcanoes, and the changes in forcing due to changes in solar irradiance. They considered 16 different combinations of these forcing mechanisms. For each combination of forcing mechanisms, they determined the changes in global mean near-surface temperature resulting from the forcing mechanisms and compared them to observations. In addition, they considered the uncertainty arising from natural variability by using a bootstrap-resampling approach.

Thus they derived probability distributions for the climate sensitivity T_{2xCO_2}. The probability distributions for the 16 different combinations of forcing mechanisms roughly fall into three classes. The class T1 does not consider the radiative forcing by aerosols, whereas the other two classes do. The T2 and T3 class estimates differ in their consideration of solar forcing. While the T2 class of estimates considers the solar irradiance forcing, the T3 class does not. Since the T1 class of estimates does not consider the aerosol forcing and it's maximum in probability density is at the very low end of the IPCC range, it will not be considered here, but the T2 and T3 class estimates will be considered.

Finally, Forest et al. (2002) derived joint probability distributions for three uncertain properties of the climate system. They used an optimal fingerprinting approach for comparing simulations of a climate model of intermediate complexity with three diagnostics of recent climate observations derived from the upper-air temperature record, the surface temperature record, and the record of ocean temperatures. In climate model simulations, they systematically varied the climate sensitivity, the rate of heat uptake by the deep ocean, and the strength of the anthropogenic aerosol forcing in order to assess, which simulations match the observed climate record. By using a Bayesian updating scheme, they utilised each diagnostic to update the probability distribution for T_{2xCO_2}, starting from either an expert prior distribution or a uniform prior distribution. Both of the posterior distributions published will be considered.

The probability distributions considered are shown in Fig. 1. The estimated probability distributions by Andronova and Schlesinger are shown as a continuous line for the T2 class of estimates and as a dashed

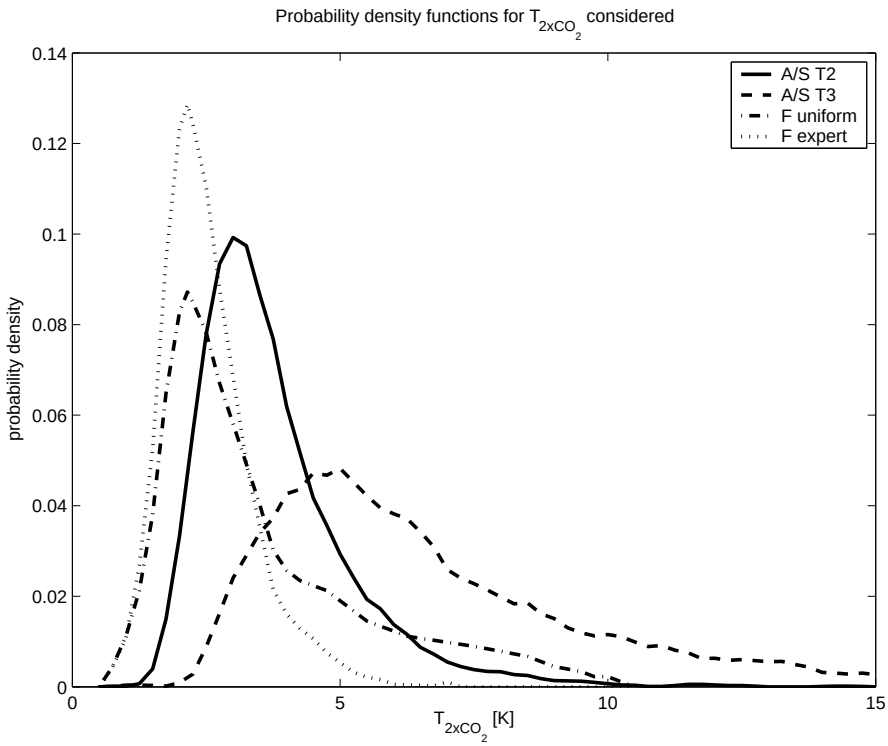

Figure 1. Estimates for climate sensitivity T_{2xCO_2} by Andronova and Schlesinger (A/S T2 and T3), and Forest et al. (F uniform and expert).

line for the T3 class, while the estimates by Forest et al. are shown as a dash-dotted line for the uniform prior and as a dotted line for the expert prior. While both distributions by Forest et al. have the maximum probability density at 2.15K, the maximum in probability density is located at 3.0K for the Andronova and Schlesinger T2 distribution and at 4.75K for the T3 distribution. For the Forest et al. estimates probability density is higher than Andronova and Schlesinger's at low values of T_{2xCO_2}, while it doesn't reach as large values at high T_{2xCO_2}. The pdf generated from a uniform prior assigns higher probabilities to high values of T_{2xCO_2} than the one generated from an expert prior.

The Andronova and Schlesinger T3 distribution gives comparatively high probabilities to high values of T_{2xCO_2}, with values as large as 15K still getting non-zero probabilities. Such high climate sensitivities appear to be quite improbable, but they cannot be ruled out with certainty so far, as was recently shown by Stainforth et al. (2005). Stainforth et al. (2005) performed a large ensemble experiment with a GCM, where they varied a number of uncertain parameters. The

climate sensitivities produced by this ensemble were in a range from 1.9 to 11.5K, highlighting that such high climate sensitivities can also be reproduced by GCMs and cannot be ruled out with certainty.

In the future, it may be possible to narrow the range of possible climate sensitivities by constraining climate sensitivity with proxy data from climate states other than the current, e. g. the last glacial maximum. For example, Schneider von Deimling et al. (2006) report that they can exclude climate sensitivities $> 4.7°$C, since these are inconsistent with current understanding of the climate at the last glacial maximum. As is apparent from a comparison with Fig. 1, these estimates may reduce the uncertainty in climate sensitivity.

2.4. Consequences of uncertainty for the TWA

The presence of probabilistic uncertainty has profound consequences for the conceptual framework of the TWA, as we will explore in the following paragraphs.

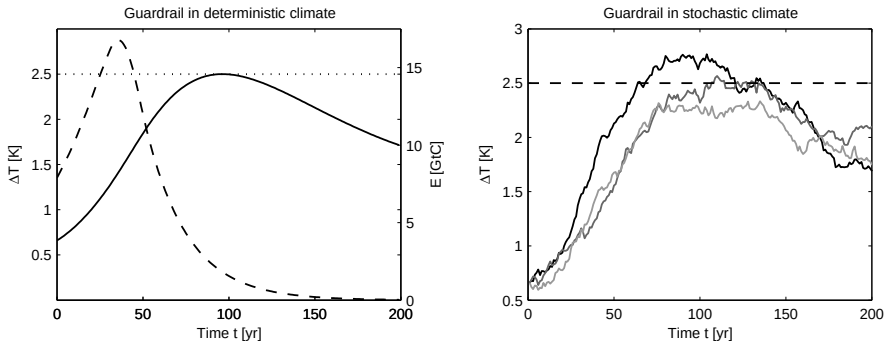

Figure 2. Consequences of natural variability for the TWA. Left: One climate trajectory observing guardrail $\Delta T \leq 2.5$K in deterministic TWA. Shown are change in global mean temperature ΔT (solid line) and CO_2 emissions E (dashed line). Right: Three realizations of the same CO_2 emissions trajectory from a stochastic climate model. While the guardrail is observed in the deterministic system, this depends on the realization of the stochastic process in the stochastic system. Therefore there is some probability of exceeding the guardrail in the stochastic case.

As a reference for comparison, we determined one emission trajectory that would lead to the observation of a temperature guardrail $\Delta T \leq 2.5$K in the deterministic model setup, i. e. the temperature change ΔT in the deterministic model was limited to $\Delta T = 2.5$K.

The emission trajectory is shown in Fig. 2, on the left, along with the corresponding temperature trajectory. The emissions, shown as a dashed line, rise quickly at first, reaching a maximum at time $t = 38$, and are then reduced in an exponential decay. The temperature change

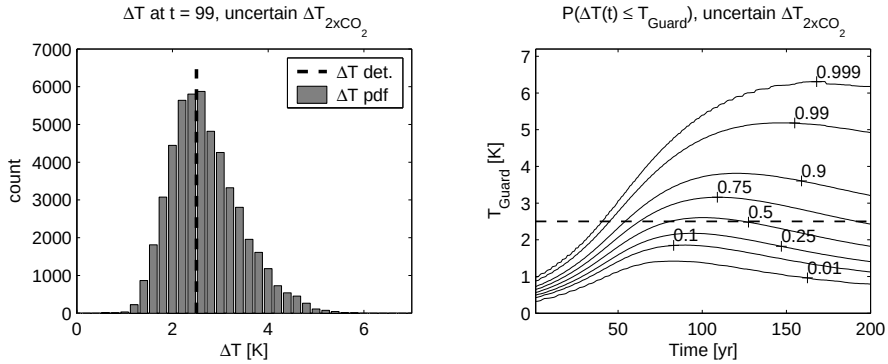

Figure 3. Left: Histogram of temperature change ΔT at time $t = 99$ with T_{2xCO_2} sampled from the Andronova and Schlesinger T2 probability distribution. Right: Cumulative probability of temperature change $\Delta T(t)$ exceeding T_{Guard} over the time horizon of the integration, shown as contours. Climate sensitivity is sampled from the T2 probability distribution by Andronova and Schlesinger, natural variability is not considered.

ΔT, shown as a solid line, also rises initially, until maximum warming is reached at time $t = 99$. Afterwards, temperature falls slowly, but temperature does not reach a stationary state at the end of the time horizon. As a temperature guardrail limiting ΔT to $\Delta T = 2.5$K was set, the maximum temperature change at $t = 99$ is $\Delta T = 2.5$K.

If uncertainty from the natural variability of climate is acknowledged, this situation changes. If the stochastic climate model that reproduces the natural variability of the global mean temperature, as in Equations 5 to 9, is driven by the same emission trajectory, the temperature guardrail will not necessarily be observed. The climate trajectories stemming from three realizations of the stochastic process ξ are shown on the right of Fig. 2. It is obvious that not all realizations of the stochastic process observe the guardrail. While the realization shown in light grey observes the guardrail, the realization shown in black grossly violates the guardrail, and the realization shown in dark grey slightly violates it. This clearly demonstrates that it is dependent on the realization whether the guardrail is observed in the presence of variability. Therefore a certain probability exists, that the guardrail is violated, which can be determined from the cumulative distribution.

A small violation of the guardrail, as in the case of the temperature guardrail shown in Fig. 2, may not be relevant to the larger problem at hand. In the case of the global mean temperature, a slight deviation will probably not be all that important, and the guardrail could also be defined in terms of e. g. ten year averages. On the other hand there are impacts of climate change, where the variability of climatic variables

plays a major role. Mearns et al. (1997) have shown that changes in the variability of temperature and precipitation may strongly affect agricultural yield. Similarly, changes in extreme precipitation events may cause changes in flooding probabilities (Becker and Grünewald, 2003; Booij, 2002; Shabalova et al., 2003). In these cases, the variability plays a major role and therefore needs to be taken into account in guardrail definitions. The need for the consideration of natural variability therefore depends on the impact category under consideration.

The second source of uncertainty we are considering is uncertainty in climate sensitivity. A probability distribution for the climate sensitivity T_{2xCO_2} now leads to a probability distribution for the parameter α in Eq. 7. In order to explore the effect of this uncertainty on the TWA, a Monte-Carlo scheme is employed to sample from the T2 probability distribution estimate by Andronova and Schlesinger (2001).

The climate model is driven by the emission scenario shown in Fig. 2 and the probability distribution is determined for temperature change ΔT at time $t = 99$, which is the time of maximum warming in the deterministic scenario shown in Fig. 2. As shown in Fig. 3, left hand side, the temperature change ΔT varies widely around the temperature guardrail $T_{Guard} = 2.5$K assumed in the deterministic scenario, and most of the probability distribution is located at higher temperatures. The temperature change at the time of maximum warming, which varies with the respective climate sensitivity, ranges from 0.72 K, relative to the preindustrial climate, to a warm 7.27 K reached at $t = 200$ for the largest climate sensitivity in the ensemble.

The consequences of this uncertainty in climate sensitivity for the TWA are profound. Fig. 3, right hand side, shows the cumulative distribution of temperature change ΔT over the time horizon of the integration. The contour plot shows $P\left(\Delta T\left(t\right) \leq T_{Guard}\right)$, the probability of staying below the temperature guardrail T_{Guard}, shown on the abscissa, at time t. According to this figure, the deterministic guardrail of 2.5K has a minimum probability $P \approx 0.44$ of being observed at $t \approx 100$.

Finally, it is also possible to consider both sources of uncertainty by using Monte-Carlo techniques. In this case, one samples from the probability distribution for climate sensitivity and from the realizations of the stochastic process representing natural variability. Since $P\left(\Delta T\left(t\right) \leq T_{Guard}\right)$, the cumulative distribution function, is virtually identical to the one shown in Fig. 3, it is not shown here. In this case the maximum probability of exceeding the deterministic guardrail is about $P \approx 0.44$ at time $t = 97$.

3. The probabilistic TWA

3.1. Probabilistic guardrails

As shown in the last section, the deterministic TWA is not able to fully cope with the uncertainty that is inherent in the climate change problem. The consideration of uncertainty leads to a certain probability that a guardrail will be violated, even though it may be observed in the deterministic case.

In order to address this problem, the TWA therefore has to be extended to a probabilistic TWA. This has two consequences:

1. the conceptual framework of the TWA has to be extended in such a way, that probabilities can be considered, especially with regard to guardrails
2. the model framework and solution algorithms have to be adapted to a probabilistic formulation.

We will address the first point in this section, and we'll come back to solution algorithms in Section 3.2.

Section 2.4 has shown that deterministic guardrails under probabilistic uncertainty lead to a non-zero probability that the guardrail will be violated. Therefore the concept of a guardrail used in the TWA has to be extended in such a way that probabilities can be considered.

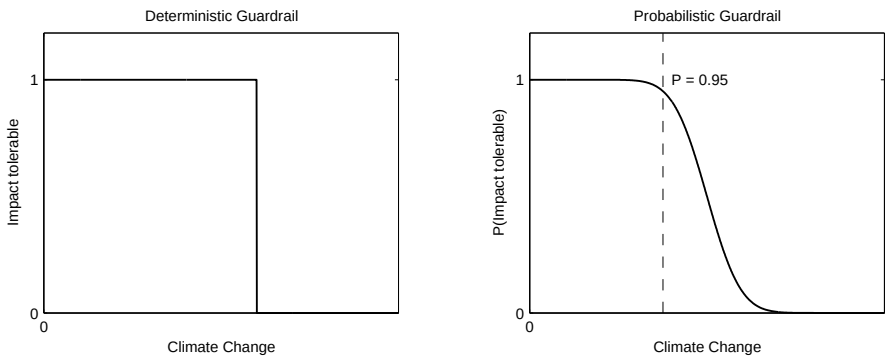

Figure 4. Conceptual visualisation of the relation of climate change and tolerability of impact. Left: deterministic guardrail. Right: probabilistic guardrail.

In the deterministic TWA, a guardrail is envisioned as a binary decision. A decisionmaker decides, which impact level is tolerable and which is intolerable, e. g. Bruckner et al. (1999). The guardrail in the deterministic TWA is then placed at the impact level where the transition between tolerability and intolerability takes place. Such a situation

is sketched in the left hand panel of Fig. 4. In this case the TWA aims to insure $I \leq I_{Guard}$, with I the impact under consideration and I_{Guard} the impact guardrail, which is set, where the tolerability of I changes from 1 to 0. Expressed in terms of probabilities, the deterministic TWA therefore assumes the probability of observing the guardrail to be

$$P\left(I \leq I_{Guard}\right) \in \{0, 1\} : \tag{10}$$

The probability of staying below the guardrail is either zero or one.

If there is uncertainty, whether a certain impact level is tolerable or not, the placement of the guardrail becomes a grave problem. One could either place the guardrail at the highest impact where one is still certain that the impact is tolerable, or one might place the guardrail at the lowest impact where one is certain that the impact is intolerable, or one might place the guardrail somewhere in between. This uncertainty in placing the guardrail may arise out of cognitive uncertainty (the decisionmaker simply doesn't know, what is (in)tolerable), but it also arises if probabilistic uncertainty is considered explicitly. If one looks further at the chain of cause and effect in climate change, this situation could also arise, if the relation between climate change and impact of climate change, the CIRF, becomes uncertain.

One solution to this conceptual problem is the introduction of a probabilistic guardrail. Contrary to a deterministic guardrail, a probabilistic guardrail is not just a single impact level dividing tolerable from intolerable, but it is a tuple of impact level and probability limit. In this situation, the decisionmaker does not just specify I_{Guard}, but also a probability guardrail P_{Guard}, a limit to the probability of reaching a certain impact level. In addition, P_{Guard} could also be derived by determining the different I_{Guard}, where a number of decisionmakers would put the guardrail, and using this information to obtain a probability distribution. This approach could therefore also extend the single-actor perspective currently employed by the TWA.

These new probabilistic guardrails can now be properly expressed conceptually. In the case of probabilistic uncertainty, Eq. 10 becomes

$$P\left(I \leq I_{Guard}\right) \in [0, 1]\,, \tag{11}$$

the probability of observing the guardrail is no longer either zero or one, but it is any value in between. The new probabilistic guardrail can then be formulated as

$$P\left(I \leq I_{Guard}\right) \geq P_{Guard}. \tag{12}$$

The guardrail now consists of the impact limit I_{Guard} and the probability limit P_{Guard}. Please note that the notation is somewhat arbitrary.

Here, we decided to determine $P\left(I \leq I_{Guard}\right)$, and therefore P must be larger than P_{Guard}, which will probably be some comparatively large value, e. g. $P_{Guard} = 0.95$. It could also be done vice versa and would be equally valid, as long as it is done consistently.

This conceptual extension of the guardrail allows the consideration of probabilistic uncertainty, i. e. uncertainty that can be expressed as a probability distribution, and of natural variability.

3.2. Calculation of emission corridors

The last sections have shown how the deterministic TWA can be extended conceptually to enable the consideration of probabilistic uncertainty. In the probabilistic TWA, the guardrail is no longer a simple limit I_{Guard} to an impact I, but the guardrail becomes a combination of impact- and probability limit, which can be expressed as $P\left(I \leq I_{Guard}\right) \geq P_{Guard}$.

Such modifications to the conceptual foundations of the approach also call for a modification of the way solution are determined. We begin this by reviewing the deterministic approach to determining solutions.

There are various possible concepts for what can be considered solutions to the TWA. In the following, it is assumed that the behaviour of the system under consideration can be described by the time evolution of a vector of state variables $\mathbf{x}\left(t\right) \in \mathbb{R}^n$. This vector might, for example, contain global mean temperature, greenhouse gas concentration, gross domestic product and agricultural yield. The time evolution of this vector $\mathbf{x}\left(t\right)$ depends on a vector $\mathbf{u}\left(t\right) \in \mathbb{R}^m$ of control variables. In the climate change problem that is considered here, these are the greenhouse gas emissions, but in principle the approach is of a generic nature, so that any other control variable could be used as well. The evolution of the system can then be modelled as a set of differential equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t; \mathbf{u}), \tag{13}$$

with a state vector $\mathbf{x} \in \mathbb{R}^n$, a control vector $\mathbf{u} \in \mathbb{U} \subseteq \mathbb{R}^m$, and an initial state $\mathbf{x}_0$. The guardrails or constraints can in the deterministic case usually be formulated as a vector of inequalities

$$\mathbf{h}\left(\mathbf{x}, t; \mathbf{u}\right) \leq \mathbf{0}. \tag{14}$$

The goal of the TWA is the determination of the set of all emission strategies $\mathbf{u}\left(\cdot\right)$ that are compatible with the predefined constraints. Mathematically, this problem is equivalent to the differential inclusion (Aubin and Cellina, 1984; Aubin and Frankowska, 1990)

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}, t) \;\; \text{with } \mathbb{F} := \{\mathbf{f}(\mathbf{x}, t; \mathbf{u}) | \mathbf{u} \in \mathbb{U}\} \tag{15}$$

under the condition

$$\mathbf{h}(\mathbf{x}, t; \mathbf{u}) \leq 0 \quad \forall\, t \in [0, t_e] \tag{16}$$

subject to $\mathbf{x}\,(t = 0) = \mathbf{x}_0$, $\mathbb{F} \in \mathbb{R}^n \times \mathbb{R}^m$.

Different concepts exist for definitions of what can be considered a solution to Equations 15 and 16. Following Bruckner et al. (2003), the following solution concepts can be identified.

A single state trajectory $\mathbf{x}\,(\cdot)$ starting from $\mathbf{x}_0$ and fulfilling simultaneously Eq. 13 and 14 is called an *admissible trajectory* driven by a corresponding *admissible control path* $\mathbf{u}\,(\cdot)$. The comprehensive solution to the problem would then be provided by the *bundle of all admissible trajectories* $\mathfrak{S}\,(\mathbf{x}_0)$, which corresponds to a *bundle of admissible control paths*. This bundle of admissible control paths is the set of all emission reduction strategies sought. The actual determination of this bundle is currently not possible, however (Bruckner et al., 2003).

While the focus for the bundles of admissible trajectories / control paths is on the different *trajectories*, the set of *admissible points* in either state or control space can also be determined and is given by

$$\begin{aligned} \Gamma\,(\mathbf{x}_0) &\equiv \{(t, \mathbf{x}\,(t)) \mid t \in [0, t_e]\,, \mathbf{x}\,(\cdot) \in \mathfrak{S}\,(\mathbf{x}_0)\} \\ &\subseteq [0, t_e] \times \mathbb{R}^n. \end{aligned}$$

$\Gamma\,(\mathbf{x}_0)$ is called the *funnel*. It is the set of points one obtains when plotting all admissible trajectories. This approach simplifies the problem, since it is no longer necessary to determine all the admissible trajectories, but only the admissible states, and it is possible to determine the boundary of the funnel without knowing $\mathfrak{S}\,(\mathbf{x}_0)$. It has to be stressed, though, that the funnel contains less information than the bundle. The funnel contains the admissible states, but the information how these states are connected is lost.

Finally, one can select one component of either the state or the control vector and project the funnel onto a plane defined by the time axis and the axis of the respective variable. These projections are called *necessary corridors*. Unfortunately, these corridors do not contain the full information contained in the bundles of admissible trajectories and control paths, but they rather are necessary conditions for trajectories and control paths to be admissible. This implies that every trajectory or control path that leaves the corridor violates at least one of the guardrails and is therefore not admissible, while not every trajectory lying completely within the corridor is necessarily admissible. The fact that an emission path lies completely within the corridor does not insure that none of the constraints is violated. This has to be verified on a case by case basis. While it is possible to derive sufficient subsets of the

emission corridor (Kriegler and Bruckner, 2004), these subsets are not complete, and it is currently not possible to derive complete sufficient subsets.

Since the emissions of CO_2 are the most important control variable in the climate change problem considered here, the typical results of TWA-based analyses are *emission corridors*, i. e. projections of the funnel of admissible emissions on the plane defined by a time and an emission axis. For the case of the deterministic TWA an algorithm for the approximation of emission corridors has been developed (Bruckner et al., 2003). In this case it is sufficient to calculate the upper and the lower boundary of the emission corridor. As a further approximation, this can be done for a finite number of points $t \in \{t_1, t_2, ..., t_n\}$ with $t_n \leq t_e$.

In this case the upper (lower) boundary of the emission corridor can be determined by successively maximising (minimising) the emissions $E(t_i)$ at time t_i subject to the dynamical constraints (Eq. 13) and the additional constraints provided by the guardrails (Eq. 14). The maximal (minimal) emissions $E(t_i)$ are then determined numerically by a constrained optimisation algorithm, such as the algorithms implemented in GAMS or MATLAB.

For the case of the probabilistic TWA, this algorithm can be used as well, with minor modifications. Within the framework described in Equations 13 and 14, two elements can be identified that may be subject to probabilistic uncertainty:

1. The system of differential equations describing the coupled socio-economic-climate system (Eq. 13) is transformed to a system of stochastic differential equations

$$d\xi = \mathbf{f}(\xi, \eta, t; \mathbf{u})\, dt + \mathbf{g}(\xi, \eta, t; \mathbf{u})\, d\mathbf{W}(t) \tag{17}$$

   with a state vector $\xi(t)$, a drift term $\mathbf{f}(\cdot)$, a diffusion term $\mathbf{g}(\cdot)$ and a Wiener process $\mathbf{W}(t)$. The terms may also contain uncertain parameters η.

2. The deterministic constraints (Eq. 14) become stochastic constraints

$$\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0} \quad \forall\, t \in [0, t_e]\,. \tag{18}$$

In this case, the trajectories $\xi(\cdot)$ that solve the system of differential equations (Eq. 17) and still fulfil the constraints (Eq. 18), are the solutions to the stochastic differential inclusion (Aubin et al., 2000)

$$d\xi \in \mathbb{F}(\xi, dt \oplus d\mathbf{W})\,, \qquad \mathbb{F} \in \mathbb{R}^n \times \mathbb{R}^m \tag{19}$$

with

$$\mathbb{F}(\xi, dt \oplus d\mathbf{W}) := \{\mathbf{f}(\xi, \eta, t; \mathbf{u})\, dt + \mathbf{g}(\xi, \eta, t; \mathbf{u})\, d\mathbf{W} \mid \mathbf{u}(t) \in \mathbb{U}(\mathbf{x})\}$$

under the constraint condition

$$P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0}) \geq P_{Guard}. \tag{20}$$

This probabilistic constraint condition limits the probability P of observing the guardrail to the limiting probability guardrail P_{Guard}.

Similar to the deterministic case it will in general not be possible to determine an exact solution, i. e. the *bundle of control paths*, to this problem, but the algorithm for approximating the *emission corridor* can be adapted to the probabilistic problem. As in the deterministic case a numerical implementation of the model describing the evolution of the coupled system is a prerequisite to the determination of emission corridors. Depending on the nature of the problem, this may either be a deterministic formulation as in Eq. 13 or a stochastic implementation as in Eq. 17.

For considering the probabilistic constraints, the probability $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ has to be determined by some method, e. g. by using Monte-Carlo techniques. If one considers a probabilistic formulation for the dynamical system, such as in the examples shown in Section 2.4, $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ can be determined by calculating the time evolution of an ensemble of realisations of either the climate sensitivity or the stochastic process ξ (or both). If, on the other hand, the guardrails in Eq. 14 become probabilistic, while the dynamical system itself remains deterministic, then an ensemble of realisations of the process considered in the guardrail can be used to determine $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$.

For the determination of the emission corridors that will be shown in Section 4, both a deterministic and a stochastic version of the simple climate model, as in Eq. 5-9, have been implemented. In order to consider a probability distribution for the climate sensitivity, the deterministic version is used and an ensemble of model configurations is generated by sampling from the probability distribution for T_{2xCO_2}. $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ can then be determined from the frequency of experiment outcomes. For the consideration of natural variability, on the other hand, the stochastic version is used, and an ensemble of realisations of the stochastic process is generated. $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ can again be determined from the frequency of experiment outcomes.

The consideration of both sources of uncertainty then becomes a straightforward task: the stochastic version of the model is used, and an ensemble of of model configurations is generated by sampling from the pdf for T_{2xCO_2}. This ensemble then samples from the realisations

of the stochastic process, and $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ is once again determined from the experiment outcomes. In this case, as in all cases where multiple sources of uncertainty are considered, care must be taken in choosing an appropriate sampling strategy to insure that the uncertainty is properly taken into account.

The algorithm described above has been implemented in MATLAB using the constrained optimisation routine provided. As in the deterministic case, the emissions $E(t_i)$ are maximised (minimised) at times $t_i \in \{t_1, t_2, ..., t_n\}$ for the determination of the upper (lower) boundary of the emission corridor. In Section 4 the probability of observing a temperature guardrail T_{Guard} is considered as a climate constraint, therefore $P(\Delta T \leq T_{Guard})$ is determined by sampling from the probability distribution for climate sensitivity *and* by sampling from the realisations of the stochastic process as described above.

In the case of the very simple system considered here, a more elegant solution to the determination of $P(\mathbf{h}(\xi, \eta, t; \mathbf{u}) \leq \mathbf{0})$ could be found, e. g. by employing a climate model expressed as a Fokker-Planck equation. Our aim here was to develop the conceptual framework of a probabilistic TWA, though, and therefore a method that can be applied to a wide range of problems was used. In addition, the consideration of different realisations also allows the propagation of uncertainty through the chain of cause and effect in climate change, which is in most cases not possible using analytical solutions.

4. Emission corridors in the probabilistic TWA

The uncertainties considered, uncertainty in climate sensitivity and natural variability in global mean temperature, lead to a probability distribution for the warming realised under a defined greenhouse gas forcing scenario. In this section, emission corridors will therefore be determined, limiting the temperature change to a temperature guardrail T_{Guard} that has to be observed with a probability $P(\Delta T \leq T_{Guard}) \geq P_{Guard}$ larger than or equal to the probability guardrail P_{Guard}.

Following Kriegler and Bruckner (2004), additional guardrails are set for these corridors. The change in emissions is parametrised as $\dot{E} = gE$, and the maximal emission reduction is limited to 4% p.a., as large emission reductions may be very costly or even impossible to obtain. Second, the rate of change in emission reduction is limited, as a certain inertia in the socio-economic system has to be assumed. We are assuming a transition timescale of $t_{trans} = 20$ yrs from the initial rate of change in emissions g_0 to the maximal emission reduction $g_{max} = -0.04$. It is also assumed that the growth rate in emissions does not rise again after

emission reductions have started, for plausibility reasons. The latter two constraints can be summarised as $0 \leq \dot{g} \leq -(g_0 + g_{max})/t_{trans}$. The initial rate of change in emissions g_0 is determined by the optimisation algorithm, but bounded to be between 1% p.a. and 3% p.a., the range of the late 20th century rise in emissions.

The probability guardrail considered has a large influence on the overall size of emission corridors. Fig. 5, upper left, shows the consequences of differing limits to the probability of exceeding the temperature guardrail. The emission corridor is shown as shaded area. Please note that the corridors shown here are additive, in the sense that the larger corridors consist of the total area between the upper boundary of the corridor and the lower boundary of the shaded area in the plot. Here, the temperature guardrail is set to $T_{Guard} = 3\text{K}$, and corridors are derived for probabilities of observing the temperature guardrail of $P(\Delta T \leq T_{Guard}) \geq P_{Guard} = 0.97$, 0.9, 0.7 and 0.5. Climate sensitivity is sampled the Andronova and Schlesinger T2 distribution. It is obvious that the emission corridor shrinks for higher probability guardrails. While a probability guardrail $P_{Guard} = 0.5$ allows a maximum in emissions of nearly 20 GtC, $P_{Guard} = 0.97$ allows less than 9 GtC.

Another important question is the influence of the probability distribution for climate sensitivity on the emission corridors. For guardrails $T_{Guard} = 3\text{K}$, $P_{Guard} = 0.9$, this is shown in Fig. 5, upper right. In this case emission corridors were obtained for all the pdfs considered. As could be expected after considering the pdfs shown in Fig. 1, the Forest et al. pdf from an expert prior yields the largest emission corridor, with maximal emissions of about 17.5 GtC allowed, while the A/S T2 and the Forest et al. uniform pdfs yield viable emission corridors, with a maximum of about 12.5 GtC and 9.4 GtC allowed, respectively. The most interesting case is the A/S T3 estimate, shown as a dotted line in Fig. 5. This dotted line shows the hypothetical upper boundary of the emission corridor, but since the upper boundary is located *below* the lower boundary, the emission corridor is an empty set. If the A/S T3 estimate had to be assumed for the probability distribution of climate sensitivity, it would therefore be impossible to keep climate change below 3K with a high probability of not exceeding this value. Compared to the other estimates, the high probability of high values for climate sensitivity leads to a low probability of observing the $T_{Guard} = 3\text{K}$ guardrail.

The main difference between the Forest et al. estimates from a uniform and an expert prior, as shown in Fig. 1, is that the distribution generated from a uniform prior has a heavy tail, i. e. comparatively high probabilities for high values of climate sensitivity, even though the

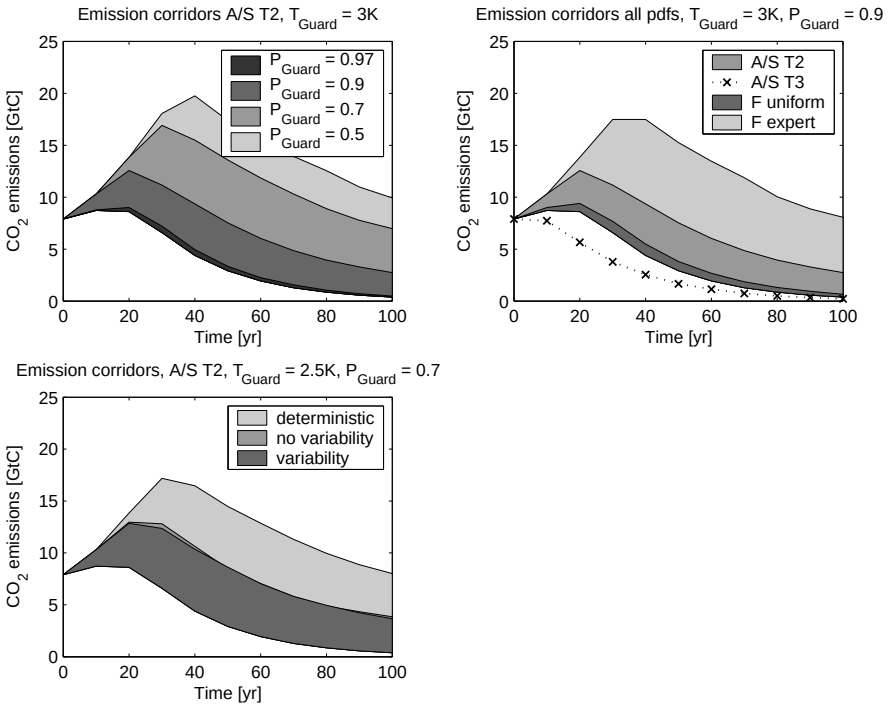

Figure 5. Upper left: Emission corridors for temperature guardrail $T_{Guard} = 3$K and probabilities $P(\Delta T \leq T_{Guard}) \geq P_{Guard} = 0.97$, 0.9, 0.7 and 0.5. Climate sensitivity is sampled from the A/S T2 distribution. Upper right: mission corridors for $T_{Guard} = 3$K, $P_{Guard} = 0.9$, all probability distributions considered. Lower left: Comparison of emission corridors. Temperature guardrail is $T_{Guard} = 2.5$K, probability guardrail is $P_{Guard} = 0.7$. Shown are deterministic corridor with $T_{2xCO_2} = 3$K, probabilistic corridor with no consideration of natural variability, and probabilistic corridor with consideration of natural variability and uncertainty in T_{2xCO_2} (A/S T2 estimate).

maximum in probability density is located at the same value of T_{2xCO_2}. The consequence of this difference is a dramatically smaller emission corridor available in the case of the heavy tailed distribution.

For comparison, Fig. 5, bottom left, shows emission corridors for the deterministic case, as well as for the probabilistic case (based on the A/S T2 estimate) with and without consideration of natural variability in. The corridor for the deterministic case was derived for a climate sensitivity $T_{2xCO_2} = 3$K, and the guardrail settings were a temperature guardrail $T_{Guard} = 2.5$K with a probability guardrail $P_{Guard} = 0.7$. The deterministic case yields a much larger corridor, but the size of this corridor is very sensitive to the choice of climate sensitivity. The difference between the probabilistic corridors, on the other hand, is very small, with the corridor that considers natural variability slightly

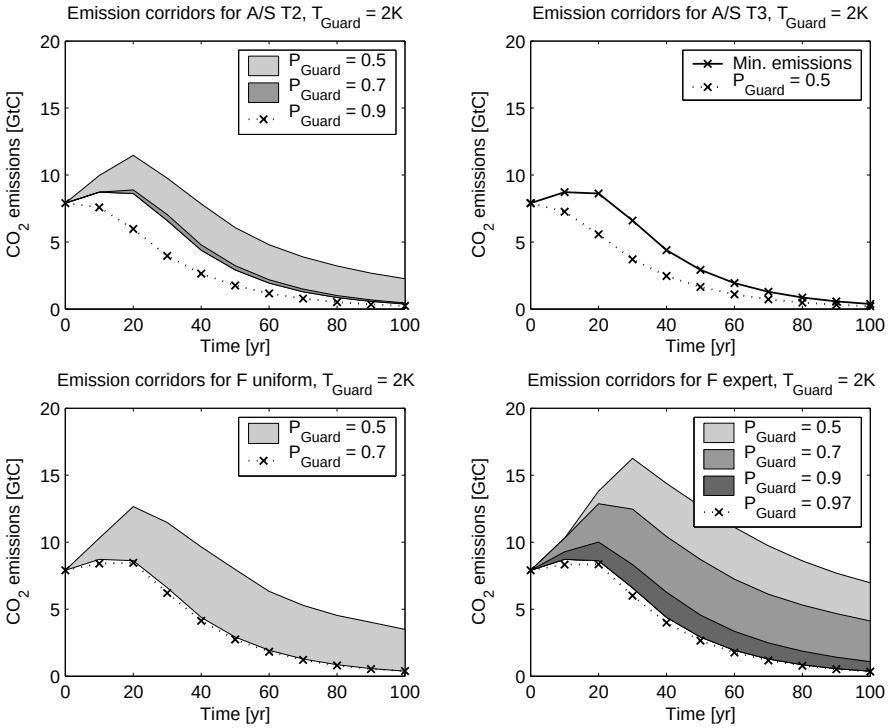

Figure 6. Emission corridors for a climate protection target $T_{Guard} = 2K$ for all probability distributions considered. Probability guardrails P_{Guard} are shown in legend.

smaller than the one that does not. Therefore the consideration of the uncertainty in climate sensitivity appears to be more important than the consideration of natural variability in this case, but this is very much dependent on the problem under consideration. As soon as guardrail settings other than limits to the global mean temperature change are considered, the natural variability may turn out to be the main factor.

Finally, the matter of emission corridors limiting temperature change to 2°C remains an interesting question. A climate protection target of limiting temperature change to 2°C above the preindustrial climate was proposed by the German advisory council on climate change in 1995 (WBGU, 1995), and this target was later adopted by the European Union. Fig. 6 shows emission corridors for a guardrail setting $T_{Guard} = 2K$ for all the probability distributions considered. For each probability distribution, the emission corridors for all probability guardrails up to the lowest setting, where the corridor was an empty set, were determined. The figure therefore allows a comparison of the consequences of the probability distributions for the 2°C climate protection target.

If A/S T2 is the "real" probability distribution, the target can be met with a probability $P = 0.7$, while it cannot be met for the A/S T3 distribution. Similarly, the Forest et al. distribution from a uniform prior yields just a $P_{Guard} = 0.5$ corridor, whereas the distribution from an expert prior yields viable corridors up to $P_{Guard} = 0.9$. Concentrating on the A/S T2 and the Forest et al. expert distributions, it becomes apparent that emission corridors that give high probabilities of staying within the temperature guardrail are quite small. Therefore emission reductions will have to happen soon, unless we are willing accept a non-negligible probability of violating the climate protection target. On the other hand, a probability guardrail $P_{Guard} = 0.9$ implies that there still is a probability $P = 0.1$ that the guardrail will be violated. Therefore even emission reduction strategies conforming to the most ambitious corridor determined do not insure that targets will be met with certainty.

5. Summary and conclusions

In this article we have demonstrated how the "traditional" deterministic tolerable windows approach can be extended to a probabilistic TWA. This extension improves the deterministic TWA because it allows the consideration of probabilistic uncertainty, i. e. uncertainty that can be expressed as a probability distribution or that arises through natural variability.

This extension of the TWA involves changes to the modelling framework and solution algorithms, but most important of all is a different understanding of guardrails. While guardrails in the deterministic TWA are single values dividing tolerable impacts from intolerable, a probabilistic TWA forces us to also consider a probability limit. Therefore the guardrail now involves two numbers, not one: An impact guardrail and a probability guardrail. The impact guardrail is – as before – an impact level that is considered a boundary that divides tolerable impacts of climate change from intolerable impacts, but in addition we need to specify a probability guardrail that specifies the minimum probability of staying below the impact guardrail that the policymaker is willing to accept.

This conceptual change is more important than it may appear, because at the current state of climate change science there is very little certainty. Therefore it just isn't possible to exclude impacts of climate change with certainty, but the maximum one can hope for is a certain probability for having excluded the impact of climate change one is

concerned about. Scientific policy advice will therefore gain from the explicit consideration of uncertainty.

We were able to demonstrate the probabilistic TWA by determining emission corridors that limit the change in global mean temperature to 2, 2.5 and 3K, with various probabilities of observing the guardrail. For this the uncertainty in climate sensitivity was included by considering various estimates of probability distributions for climate sensitivity, and natural variability was also included as an additional source of uncertainty.

In general, emission corridors shrink, if uncertainty is considered and higher probabilities of observing the guardrail are enforced. The higher the probability of observing the guardrail, the smaller the corridor. This may be obvious to the reader who has already given some thought to this problem, but the finding is important enough to be repeated here.

While the guardrails used here may not be the most interesting – or the most relevant – ones, this article serves as an illustration of the conceptual framework. Applications of the probabilistic TWA to more pressing issues will surely follow, since the groundwork has now been laid.

One observation with respect to the emission corridors shown needs to be made, though. The European Union has repeatedly stated a goal of limiting global warming to 2°C above preindustrial. Fig. 6 shows emission corridors for a temperature guardrail $T_{Guard} = 2$K and all the probability distributions considered. Depending on the probability distribution of climate sensitivity, this goal can be met with varying probabilities of staying within the tolerable window, but high probabilities can only be assured if the probability distribution is one of the more benign ones. In addition, strong efforts to curb global warming have to made soon, since the emission corridors, the "maneuvering space" of humanity, are comparatively small.

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