

# Prices vs. Quantities and the Intertemporal Dynamics of the Climate Rent

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*Comments Welcome*

## **Abstract:**

This paper provides a formal survey of price and quantity instruments for mitigating global warming. The recent work of Sinn (2008) has started a discussion about the performance of price and quantity instruments when suppliers of exhaustible resources can act strategically. We explicitly consider policies' impact on the incentives of resource owners who maximize their profits intertemporally. We focus on the informational and commitment requirements of the regulator. Furthermore, we study the interplay between (private) resource extraction rent and (public) *climate rent* and ask how property and management of the climate rent can be assigned between regulator and resource sector. There are only two instruments that unburden the regulator from the complex intertemporal management of the climate rent and associated commitment problems: in the cost-benefit world, we derive a stock-dependent tax rule; in the cost-effective (carbon budget) world, only an emissions trading scheme with free banking and borrowing can shift intertemporal timing decisions completely to the market. If risk premiums are added to the discount rate due to insecure property rights of resources or permits, however, emission trading schemes without intertemporal flexibility turn out to be the most robust instrument.

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## 1 Introduction

There has been much research about policy instruments to overcome global warming as the “greatest market failure” of mankind (Stern, 2007). A main stand of a successful climate policy is seen in pricing global emissions. This price signal can be obtained by taxes or quantity instruments like emission trading schemes (ETS). While both instruments are equivalent in an idealized world of perfect information, the symmetry breaks down when a social planner is confronted with uncertainties in marginal costs and marginal benefits (Weitzman, 1974). Newell and Pizer (2003) analyze the comparative advantage of price instruments over quantity instruments for stock pollution problems like global warming. The comparative advantage of price instruments reverses over time when damage functions become steeper due to accumulated stock-pollutants. They conclude that in the short-run tax policies are superior to quantity instruments; in the long-run quantity instruments are more efficient when climate damages become more severe.

Beside these approaches where a social planner plays against the uncertainty of nature, one of the most challenging problems seems to be the management of intertemporal carbon pricing when owners of the exhaustible resources can act strategically. Sinn (2008) suggests with his ‘green paradox’ that carbon taxes are likely to fail to achieve socially optimal emission paths - even if such taxes are imposed globally and, hence, cover all countries. By linking the problem of global warming to the intertemporal extraction problem of fossil resources, he showed within a simple Hotelling model that increasing resource taxes can lead to an acceleration of resource extraction which worsens global warming. The green paradox occurs because resource owners fear a devaluation of their resource rent by future ‘green policies’. Sinn (2008) emphasizes that quantity instruments are superior to price instruments even in a deterministic setting due to strategic behavior of the suppliers of fossil fuels.

The work of Sinn constitutes an important change from the demand perspective to the intertemporal supply perspective in the context of global warming. In contrast to existing works on resource extraction and global warming that focus on a social planner perspective (eg. Hoel and Kverndokk, 1996; Farzin, 1996) we explicitly consider the incentive, information and rent structure of this optimization problem. We go beyond Sinn’s analysis by providing a systematic comparison of optimal price and quantity instruments. In particular, we draw on the literature on the intertemporal management of exhaustible resources (eg. Hotelling, 1931; Dasgupta and Heal, 1979; Dasgupta et al., 1981) and intertemporal emissions trading (eg. Kling and Rubin, 1997; Leiby and Rubin, 2001) when exploring designs of efficient and effective climate policy instruments in presence of strategic behavior on the resource supply side.

We discuss several Hotelling-like models from a social planner and decentralized market perspective. The social planner model serves as a benchmark for the socially optimal solution. In the decentralized model, we study the strategic reaction of the resource sector that anticipates the policy instrument of the regulator and its implication on the intertemporal resource rent. As it turns out, one crucial aspect for effective climate policy is the creation and distribution of dynamic economic rents arising from increasing damages and environmental scarcities. The main policy design decision of the regulator concerns the choice between implementing a price or quantity instrument. Due to the intertemporal dynamics of the extraction–pollution problem, the regulator usually has to commit *ex ante* to a tax path or quantity path for the entire time horizon. We analyze, whether there are instruments that require less commitment and information about optimal extraction paths for the entire time horizon. In particular,

we will introduce and discuss taxes that depend on the stock size of resource owners. Furthermore, we consider the robustness of price and quantity instruments if (from a social perspective) suboptimal risk premiums are added to the discount rates. Insecure property rights in fossil resources and incomplete future markets provoke resource owners to use a higher effective discount rate than the representative household (eg. Sinn, 2008).

The remainder of the paper is structured as follows: Section 2 starts with an analysis of optimal instruments within a cost-benefit analysis. Section 3 provides a similar analysis of cost-effective instruments within a defined environmental target – a so-called ‘carbon budget’ – which has high relevance for the policy arena. Finally, we close the discussion with a brief summary of the main findings and the implications for the scientific debate about optimal policy instruments.

## 2 The Cost-Benefit-Approach

The analysis in this section is based on the modified Hotelling model presented in Sinn (2008). Production  $f(R)$  is based on the extraction of fossil resources  $R$  from a (finite) resource stock  $S$  which can be exploited with marginal extraction costs  $c(S)$ .<sup>1</sup> We use the common assumption that production is increasing and concave in  $R$ , i.e.  $f_R > 0$  and  $f_{RR} < 0$ .<sup>2</sup> As easily accessible resource sites are exploited first, we assume that extraction costs rise with depletion and are convex, thus  $c_S < 0, c_{SS} \geq 0$ . In order to focus on the supply side, we neglect decay rates of carbon dioxide in the atmosphere and carbon dioxide storage technologies. We assume that by burning fossil resources a proportional amount of carbon dioxide is emitted into the atmosphere and, thus, we describe damages  $d(S)$  as function of cumulative extraction of fossil fuels. Furthermore, damages increase with the amount of carbon in the atmosphere which is proportional to cumulative extraction, implying  $d_S < 0$ . We abstract from considerations about private or social scrap values of the resource stock in the final period  $T$  in the business-as-usual world (without climate damages). Irreversible and persistent damages of global warming, however, can be considered by the social scrap value function  $F(S(T))$ ,  $F_S \geq 0$  which is not considered by individual resource owners. In this paper, we always assume that fossil reserves are abundant in the sense that they are not fully extracted within the planning horizon. This can be justified by convex marginal extraction costs (Farzin, 1992; Hoel and Kverndokk, 1996) or by the relatively abundant resources of fossil carbon in the ground (eg. BGR, 2009) compared to the expected consumption within the planning horizon relevant for policy making (about some decades or one century).

**Assumption 1.** *The stock of fossil resources is not fully extracted within the planning horizon, i.e.  $S(T) > 0$ .*

This set of assumptions helps to clarify and highlight the supply-side dynamics by pointing out the intertemporal dimension of the control problem.

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<sup>1</sup>To improve the readability of this paper, we will usually suppress the time-dependency of flow and stock variables like  $R(t)$ ,  $S(t)$  and so on.

<sup>2</sup>In the following, we use the notation  $g_x$  for the partial derivative of  $g$  with respect to  $x$ , thus:  $g_x := \frac{\partial g(x)}{\partial x}$ . Furthermore, we denote with  $\dot{g} := \frac{dg}{dt}$  the derivative of  $g$  with respect to time.

## 2.1 The model

### 2.1.1 The social planner's problem

The social planner maximizes the net present value of output  $f(R)$  minus extraction costs  $c(S)R$  and damages  $d(S)$  with respect to the discount rate  $r$ . The optimization problem with scrap value function  $F(S(T))$  and initial resource stock size  $S(0) = S_0$  reads:

$$\max_R \int_0^T (f(R) - c(S)R - d(S)) e^{-rt} dt + F(S(T))e^{-rT} \quad (1)$$

subject to:

$$\dot{S} = -R \quad (2)$$

$$S(0) = S_0 \quad (3)$$

The solution of the intertemporal optimization problem is characterized by the following proposition:

**Proposition 1.** (*Socially optimal resource extraction*) *If a social planner maximizes intertemporal output according to (1–3), then:*

(a) *the optimal solution  $(R^*, S^*)$  is determined by the following system of equations:*

$$r = \frac{\dot{f}_R - d_S}{f_R - c(S)} \quad (4)$$

$$\dot{S} = -R \quad (5)$$

$$F_S(S(T)) = f_R(R(T)) - c(S(T)) = \lambda(T) \quad (6)$$

$$S(0) = S_0 \quad (7)$$

(b) *the shadow price  $\lambda$  for the stock  $S$  is given by:*

$$\lambda(t) = F_S(S(T))e^{-r(T-t)} - \int_t^T (c_S R + d_S) e^{r(t-\xi)} d\xi \quad (8)$$

*Proof.* (a) We set up the corresponding Hamiltonian function  $H = f(R) - c(S)R - d(S) - \lambda R$ . Application of the maximum principle then leads to the first-order condition with respect to  $R$ , the equation of motion for the shadow price  $\lambda$ , and the transversality condition:

$$\lambda = f_R - c(S) \quad (9)$$

$$\dot{\lambda} = r\lambda - H_S = r\lambda + c_S R + d_S \quad (10)$$

$$0 = (\lambda(T) - F_S(S(T)))S(T) \quad (11)$$

By substituting (9) and its derivative with respect to time into (10) we obtain the social Hotelling rule (4). Furthermore, the transversality condition (11) together with Assumption 1 implies that  $\lambda(T) = F_S(S(T))$ .

(b) Solving the differential equation (10) for given  $\lambda(T)$  yields:

$$\lambda(t) = \lambda(T)e^{-r(T-t)} - \int_t^T (c_S R + d_S) e^{r(t-\xi)} d\xi \quad (12)$$

□

For a zero scrap value function ( $F(S(T)) \equiv 0$ ), Proposition 1 implies that marginal extraction costs increase up to marginal resource productivity. If the marginal scrap value is positive ( $F_S(S(T)) > 0$ ), however, resources in the ground are additionally valued when the final period has been reached. This may be the case if society considers persistent and irreversible damages due to resource extraction after the planning period  $T$ .

Equation (8) resembles the well-known rent dynamic for exhaustible resources with stock-dependent extraction costs (eg. Farzin, 1992). However, the familiar formula is extended by the term  $d_S$  under the integral reflecting the stock-pollutant dynamics of resource extraction and the marginal scrap value term  $F_S(S(T))$ . As we will show below, this rent dynamic has to be reproduced by policy instruments in order to achieve an optimal decentralized solution.

### 2.1.2 The decentralized resource sector's problem

The resource sector takes resource prices  $p(t)$  and resource taxes  $\tau(t)$  as given and maximizes intertemporal profit according to:

$$\max_R \int_0^T (p - c(S) - \tau)R e^{-rt} dt \quad (13)$$

subject to:

$$\dot{S} = -R \quad (14)$$

$$S(0) = S_0 \quad (15)$$

In contrast to the social objective function (1), the resource sector does not consider social damages due to extraction during and after the planning horizon.

By applying the maximum principle with  $\lambda$  as shadow price for the resource stock, we obtain (just along the lines in the proof of Proposition 1):

$$0 = p - c(S) - \tau - \lambda \quad (16)$$

$$\dot{\lambda} = r\lambda + c_S R \quad (17)$$

$$0 = \lambda(T)S(T) \quad (18)$$

which leads to the private Hotelling rule and terminal condition:

$$r = \frac{\dot{p} - \dot{\tau} + r\tau}{p - c(S)} \quad (19)$$

$$\tau(T) = p(T) - c(S(T)) \quad (20)$$

because  $S(T) > 0$ .

The crucial question is how to bring the private extraction dynamics in accordance with the socially optimal extraction as described in Proposition 1. Clearly, the social and private Hotelling rules diverge when the tax  $\tau(t) \equiv 0$  and marginal damages exist. It is therefore the task of a government to tax the resource sector such that the social and private Hotelling rules coincide, thus reproducing the social planner equilibrium.

## 2.2 Optimal resource tax

By equating private and social Hotelling rule, we find the optimal resource tax.

**Proposition 2.** (*Optimal resource tax*) *If the regulator knows the socially optimal extraction path  $S^*$  according to Proposition 1 and if she can commit at  $t = 0$  to the tax path  $\tau(t)$  over the entire planning horizon, then*

(a) *the resource tax*

$$\tau(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi \quad (21)$$

*achieves the optimal extraction path;*

(b) *the rent in the resource sector is given by:*

$$\lambda(t) = - \int_t^T c_S^* R^* e^{r(t-\xi)} d\xi \quad (22)$$

*Proof.* (a) Differentiating (21) with respect to time, we get  $\dot{\tau} = r\tau - d_S^*$ . Substituting this into the private Hotelling rule (19) and considering the fact that in the market equilibrium prices equal marginal productivities, i.e.  $p = f_R$ , we obtain the socially optimal Hotelling rule (4). Furthermore,  $\tau(T) = F_S(S^*(T))$  ensures that the private transversality condition (20) equals the social transversality condition (11).

(b) The equation for  $\lambda$  follows from the solution of the differential equation (17) with  $\lambda(T) = 0$  due to  $S(T) > 0$ .  $\square$

Note that the sum of (private) resource rent  $\lambda$  and resource tax  $\tau$  describes the entire rent dynamics and is expressed by:

$$\tau(t) + \lambda(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T (c_S^* S^* + d_S^*) e^{r(t-\xi)} d\xi \quad (23)$$

which is exactly the resource shadow price in the social planner model as expressed in Eq. (8). The first summand denotes the (cumulative) scarcity of resources due to high stock externalities ( $F_S(S^*(T)) > 0$ ). The second summand describes the dynamics of extraction costs and climate damages. Hence, the optimal resource tax can be decomposed into a pure resource extraction rent  $\lambda$  and a stock externality rent  $\tau$ . In the following, we will call this rent component also *climate rent* as it evolves due to the stock-pollutant dynamics  $d_S$  and the cumulative scarcity by future damages expressed in  $\tau(T) = F_S(S(T))$ .

Hoel and Kverndokk (1996) find a similar result within a social planner model. However, they use an infinite time horizon and assume extraction costs that rise without bound implying that the optimal tax converges to zero in the long run:  $\lim_{T \rightarrow \infty} \tau(T) = 0$ . Within the infinite time horizon, such a resource tax aims at reallocating resource extraction, shifting it towards the future – within the finite time horizon, it is in addition necessary to limit cumulative extraction – at least when  $F_S(S(T)) > 0$ .<sup>3</sup>

Proposition 2 confirms that  $\tau$  in fact is incentive-compatible in a decentralized economy as suggested by the social planner model of Hoel and Kverndokk (1996): The tax attains that intertemporally maximizing resource owners adjust their extraction

<sup>3</sup>As consideration of climate damages leads to a slower extraction (Sinn, 2008), cumulative extraction at each point of time is lower than in the business-as-usual case.

path to the social optimum. However, this kind of tax requires extensive amounts of information as well as a great ability to commit on the regulator's part, both of which are difficult to achieve:

- Calculating the optimal tax requires a full assessment of social costs and benefits of fossil resource extraction from now until forever, as the regulator has to calculate marginal damages  $d_S^* = d_S(S^*(t))$  along the entire socially optimal resource stock path  $S^*(t)$ .
- Furthermore, the regulator would have to commit to this tax for now and forever to incentivize the resource sector correctly.

Thus, the informational and commitment requirements for the regulator are quite unrealistic which makes deviations from the social optimum very likely. As the tax is always positive and increasing in the beginning (Hoel and Kverndokk, 1996), an incorrect tax may lead to an acceleration of extraction if the tax growth rate is high and the initial tax level is too low (Edenhofer and Kalkuhl, 2010). Hence, only a wrongly calculated tax could provoke Sinn's green paradox (Sinn, 2008).

### 2.3 Optimal stock-dependent resource taxes

Usually, regulators cannot and do not commit *ex ante* to a time-dependent tax path  $\tau(t)$  for large time horizons. Instead, regulation is more an iterative process where the resource tax is dependent on the estimation of marginal damages from the cumulative resource extraction. When concentrations rise, the regulator increases the tax in order to price in higher social damages.

In this section we ask whether the regulator can achieve the optimal extraction path by implementing a resource tax  $\tau(S)$  which is adjusted to the current concentration of carbon in the atmosphere. The regulator announces explicitly how she modulates the tax and the resource sector responds to this tax adjustment rule.

**Proposition 3.** (*Stock dependent tax*) *If the regulator imposes the resource tax*

$$\tau(S) = \frac{-d_S(S)}{r} \quad (24)$$

*which depends explicitly on the cumulative amount of extracted resources  $S$ , then*  
 (a) *if there are  $n > 1$  resource owners, the tax rule (24) leads to a steeper (flatter) resource price path compared to the optimal extraction if damages are convex (concave). The private Hotelling rule is as follows:*

$$r = \frac{\dot{p} - d_S - \frac{d_{SS}}{r} \sum_{j=1, j \neq i}^n R^j}{p - c^i(S^i)} \quad (25)$$

*where  $R^i$  and  $S^i$  denote the resource flow and stock of the  $i$ -th resource owner.*

(b) *The socially optimal Hotelling rule is achieved if there is only one (competitive) resource owner. In order to meet the socially optimal transversality condition, the regulator has furthermore to commit to the terminal-period payment rule  $\varsigma(S(T))$*

$$\varsigma(S(T)) = \frac{d(S(T))}{r} - F(S(T)) \quad (26)$$

The combined rent and tax dynamics is as follows:

$$\lambda(t) + \tau(t) = F_S(S(T))e^{-r(T-t)} - \int_t^T (c_S^* R^* + d_S^*)e^{r(t-\xi)} d\xi \quad (27)$$

*Proof.* (a) Formula (25) is derived in Appendix A.1. If damages are convex ( $d_{SS} > 0$ ), the existence of more than one resource owner (i.e.  $\sum_{j=1, j \neq i}^n R^j \neq 0$ ) makes the price path steeper compared to the socially optimal Hotelling rule. If damages are concave ( $d_{SS} < 0$ ), the price path is flattened even more and resource extraction becomes too conservative. Only if damages are linear in  $S$  ( $d_{SS} = 0$ ), the optimal price path is achieved.

(b) The social optimality follows directly from Eq. (25) as  $\sum_{j=1, j \neq i}^n R^j = 0$  for  $n = 1$  and the private Hotelling rule equals the social Hotelling (see also Appendix A.2). In Appendix A.2, there is also shown, that the private transversality condition equals the socially optimal transversality condition and that the rent dynamics follows (27).  $\square$

A resource tax that is adjusted to the current resource stock suffers from an additional externality within the resource sector. If damages are convex, a high aggregated stock  $S$  leads to a low resource tax which benefits all resource owners in the same way. Thus, if the  $i$ -th resource owner postpones extraction, all resource owners will benefit from lower resource taxes. But at the same time, he has to extract his resources later and then he has to pay high taxes that are caused by all resource owners together. Hence, he has an incentive to extract as fast as possible (as long as taxes are low).

Thus, proposition 3 gives an explanation, how resource taxes lead to inefficient extraction paths and how a green paradox appears as an externality problem within the resource sector.

There is, however, a possibility to design a stock-dependent tax on resource extraction that is linked to the *individual* resource stock of each resource owner. At least for a specific set of extraction functions, we can give a tax rule that achieves the social optimum:

**Proposition 4.** (*Individually adjusted optimal stock-dependent taxes*) *If there are  $n$  identical resource owners (i.e. with the same extraction cost function and initial resource stock) and the regulator announces to the  $i$ -th resource owner the resource tax rule  $\tau^i(S^i)$  and the terminal-period payment rule  $\varsigma^i(S^i)$*

$$\tau^i(S^i) = \frac{-d_S(nS^i)}{r} \quad (28)$$

$$\varsigma^i(S^i(T)) = \frac{1}{n} \left( \frac{d(nS^i(T))}{r} - F(nS^i(T)) \right) \quad (29)$$

*which depends explicitly on the  $i$ -th resource owners' cumulative extraction  $S^i$ , resource owners extract along the socially optimal extraction path.*

*Proof.* The proof is along the lines of Appendix A.2. The individual tax rule leads for each resource owner to the Hotelling rule (cf. Eq. 106)

$$r = \frac{\dot{p} + r\tau^i(S^i)}{p - c(S^i)} = \frac{\dot{p} - d_S(nS^i)}{p - c(S^i)} \quad (30)$$

As all resource owners are identical,  $S = nS^i$  and the social Hotelling rule (4) follows. Furthermore, the terminal-period payment guarantees the socially optimal transversality condition.  $\square$

The tax rule extrapolates the stock-damage caused by each resource owner's extraction behavior by multiplying with factor  $n$ . Although each resource owner only causes the fraction  $1/n$  of social damage, he internalizes the entire stock-pollutant dynamic as if timing and extend of the externality would only depend solely on himself.

To conclude, increasing resource taxes dependent on the individual cumulative resource extraction could achieve an extraction pathway according to the social optimal time profile. In addition, the regulator does neither need to know the optimal stock size  $S^*(t)$  in advance nor marginal productivity and extraction costs of resources along the optimum. She only has to estimate the damage function and to commit to the tax and terminal-period payment rule which determines the tax in dependence of the individual extraction behavior. The calculation of an optimal extraction pathway has to be carried out by the private sector. This could be seen as advantage when the private sector's capability to perform this computation is perceived as relatively high compared to the regulator's capability. Their huge practical problem lies in the high transaction costs due to the dependence of the tax rate on each firm's individual cumulative resource extraction. The regulator would have to assess the distribution of fossil reserves between resource owners and adjust his taxes to the individual extraction behavior. In the more realistic case of heterogeneous resource owners, there is no simple tax rule that internalizes the stock externality appropriately.<sup>4</sup>

## 2.4 Optimal emissions trading scheme

So far, we have seen that the informational requirements to implement a socially optimal resource tax are daunting. The regulator could implement an incorrect resource tax which could lead to the green paradox under certain circumstances (Edenhofer and Kalkuhl, 2010). Sinn (2008) promotes a global emissions trading scheme which does not suffer from the green paradox. Below, we elaborate how an efficient emissions trading scheme (ETS) should be designed and whether intertemporal flexibility could be left to the market.

### 2.4.1 Emissions trading without banking and borrowing

The regulator issues in each period permits  $C$  for resource extraction. If a resource owner wants to sell a unit of resource, he has to use one permit. Thus, the regulator can effectively limit the resource use to  $C$ . Introducing a cap to resource extraction restricts the resource amount that can be extracted from above. It does, however, not imply that resource extraction always equals the permit path (as it could be profitable for resource owners to extract less than the cap allows). We do not study the conditions under which such an undersupply of resource can occur as it requires quite tedious calculations. Instead, we assume that optimal extraction under climate policy is always lower than the business-as-usual extraction:

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<sup>4</sup>The reason is that the share of each resource owner's cumulative extraction  $S^i$  on total cumulative extraction  $S$  is in general not constant. This, however, makes it impossible to assign the contribution of individual resource owners to global damages (as in (28)) without using information about other resource owners' extraction paths.

**Assumption 2.** (*Scarcity of permits*) In each period, there are fewer permits issued than resources extracted in the no-policy (BAU) case, i.e.

$$C(t) < R^{BAU}(t) \quad (31)$$

As we will show, this assumption guarantees that all permits are used at each time and no undersupply of resources occurs. The optimal ETS is characterized by the following proposition:

**Proposition 5.** (*Optimal ETS without banking*) If the regulator issues permits  $C(t) = R^*(t)$  along the socially optimal extraction path from Proposition 1, then

- (a) the optimal extraction is achieved,
- (b) the resource rent is given by  $\lambda + \theta$  according to:

$$\lambda(t) = - \int_t^T c_S^* R^* e^{r(t-\xi)} d\xi \quad (32)$$

$$\theta(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi \quad (33)$$

*Proof.* (a) We have to show that all permits are used, i.e. that  $R(t) = C(t) = R^*(t)$ . The optimization problem of the resource sector is given by  $\max_R \int_0^T (p - c(S)) R e^{-rt} dt$  subject to the constraints  $\dot{S} = -R$ ,  $S(0) = S_0$ ,  $R(t) \leq C(t)$ . The Hamiltonian function then reads  $H = (p - c(S))R - \lambda R - \theta(C - R)$ , where  $\theta$  denotes the shadow price for the binding constraint  $R \leq C$ . Applying the maximum principle leads to the following first-order condition, equation of motion, transversality and Kuhn-Tucker condition, respectively:

$$0 = p - c(S) - \lambda - \theta \quad (34)$$

$$\dot{\lambda} = r\lambda + c_S R \quad (35)$$

$$0 = \lambda(T)S(T) \quad (36)$$

$$0 = \theta(C - R) \quad (37)$$

Assumption 1 and Eq. (36) imply that  $\lambda(T) = 0$ . Solving the differential equation (35) with  $\lambda(T) = 0$  we obtain

$$\lambda(t) = - \int_t^T c_S R e^{r(t-\xi)} d\xi \quad (38)$$

From assumption 2 follows that  $R \leq R^* < R^{BAU}$  and therefore  $S > S^{BAU}$  and  $c_S > c_S^{BAU}$  as  $c_{SS} > 0$ . This implies that

$$\lambda(t) = - \int_t^T c_S R e^{r(t-\xi)} d\xi < - \int_t^T c_S^{BAU} R^{BAU} e^{r(t-\xi)} d\xi = \lambda^{BAU}(t) \quad (39)$$

With (34) we obtain  $\lambda = p - c(S) - \theta$  and with (16) and  $\tau = 0$  (in BAU) we have  $\lambda^{BAU} = p^{BAU} - c(S^{BAU})$ . The inequality (39) therefore reads:

$$p - c(S) - \theta < p^{BAU} - c(S^{BAU}) \quad (40)$$

which can be rearranged to

$$(p - p^{BAU}) + c(S^{BAU}) - c(S) < \theta \quad (41)$$

As  $p$  falls with higher  $R$  (because  $p = f_R$  and  $f_{RR} < 0$ ) and  $R < R^{BAU}$  it follows  $p > p^{BAU}$ . Likewise,  $S^{BAU} < S$  and  $c_S < 0$  imply  $c(S^{BAU}) > c(S)$ . Therefore, (41) leads to  $\theta > 0$  and due to the Kuhn-Tucker condition (37), we have  $R(t) = C(t)$ .

(b) As  $R$  follows the socially optimal path  $R^*$ , (32) directly follows from (38). From (34) follows that the rent in the resource sector is given by  $p - c(S) = \lambda + \theta$ . In particular,  $p(T) = c(S(T)) + \theta(T)$ . As  $R(t) = R^*(t)$  and  $p = f_R$ , the difference  $p - c(S)$  is the same as in the social Hotelling model (9) which implies together with (8):

$$\lambda + \theta = p^* - c(S^*) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T (c_S^* R^* + d_S^*)e^{r(t-\xi)} d\xi \quad (42)$$

Substituting  $\lambda$  from (32) into (42), we finally obtain (33).  $\square$

The shadow price  $\theta$  for permits exactly equals the optimal resource tax (21) and thus reflects the climate rent. It is worthwhile to note that it has been left open which party gets the new climate rent – the resource sector or the regulator. If the regulator issues permits for free to the resource sector, the resource sector receives the extraction rent  $\lambda$  and adds the user cost  $\theta$  to the resource price. His rent is then given by  $\lambda + \theta$ . Alternatively, the regulator can sell (or auction) the permits with a price up to  $\theta$  and absorb the climate rent completely. In accordance with conventional wisdom this rent can be captured by the regulator without any intertemporal efficiency losses.

#### 2.4.2 Emissions trading with banking and borrowing

Again, one might be tempted to argue that a regulator cannot successfully commit herself to the optimal time path  $C(t) = R^*(t)$ . Instead of controlling the time path of permits, banking and borrowing on markets might allow the regulator to leave the intertemporal timing to the markets. However, it can be shown that a free intertemporal permit trade between periods would result in a Hotelling path. Within this market, permits are treated like an exhaustible resource – one permit used now is not available in the future. This Hotelling-path is not socially optimal because the intertemporal allocation of marginal damages is not taken into account properly (Kling and Rubin, 1997). This problem could be resolved by introducing intertemporal trading rates. Thus, Leiby and Rubin (2001) have calculated intertemporal trading rates (ITR) which change the effective size of the pollution allowance held from one period to the next and lead to an optimal intertemporal reallocation of permits. We apply this approach to our problem in order to prove that the regulator cannot shirk the information and commitment problems as raised under the previous ETS without banking and borrowing.

In order to analyze banking and borrowing within our framework, only small modifications are required. The objective function and equation of motion for the resource stock remain unchanged. However, we add an equation of motion for the permit stock  $b$ . The permit stock decreases by one unit with one unit of resource use and increases at  $r_b$  – the intertemporal trading rate (ITR).

$$\dot{b} = -R + r_b b \quad (43)$$

To keep our analysis simple, we restrict it to the case where the regulator issues  $b_0$  permits only at the initial period for the entire time horizon.

As it turns out, the formula for the ITR  $r_b$  is in accordance with the formula given by Leiby and Rubin (2001). We extend their analysis by giving a formula for the optimal size of the initial permit stock.

**Proposition 6.** (Optimal ETS with banking) *If the regulator knows the optimal extraction path  $S^*$  from Proposition 1, then*

(a) *she can achieve the socially optimal extraction path by issuing  $b_0$  permits in the beginning and allowing for banking of permits with the intertemporal trading rate  $r_b$  according to:*

$$b_0 = S_0 + \frac{\int_0^T e^{-r\xi} d_S^* S^* d\xi - S^*(T) F_S(S^*(T)) e^{-rT}}{-\int_0^T e^{-r\xi} d_S^* d\xi + F_S(S^*(T)) e^{-rT}} \quad (44)$$

$$r_b = \frac{-d_S^*}{F_S(S^*(T)) e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi} \quad (45)$$

(b) *the rent in the resource sector is given by  $\lambda + \mu$  where:*

$$\lambda = - \int_t^T c_S^* R^* e^{r(t-\xi)} d\xi \quad (46)$$

$$\mu = F_S(S^*(T)) e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi \quad (47)$$

*Proof.* See Appendix B. □

In principle, optimal intertemporal permit trading requires two regulating screws. In addition to the ITR, the regulator has to issue the optimal number of permits in the first period which can be traded over the entire time horizon. While the ITR enforces the optimal timing of extraction,  $b_0$  enforces the optimal cumulative resource consumption in accordance with the transversality condition of the social problem. Thus, the regulator has to calculate ex ante the damages and the extraction along the social optimum  $d_S(S^*(t))$  and  $S^*(t)$ , respectively. Regarding Eq. (44) and (45), one can get an impression of the information requirements that the regulator as well as the market is confronted with. This result confirms the insight which has been gained in the previous sections. So far, there is little evidence that banking and borrowing can increase efficiency within this framework or discharge the regulator from difficult intertemporal optimization decisions by using market mechanisms.

The ETS with banking resembles the resource rent dynamic with stock externality as given by (8). It becomes apparent that the regulator could absorb the rent associated to the shadow price of permits  $\mu$ . By an auctioning mechanism, she could sell permits in the first period at maximum price  $\mu_0$ , which equals the discounted value of the cumulative tax income from the optimal resource tax (21).

## 2.5 Comparison between price and quantity instruments within a CBA framework

### 2.5.1 Suboptimal discount rates

The analysis above always assumed that the regulator has only to care about the climate externality. Due to the intertemporal dynamic of the problem, however, discount rates of agents and of the society play a crucial role. In particular, when property rights for resources are insecure and capital or future markets incomplete, agents' effective discount rate could be higher than in the representative-household economy (eg. Sinn, 2008). Thus, policy instruments may be subject to these secondary distortions and turn out to be suboptimal if not adjusted.

**Proposition 7.** (*Suboptimal discount rates*) *If the resource sector discounts profits with rate  $\rho$  which differs from the discount rate  $r$  from the social planner's problem, then:*

(a) *the optimal resource tax from Proposition 2 has to be modified according to*

$$\begin{aligned} \tau(t) = & F_S(S^*(T))e^{-\rho(T-t)} - \int_t^T d_S^* e^{\rho(t-\xi)} d\xi \\ & - (r - \rho) \int_t^T (p^* - c(S^*))e^{\rho(t-\xi)} d\xi \end{aligned} \quad (48)$$

(b) *the efficiency of the ETS without banking is not affected; the shadow price for permits, however, changes according to:*

$$\begin{aligned} \theta^\rho(t) = & F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi \\ & + \int_t^T c_S^* R^* (e^{\rho(t-\xi)} - e^{r(t-\xi)}) d\xi \end{aligned} \quad (49)$$

*In particular,  $\theta^\rho$  increases in  $\rho$  for  $0 \leq t < T$ .*

*Proof.* (a) See Appendix C.1. (b) The permit path  $C(t)$  enforces the resource extraction path  $R(t) = C(t)$  as permits are scarce (Assumption 2). Thus, the final price for resources  $p^* = p(R^*)$  and the marginal extraction costs  $c(S^*)$  follow the socially optimal path. The shadow price for resources  $\lambda$ , however, changes to:

$$\lambda^\rho(t) = - \int_t^T c_S^* R^* e^{\rho(t-\xi)} d\xi \quad (50)$$

With (34) follows  $\theta^\rho = p^* - c(S^*) - \lambda^\rho$ . For  $p^* - c(S^*)$  we can finally substitute the right-hand-side of Eq. (42) which gives us together with (50) the shadow price for  $\theta$  with private discount rate  $\rho$  (49).

Finally, for  $0 \leq t < T$ :

$$\frac{\partial \theta^\rho}{\partial \rho} = \int_t^T \rho(t - \xi) c_S^* R^* e^{\rho(t-\xi)} d\xi > 0 \quad (51)$$

as  $c_S^* < 0$ . □

Thus, the most robust instrument against suboptimal discount rates is the ETS with-

		Management	
		Regulator	Resource Owner
Property	Regulator	ETS with auctioning; Resource tax	Stock-dependent tax
	Resource Owner	ETS with grandfa- thering	

Table 1: Management and property of the climate rent within the cost-benefit approach

out banking and borrowing. As long as the permit constraint is binding only the user cost for permit scarcity is affected. A higher (lower) private discount rate  $\rho > r$  leads only to a higher (lower) valuation of the user cost. If permits are grandfathered, suboptimal discount rates make no difference in final resource prices. If permits are auctioned, the resource sector's willingness to pay for permits may now change to the modified user costs  $\theta^\rho(t)$ . Although suboptimal discount rates do not change the efficient extraction path, they may lead to a slightly different distribution of the climate rent.

Suboptimal discount rates in the ETS with banking and borrowing, however, are hard to cure as they affect both intertemporal arbitrage conditions for permit as well as resource path. In principle, a higher ITR gives an incentive to postpone permit and resource use.

It is worthwhile to note here that a tax can in general cure both market failures. However, the information the regulator requires for implementation are more demanding as in the case of an ETS without banking and borrowing: The regulator has to consider the impact of a distorted discount rate on the entire extraction process.

### 2.5.2 Assessment of price and quantity instruments

The considerations above have shown that efficient climate policy has to introduce an additional climate rent term into the resource rent dynamic by specific policy mechanisms. The *climate rent* in the cost-benefit approach under each of the instruments is given by:

$$\theta(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi \quad (52)$$

The complexity, however, arises due to the stock externality: For a flow pollutant,  $d_S \equiv 0$ , we observe only the classical resource rent. However, it should be noted that all instruments have to create this *climate rent* irrespective how this rent is distributed in the end. The capitalization of rents allows the optimal provision of public goods or avoiding public bads when a limited resource can be imposed on private firms. It is well-known in urban economics, that limited land creates a rent which enables cities to provide an optimal amount of local public goods (Fujita and Thisse, 2002). In principle, the same mechanism is at work in this context: The resource tax or the direct quantity control allows for creating a rent ensuring the optimal provision of a public good.

Tab. 1 summarizes how property and management of the climate rent can be assigned to the regulator or to the resource owners by choosing different policy instruments. Even in a world with perfect information and without transaction costs, price and quantity instruments differ in their institutional and informational requirements. For the stock-dependent resource tax the regulator needs only to know the damage function but not the optimal pathway. She furthermore does not need to commit to the entire time path of the tax but only to a rule to adjust the tax and the terminal-period payment. For all other policy instruments, the regulator has to calculate the optimal extraction path for the entire time horizon and to commit to the time path of his price or quantity instrument. If the regulator would know the socially optimal extraction path, the regulator could control extraction directly or by an emission trading scheme without banking and borrowing. In addition, emissions trading without banking and borrowing might be quite effective when additional market distortions like insecure property rights, incomplete future markets or liquidity constraints have to be taken into account.

### 3 The Carbon-Budget-Approach

The cost-benefit-approach requires a balancing of the damages of the use of fossil resources against the opportunity costs of postponed resource extraction. Quantifying the damages of climate change, however, is a difficult and controversial task. It is confronted with deep uncertainties in the climate system, regional market and non-market impacts and in normative parameters like discount rates, risk aversion or assumed substitution possibilities between physical capital and ecosystem services. Tipping points in the earth systems can lead to irreversible and catastrophic impacts when certain thresholds in the temperature increase are crossed. Last but not least, the loss of human lives and many ecosystems cannot always be quantified reasonably well in monetary terms.

These are some of the reasons for why cost-benefit-analysis for global warming is so controversial in science and politics. In practice, however, there is a growing consensus to limit global warming to a certain temperature limit, e.g. to two degrees above the pre-industrialized level. As Meinshausen et al. (2009) showed achieving such temperature limits with certain likelihood depends mainly on the cumulative emissions until 2050. Hence, a more practical way of communicating and negotiating climate targets could be based on (global or national) caps for cumulated emissions – a so-called “carbon budget” (WBGU, 2009; Edenhofer et al., 2009).

#### 3.1 The model

The carbon budget approach, however, does not directly imply an option for a policy instrument in order to achieve the temperature limit in a cost-effective way. The purpose of this part is to clarify the precise requirements for the design of policy instruments. In the following, we denote with CB the carbon budget, i.e. the politically set cumulative amount of carbon (i.e. extracted resources) in the atmosphere:

### 3.1.1 The social planner's problem

Removing the damage term and adding the budget constraint to the intertemporal social optimization problem is described as follows:

$$\max_R \int_0^T (f(R) - c(S)R) e^{-rt} dt \quad (53)$$

subject to:

$$\dot{S} = -R \quad (54)$$

$$\dot{C} = -R \quad (55)$$

$$S(0) = S_0 \quad (56)$$

$$C(0) = CB \quad (57)$$

Implementing a carbon budget does only make sense, if it exhibits a binding constraint. We formulate a similar, but more general assumption than Assumption 2:

**Assumption 3.** (*Scarcity of the carbon budget*) Cumulative extraction in the absence of the budget (BAU) exceeds the carbon budget:

$$CB < \int_0^T R^{BAU} dt < S_0 \quad (58)$$

**Proposition 8.** (*Socially optimal resource extraction*) If a social planner maximizes intertemporal output according to (53–57), then:

(a) the optimal solution  $(R^*, S^*)$  is determined by the following system of equations:

$$r = \frac{\dot{f}_R}{f_R - c(S)} \quad (59)$$

$$\dot{S} = -R \quad (60)$$

$$S(0) = S_0 \quad (61)$$

$$S(T) = S_0 - CB \quad (62)$$

(b) The shadow prices  $\lambda$  and  $\mu$  for  $S$  and  $C$ , respectively, are given by:

$$\lambda(t) = - \int_t^T c_S R e^{r(t-\xi)} d\xi \quad (63)$$

$$\mu(t) = \mu_T^{CB} e^{-r(T-t)} \quad (64)$$

where  $\mu_T^{CB} = f_R(R(T)) - c(S(T)) = f_R(R(T)) - c(S_0 - CB)$ .

*Proof.* (a) We set up the corresponding Hamiltonian function  $H = f(R) - c(S)R - \lambda R - \mu R$ . Applying the maximum principle leads to the following first-order and

transversality conditions:

$$\lambda + \mu = f_R - c(S) \tag{65}$$

$$\dot{\lambda} = r\lambda + c_S R \tag{66}$$

$$\dot{\mu} = r\mu \tag{67}$$

$$0 = \lambda(T)S(T) \tag{68}$$

$$0 = \mu(T)C(T) \tag{69}$$

Differentiating (65) with respect to time and rearranging with (66) and (67), we obtain the social Hotelling rule (59). Assumption 1 and (68) imply that  $\lambda(T) = 0$ . As shown in the Appendix D, Assumption 3 implies that the entire budget is used up, i.e.  $C(T) = 0, \mu(T) > 0$  and, hence,  $S(T) = S_0 - CB$ .

(b) Solving (66) with  $\lambda(T) = 0$ , we obtain (63). From  $\lambda(T) = 0$  and (65) follows  $\mu(T) = f_R(T) - c(S(T))$  – and with (67) we obtain (64).  $\square$

### 3.1.2 The decentralized resource sector’s problem

As the decentralized market dynamics equals the one described in the CBA Sec. 2.1.2, we only restate the private Hotelling rule and the terminal condition:

$$r = \frac{\dot{p} - \dot{\tau} + r\tau}{p - c(S)} \tag{70}$$

$$\tau(T) = p(T) - c(S(T)) \tag{71}$$

## 3.2 Optimal resource tax

**Proposition 9.** (*Optimal resource tax*) *If the regulator knows  $\mu_T^{CB}$  according to Proposition 8 and if she can commit at  $t = 0$  to the tax path  $\tau(t)$  over the entire planning horizon, then*

(a) *the resource tax*

$$\tau(t) = \mu_T^{CB} e^{-r(T-t)} \tag{72}$$

$$\mu_T^{CB} = f_R(R^*(T)) - c(S_0 - CB) \tag{73}$$

where  $R^*(T)$  denotes the final resource extraction from the social optimum (Proposition 8) achieves the optimal extraction path.

(b) *The rent in the resource sector is given by:*

$$\lambda(t) = - \int_t^T c_S S e^{r(t-\xi)} d\xi \tag{74}$$

*Proof.* (a) Plugging  $\tau$  from (72) and its derivative into the private Hotelling rule (70) and utilizing that in the market equilibrium  $p = f_R$ , we obtain the social Hotelling rule (59). The transversality condition of the decentralized resource sector (71) implies that  $p(T) - c(S(T)) = \mu_T^{CB}$  which equals the social transversality condition derived in Proposition 8. Hence,  $S(T) = S_0 - CB$ .

(b) Same proof as in Proposition 2 (b).  $\square$

Hence, the regulator has to solve the social planner model in order to calculate the initial tax level  $\mu_T^{CB}$ . Although the regulator could theoretically impose the correct

tax, an incorrect initial tax level or tax growth rate will lead to an exceeding or exacerbating of the budget.

The optimal resource tax is a pure budget scarcity price that reflects the scarcity of the (exhaustible) carbon budget according to the Hotelling rule. There is only a rent for reserves with low extraction costs which would also diminish if extraction costs were constant. Within the carbon budget approach, we call the rent which is associated to the budget scarcity as *climate rent*. This rent is completely incorporated by the resource tax.

### 3.3 Optimal emissions trading scheme

#### 3.3.1 Emissions trading without banking and borrowing

The optimal intertemporal use of the carbon budget requires that permits are issued according to the optimal resource path  $R^*$  from Proposition 8.

**Proposition 10.** (*Optimal ETS without banking*) *If the regulator issues permits  $C(t) = R^*(t)$  along the socially optimal extraction path from Proposition 8, then*

- (a) *the optimal extraction is achieved,*
- (b) *the resource rent is given by  $\lambda + \theta$  according to:*

$$\lambda(t) = - \int_t^T c_S R e^{r(t-\xi)} d\xi \quad (75)$$

$$\theta(t) = \mu_T^{CB} e^{-r(T-t)} \quad (76)$$

$$\mu_T^{CB} = f_R(R^*(T)) - c(S_0 - CB) \quad (77)$$

*Proof.* The proof is basically along the lines of the proof of Proposition 5. □

Proposition 10 requires that the regulator can calculate the socially optimal resource extraction path for the entire time horizon. She has to issue permits in each time period according to this path.

The shadow price for permits  $\theta$  (which would be observed on a market for tradable permits) equals the optimal tax in each period. Similar to the previous section where we studied CBA compatible instruments, we denote the scarcity price for carbon  $\theta$  as *climate rent*. The regulator could absorb this rent by auctioning permits or she could shift this rent to resource owners by a grandfathering scheme.

#### 3.3.2 Emissions trading with banking and borrowing

Alternatively, the regulator could allocate the permits from the carbon budget in the first period to the resource owners and allow for intertemporal flexibility when to use the permits. As objective function and constraints then equal the social problem, the market reproduces the socially optimal solution.

**Proposition 11.** (*Optimal ETS with banking*) *If the regulator issues CB permits in the initial period which can be banked by resource owners, then*

- (a) *the optimal extraction is achieved,*

(b) the resource rent is given by  $\lambda + \theta$  according to:

$$\lambda(t) = - \int_t^T c_S R e^{r(t-\xi)} d\xi \quad (78)$$

$$\theta(t) = \mu_T^{CB} e^{-r(T-t)} \quad (79)$$

$$\mu_T^{CB} = f_R(R^*(T)) - c(S_0 - CB) \quad (80)$$

*Proof.* (a) and (b) follow directly from Proposition 8 with resource rent  $p - (c(S)) = \lambda + \theta$  and  $\theta = \mu$ .  $\square$

The initial permit price  $\theta_0$  has to be set at the level which equals cumulative permit (=resource) demand with the carbon budget  $CB$ . As it turns out, the problem is equivalent to the emission tax problem (72) and  $\theta_0 = \tau_0$ . But in contrast to the taxation scheme, the market has to determine  $\theta_0$  or  $\mu_T^{CB}$  by estimating the demand function and the extraction cost curve. This, however, requires a complete set of future markets to achieve an intertemporal market equilibrium (Dasgupta and Heal, 1979, pp. 100–110).

The regulator could issue permits for free (e.g. in a grandfathering mode to resource owners) or sell them at maximum price  $\theta(t)$  – thus she can divide the scarcity rent in a non-distortionary way between several economic actors. As the regulator may not estimate  $\theta(t)$  correctly, she could auction the entire permit stock in the first period. The rent left to the resource owner then reduces to  $\lambda$ .

### 3.4 Comparison between price and quantity instruments

#### 3.4.1 Suboptimal discount rates in the resource sector

Equal to the analysis in the CBA section, we want to find out how suboptimal discount rates influence the performance of the previously studied policy instruments.

**Proposition 12.** (*Suboptimal discount rates*) *If the resource sector discounts profits with rate  $\rho$  which differs from the discount rate  $r$  from the social planner's problem and if the regulator furthermore knows the socially optimal extraction and price paths  $S^*$ ,  $R^*$ ,  $p^*$  and  $\mu_T^{CB}$  from Proposition 8, then:*

(a) the resource tax from Proposition 9 has to be modified according to

$$\tau(t) = \mu_T^{CB} e^{-\rho(T-t)} - (r - \rho) \int_t^T (p^* - c(S^*)) e^{\rho(t-\xi)} d\xi \quad (81)$$

(b) the efficiency of the ETS without banking is not affected; the shadow price for permits, however, changes according to:

$$\theta(t) = \mu_T^{CB} e^{-r(T-t)} + \int_t^T c_S^* R^* \left( e^{\rho(t-\xi)} - e^{r(t-\xi)} \right) d\xi \quad (82)$$

In particular,  $\theta$  increases in  $\rho$ .

(c) under the ETS with banking the regulator has to introduce an additional resource tax according to:

$$\tau(t) = (\rho - r) \int_t^T (p^* - c^*(S)) e^{\rho(t-\xi)} d\xi \quad (83)$$

*Proof.* For (a) and (c) see Appendix C.2 and C.3; (b) follows basically along the lines of the proof of Proposition 7 (b).  $\square$

If the discount rate in the resource sector exceeds the social discount rate ( $\rho > r$ ), the resource tax has to increase at a lower rate compared to the case where  $\rho = r$  in order to provide an incentive for future extraction. Equal to the findings in the CBA framework, the ETS without banking and borrowing is the most robust instrument – as long as the regulatory institution uses the ‘right’ discount rate. In this case, suboptimal discount rates only affect the shadow price for permits and, thus, the distribution of the permit rent if permits are auctioned by the regulator. In particular, the optimal permit price does not increase exponentially at a constant rate and is therefore not consistent with intertemporal maximization of the permit rent. This is the reason why an ETS with banking and borrowing is suboptimal. High discount rates of permit owners lead to a steeper permit price path and, thus, to an accelerated extraction. Within the banking-and-borrowing ETS, the regulator now additionally has to tax resource extraction. This, however, requires for the regulator to have all the necessary information about optimal timing and demand for resources for the entire time horizon. If intertemporal markets do not perform well, the regulator cannot leave the timing decision to the market.

### 3.4.2 Assessment of price and quantity instruments

With the emissions trading scheme the scarcity value of the carbon budget

$$\mu(t) = \tau(t) = \theta(t) = \mu_T^{CB} e^{-r(T-t)} = (f(R^*(T)) - c(S_0 - CB))e^{-r(T-t)} \quad (84)$$

is made explicit (by tax or permit price) and separated from the resource price which covers extraction cost and an extraction cost rent  $\lambda$ .

The budget approach transforms the intertemporal resource scarcity rent into a climate rent by imposing a cumulative budget on resource extraction. This new scarcity rent can be distributed through permits arbitrarily and in a non-distorting way. The resource tax mimics the permit scarcity price and implies a total transfer of the scarcity rent to the regulator. In contrast to policy instruments in the cost-benefit-approach, the climate rent in the carbon budget does not need to be modified by complex stock-externality dynamics as it simply follows the Hotelling rule.

The budget approach does not require an explicit estimation of damages for given carbon budget. In order to achieve an optimal intertemporal ‘exhaustion’ of the carbon budget, extraction costs and demand for fossil resources have to be known for the entire planning horizon.

Tab. 2 shows management and property of the climate rent for the considered policy instruments. By issuing tradable permits with full intertemporal flexibility (free banking and borrowing), the regulator could delegate this estimation problem to the market. An efficient market solution, however, relies on competitive markets and the existence of functioning futures markets. As an additional market is created by the quantity instrument, there could occur several market failures due to competition problems, market power or information asymmetries. The functioning of the permit market hinges on the performance of related markets – in particular on resource, technology and capital markets (the latter is crucial for intertemporal arbitrage decisions). The quantity instruments can be seen (by definition) as the fool-proof instrument to achieve the carbon budget.

Climate Rent		Management	
		Regulator	Resource Owner
Property	Regulator	ETS w/o banking and with auctioning; Resource tax	ETS with banking and auctioning
	Resource Owner	ETS w/o banking and with grandfathering	ETS with banking and grandfathering

Table 2: Management and property of the climate rent within the carbon budget approach.

If the regulator sets an incorrect tax path, she could provoke cumulative extraction higher or lower than the budget. If the regulator sets the initial tax too low and the tax growth rate too high, she could provoke a green paradox and incentivate resource owners to accelerate extraction compared to the no-policy case (Sinn, 2008; Edenhofer and Kalkuhl, 2010). The budget approach, however, says nothing about how costly deviation from the budget is (e.g. due to higher damages).

## 4 Conclusion

From the analysis above we draw the following conclusions. First, in a deterministic world price and quantity instruments differ with respect to the distribution of informational requirements between market and regulator (see Tab. 1 and 2) and their robustness against additional market failures. In particular, the cost-benefit approach has to deal with more complex intertemporal rent dynamics as the carbon budget approach due to its aim to allocate climate damages efficiently in time.

Second, due to the complexity of the stock-pollutant problem markets are hardly able to manage the climate rent intertemporally in an efficient way. It seems to be unavoidable to entrust a regulatory institution with the challenging task to find an extraction path that is ‘close’ to the social optimum. In a cost-benefit framework, only the stock-dependent resource tax which is dependent on each resource owner’s reserve size could discharge the regulator from this task. But the necessary condition of homogenous resource owners is highly unrealistic and, furthermore, the implementation requires the commitment to a terminal-period payment rule. In the carbon-budget approach, only an intertemporally flexible permit trade could dispense the regulator from finding the intertemporally efficient extraction path. All other instruments rely crucially on the performance of the regulatory institution to implement an intertemporally efficient allocation plan (see Tab. 3 for a summary).

Third, leaving the task of optimal timing to the market requires complete future markets if the costs of misallocation are to be avoided. Until now, future markets for commodities or resources have not been established for planning horizons of many decades or even an entire century. Existing future markets for several decades (e.g. for fossil resources) are often thin and suffer from volatile prices due to high uncertainties and speculations.

	<b>Cost-Benefit Framework</b>	<b>Carbon-Budget Framework</b>
<b>Price Instruments</b>	<p><i>Resource tax:</i></p> <ul style="list-style-type: none"> <li>regulator needs complete information about optimal paths and damages</li> <li>regulator has to commit <i>ex ante</i> to the entire tax path</li> <li>suboptimal discount rates in the resource sector make complex modifications necessary.</li> </ul> <p><i>Stock dependent tax:</i></p> <ul style="list-style-type: none"> <li>regulator does only need to know damage function and not optimal paths</li> <li>regulator has to commit to a rule how to adjust the tax and make terminal-period payment</li> <li>hard to implement due to high transaction costs</li> <li>high vulnerability to suboptimal discount rates in the resource sector</li> <li>restrictive assumptions about resource sector.</li> </ul>	<p><i>Resource tax:</i></p> <ul style="list-style-type: none"> <li>regulator needs complete information about optimal paths within the carbon budget</li> <li>regulator has to commit <i>ex ante</i> to the entire tax path</li> <li>suboptimal discount rates in the resource sector make complex modifications necessary.</li> </ul>
<b>Quantity Instruments</b>	<p><i>ETS without banking and borrowing:</i></p> <ul style="list-style-type: none"> <li>regulator needs complete information about optimal paths and damages</li> <li>robust against suboptimal discount rates in the resource and permit sector</li> </ul> <p><i>ETS with banking and borrowing:</i></p> <ul style="list-style-type: none"> <li>regulator needs complete information about optimal paths and damages</li> <li>regulator has to commit <i>ex ante</i> to the optimal time-path of the intertemporal trading rate</li> <li>intertemporal arbitrage is highly vulnerable to distortions linked to the permit market (incomplete future or capital markets)</li> </ul>	<p><i>ETS without banking and borrowing:</i></p> <ul style="list-style-type: none"> <li>regulator needs complete information about optimal paths within the carbon budget</li> <li>robust against suboptimal discount rates in the resource and permit sector</li> </ul> <p><i>ETS with banking and borrowing:</i></p> <ul style="list-style-type: none"> <li>regulator needs no information</li> <li>market has to make entire assessment about optimal Hotelling path within carbon budget</li> <li>vulnerable to distortions linked to the permit market (incomplete future or capital markets)</li> <li>suboptimal discount rates make complex modifications necessary (which diminishes informational advantage for the regulator)</li> </ul>
<b>Climate Rent</b>	<p>Complex rent dynamics due to optimal marginal damage path <math>d_S^*</math>:</p> $\theta(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi$	<p>Pure Hotelling price determined by the scarcity of the budget:</p> $\theta(t) = (f(R^*(T)) - c(S_0 - CB))e^{-r(T-t)}$

Table 3: Comparison of policy instruments

Forth, secondary market failures play a crucial role when discussing the robustness of climate policy instruments. We studied the implications of suboptimal discount rates due to insecure property rights in fossil resources, liquidity constraints in capital markets or incomplete capital and future markets. Resource taxes can in theory cure this additional market failure but depend on the exact assessment of the extraction dynamic which becomes even more complex when discount rates differ. Emissions trading schemes with intertemporal flexibility may suffer from suboptimal discount rates. In particular, under a CBA approach where intertemporal trading rates aim to achieve an efficient re-allocation of stock-externalities, suboptimal discount rates do not only change the timing but also the cumulative amount of emissions. Thus, emissions become hardly controllable by the regulator. Emissions trading schemes without intertemporal flexibility about large time periods are the most robust instrument against secondary market failures in resource and permit markets. Suboptimal discount rates do only affect the willingness to pay for auctioned permits but do influence neither final resource prices nor extraction paths. The higher the risk premium (and thus, the effective discount rate) is, the higher is the equilibrium permit price and the lower is the rent for the resource owner.

These considerations show the need for an institution enabling a reasonable intertemporal management of the climate rent which is linked to the management of exhaustible resources. A ‘carbon bank’ could – similar to central banks – improve the commitment to a fixed budget of permits and allow markets to find an optimal intertemporal pathway. However, it should be noted that this is part of a larger discussion resembling how the division of labor between market and state should be designed under real-world conditions (cf. Stiglitz, 1994). It is beyond this modeling framework to do a meta-cost-benefit analysis of these options. It is not clear how costly the implementation of such an institution really is and whether these costs are less than the welfare losses due to incomplete future markets.

Our analysis might suggest that emissions trading without banking and borrowing is the least susceptible instrument when resource owners use suboptimal discount rates. Together with the findings of Sinn (2008) that suboptimal taxes could worsen global warming, quantity instruments seem to be the fool-proof instrument in order to achieve a certain emission path (or carbon budget). However, this conclusion should be treated more as a hypothesis to overcome green paradoxes than as a robust policy recommendation for real world applications. For a robust assessment, an extension of the model in two directions is necessary. First, uncertainty about costs and damages should be included. It seems reasonable to assume that over time regulators and market agents learn more about their potential to reduce mitigation costs. In addition, they will learn more on the impact of climate change. Such an analysis, however, requires a functional description of agent’s behavior and involved uncertainties (e.g. in resource stock, damage curves, substitutes, economic growth, discount rates etc.) which cannot be incorporated by static marginal benefit and cost curves. Thus, the Weitzman approach has to be extended by considering the impact of uncertainties on policy instruments and the strategic reaction that such suboptimal policy instruments provoke. Second, additional market distortions in the markets for permits and goods need a more careful analysis. This also includes an endogenous treatment of information asymmetries within a principal-agent framework as a promising pathway. We assumed that resource owners are themselves able to carry out intertemporal optimization. However, it seems more realistic to assume that ownership and management is separated. This could lead to a situation where the management tries to maximize profits on the spot-market which might reverse the impact of price and quantity instruments

on the overall time path. Taking into account these aspects might allow for a robust assessment whether expected losses of price or quantity instruments are greater when regulators choose wrong instruments and market participants act strategically.

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## Appendix

### A Stock-dependent Resource Taxes

#### A.1 Many resource owners

The optimization problem is given as follows:

$$\max_{R^i} \int_0^T (p - c^i(S^i) - \tau(S)) R^i e^{-rt} dt \quad (85)$$

$$\dot{S}^i = -R^i \quad (86)$$

$$S = \sum_{i=1}^n S^i, \quad R = \sum_{i=1}^n R^i \quad (87)$$

$$S^i(0) = S_0^i \quad (88)$$

Hamiltonian with first-order-conditions:

$$H^i = (p - c^i(S^i) - \tau(S)) R^i - \lambda^i R^i \quad (89)$$

$$\lambda^i = p - c^i(S^i) - \tau(S) \quad (90)$$

$$\dot{\lambda}^i = r\lambda^i + c_S^i R^i + \tau_S R^i \quad (91)$$

Differentiating (90) yields:

$$\dot{\lambda}^i = \dot{p} + c_S^i R^i + \tau_S R^i \quad (92)$$

$$= \dot{p} + c_S^i R^i + \tau_S \sum_{j=1}^n R^j \quad (93)$$

Substituting this into (91), we obtain as Hotelling rule:

$$r = \frac{\dot{p} + \tau_S \sum_{j=1, j \neq i}^n R^j}{p - c^i(S^i) - \tau(S)} = \frac{\dot{p} + r\tau(S) + \tau_S \sum_{j=1, j \neq i}^n R^j}{p - c^i(S^i)} \quad (94)$$

For the tax rule  $\tau(S) = -\frac{d_S(S)}{r}$  we finally obtain:

$$r = \frac{\dot{p} - d_S - \frac{d_{SS}}{r} \sum_{j=1, j \neq i}^n R^j}{p - c^i(S^i)} \quad (95)$$

## A.2 One (competitive) resource owner

The optimization problem is given as follows:

$$\max_R \int_0^T (p - c(S) - \tau(S)) R e^{-rt} dt + \varsigma(S(T)) e^{-rT} \quad (96)$$

$$\dot{S} = -R \quad (97)$$

$$S(0) = S_0 \quad (98)$$

$$\tau(S) = \frac{-d_S(S(T))}{r} \quad (99)$$

$$\varsigma(S(T)) = F(S(T)) + \frac{d(S(T))}{r} \quad (100)$$

Hamiltonian with first-order-conditions and transversality condition:

$$H = (p - c(S) - \tau(S))R - \lambda R \quad (101)$$

$$\lambda = p - c(S) - \tau(S) \quad (102)$$

$$\dot{\lambda} = r\lambda + c_S R + \tau_S R \quad (103)$$

$$0 = S(T) \left( \lambda(T) - F_S(S(T)) - \frac{d_S(S(T))}{r} \right) \quad (104)$$

Differentiating (102) yields:

$$\dot{\lambda} = \dot{p} + c_S R + \tau_S R \quad (105)$$

and substituting this into (103) and using (99), we obtain as Hotelling rule:

$$r = \frac{\dot{p}}{p - c(S) - \tau(S)} = \frac{\dot{p} - d_S(S)}{p - c(S)} \quad (106)$$

From (99), (102), (104) and  $S(T) > 0$  (Assumption 1) follows for  $t = T$ :

$$p(T) - c(S(T)) = F_S(S(T)) \quad (107)$$

**The resource rent:** Solving (103), we obtain:

$$\lambda(t) = \lambda(T) e^{-r(T-t)} - \int_t^T c_S R e^{-r(\xi-t)} d\xi - \int_t^T \tau_S R e^{-r(\xi-t)} d\xi \quad (108)$$

We can solve the integral over  $\tau_S$  using partial integration:

$$\int_t^T \tau_S R e^{-r(\xi-t)} d\xi = e^{rt} \int_t^T \dot{\tau}_S e^{-r\xi} d\xi \quad (109)$$

$$= e^{rt} \left[ \tau(T) e^{-rT} - \tau(t) e^{-rt} + r \int_t^T \tau e^{-r\xi} d\xi \right] \quad (110)$$

Hence, we obtain together with (108):

$$\lambda(t) + \tau(t) = (\lambda(T) + \tau(T))e^{-r(T-t)} - \int_t^T c_S R e^{-r(\xi-t)} d\xi + r \int_t^T \tau e^{-r(\xi-t)} d\xi \quad (111)$$

Substituting the tax rule  $\tau = -d_S/r$  and using  $F_S(S(T)) = \lambda(T) + \tau(T)$  due to (107) and (102), we obtain the common formula for the intertemporal rent dynamics:

$$\lambda(t) + \tau(t) = F_S(S(T))e^{-r(T-t)} - \int_t^T c_S R e^{-r(\xi-t)} d\xi - \int_t^T d_S e^{-r(\xi-t)} d\xi \quad (112)$$

## B CBA-ETS with Banking and Borrowing

The quantity trading ratio changes the effective volume of emissions through banked permits  $b$  by rate  $r_b(t)$ . The optimization problem for the resource sector reads:

$$\max_R \int_0^T (p - c(S)) R e^{-rt} dt \quad (113)$$

$$\dot{S} = -R \quad (114)$$

$$\dot{b} = -R + r_b b \quad (115)$$

$$S(0) = S_0 \quad (116)$$

$$b(0) = b_0 \quad (117)$$

Thus, the Hamiltonian with first-order-conditions reads:

$$H = (p - c(S))R - \lambda R - \mu(R - r_b b) \quad (118)$$

$$\lambda = p - c(S) - \mu \quad (119)$$

$$\dot{\lambda} = r\lambda + c_S(S)R \quad (120)$$

$$\dot{\mu} = r\mu - r_b \mu \quad (121)$$

Transversality conditions are given as follows:

$$S(T)\lambda(T) = 0 \quad (122)$$

$$b(T)\mu(T) = 0 \quad (123)$$

Now, we want to derive the optimal value for  $b_0$  and the optimal *policy trajectory* for  $r_b(t)$  that guarantees a socially optimal solution as characterized in Proposition 1.

### B.1 Determine optimal intertemporal trading rates $r_b(t)$

Differentiating (119) and substituting (121), we obtain:

$$\dot{\lambda} = \dot{p} + c_S(S)R - (r - r_b)\mu \quad (124)$$

Equating with (120) and using (119) yields:

$$\dot{p} = r(p - c(S)) - r_b\mu \quad (125)$$

The socially optimal price path, however, from (4) is given by:

$$\dot{p} = r(p - c(S)) + d_S(S) \quad (126)$$

By equating (126) with (125) and using (121), we obtain:

$$-d_S(S) = r_b\mu = r\mu - \dot{\mu} \quad (127)$$

Solving for  $\mu$ , we obtain:

$$\mu(t) = e^{rt} \int_0^t d_S(S) e^{-r\xi} d\xi + \mu_0 e^{rt} \quad (128)$$

For known  $\mu(T)$  we can calculate  $\mu_0 := \mu(0)$  and obtain for  $\mu$ :

$$\mu_0 = - \int_0^T e^{-r\xi} d_S(S) d\xi + \mu(T) e^{-rT} \quad (129)$$

$$\mu = \mu(T) e^{-r(T-t)} - \int_t^T d_S(S) e^{r(t-\xi)} d\xi \quad (130)$$

Now, we can calculate  $r_b$  by using (127) and (130):

$$r_b = \frac{-d_S(S)}{\mu} \quad (131)$$

$$= \frac{-d_S(S)}{\mu(T) e^{-r(T-t)} - \int_t^T d_S(S) e^{r(t-\xi)} d\xi} \quad (132)$$

### B.2 Determine the optimal initial permit stock $b_0$

Solving (115) yields

$$b(t) = e^{\int_0^t r_b d\xi} \int_0^t \left( -R e^{-\int_0^\xi r_b du} \right) d\xi + b_0 e^{\int_0^t r_b d\xi} \quad (133)$$

By using the substitution  $\phi$  as follows:

$$r_b = \frac{\partial}{\partial t} (-\ln(-\phi)) = -\frac{\dot{\phi}}{\phi} \quad (134)$$

$$\phi := - \int_0^t \mu_0^{-1} e^{-rs} d_s(S) ds - 1 \quad (135)$$

it follows (note that  $\phi^*(0) = -1$ )

$$e^{\int_0^t r_b d\xi} = e^{-\ln(-\phi(t)) + \ln(-\phi(0))} = \frac{-1}{\phi(t)} \quad (136)$$

Thus, (133) reduces to:

$$b(t) = \frac{-1}{\phi(t)} \left( \int_0^t \phi(\xi) R(\xi) d\xi + b_0 \right) \quad (137)$$

For known terminal value  $b(T)$  we can calculate  $b_0$  as follows:

$$b_0 = -b(T)\phi(T) - \int_0^T \phi(\xi) R(\xi) d\xi \quad (138)$$

$$= -b(T)\phi(T) + \int_0^T \phi(\xi) \dot{S}(\xi) d\xi \quad (139)$$

$$= -b(T)\phi(T) + \phi(T)S(T) - \phi(0)S(0) - \int_0^T \dot{\phi}(\xi) S(\xi) d\xi \quad (140)$$

$$= -b(T)\phi(T) + \phi(T)S(T) + S_0 + \frac{\int_0^T e^{-r\xi} d_S(S) S d\xi}{\mu_0} \quad (141)$$

$$= -b(T)\phi(T) + \phi(T)S(T) + S_0 + \frac{\int_0^T e^{-r\xi} d_S(S) S d\xi}{-\int_0^T e^{-r\xi} d_S(S) d\xi + \mu(T)e^{-rT}} \quad (142)$$

An optimal solution requires that  $\mu(t) > 0$  for  $t \in [0, T]$  as otherwise the trading ratio  $r_b(T)$  in (131) is not defined. From the transversality condition (123) then follows that  $b(T) = 0$ .

By substituting  $\phi(T)$ , the initial permit stock is finally described by:

$$b_0 = S_0 + S(T) \frac{-\mu(T)e^{-rT}}{-\int_0^T e^{-r\xi} d_S(S) d\xi + \mu(T)e^{-rT}} + \frac{\int_0^T e^{-r\xi} d_S(S) S d\xi}{-\int_0^T e^{-r\xi} d_S(S) d\xi + \mu(T)e^{-rT}} \quad (143)$$

$$b_0 = S_0 + \frac{\int_0^T e^{-r\xi} d_S(S) S d\xi - S(T)\mu(T)e^{-rT}}{-\int_0^T e^{-r\xi} d_S(S) d\xi + \mu(T)e^{-rT}} \quad (144)$$

### B.3 Determine terminal shadow price $\mu(T)$

As  $S(T) > 0$  the transversality condition (122) implies  $\lambda(T) = 0$  and with (119):

$$\mu(T) = p(T) - c(S(T)) \quad (145)$$

As in the optimum  $f_R(R(T)) - c(S(T)) = F_S(S(T))$  (see Proposition 1), it follows with  $p = f_R$  that:

$$\mu(T) = F_S(S(T)) \quad (146)$$

### B.4 Determine the resource rent

The rent  $\pi$  in the resource sector is determined by resource prices minus extraction costs, i.e.  $p - c(S)$ , and from (119) by:

$$\pi = \lambda + \mu \tag{147}$$

With the solution of the differential equation for  $\lambda$  (120) and the equation for  $\mu(t)$  (130), profits read:

$$\pi = (\mu(T) + \lambda(T))e^{-r(T-t)} - \int_t^T (d_S + c_S R)e^{r(t-\xi)} d\xi \tag{148}$$

## C Suboptimal Discount Rates

### C.1 Optimal resource tax in the cost-benefit approach

If the resource sector uses the discount rate  $\rho$  instead of the socially optimal discount rate  $r$ , the re-arranged private Hotelling rule (19) reads:

$$\rho(p - c(S)) = \dot{p} - \dot{\tau} + \rho\tau \tag{149}$$

The re-arranged socially optimal Hotelling rule (4) with  $p = f_R$  in the market equilibrium is:

$$r(p^* - c(S^*)) = \dot{p}^* - d_S^* \tag{150}$$

Substituting  $\dot{p}$  from (150) into (149), we obtain for the optimal solution:

$$\dot{\tau} = \rho\tau + d_S^* + (r - i)(p^* - c(S^*)) \tag{151}$$

Solving the ODE for given  $\tau(T)$  yields:

$$\tau(t) = \tau(T)e^{-\rho(T-t)} - \int_t^T d_S^* e^{\rho(t-\xi)} d\xi - (r - \rho) \int_t^T (p^* - c(S^*)) e^{\rho(t-\xi)} d\xi \tag{152}$$

In order to achieve the social transversality condition (11), we set  $\tau(T) = F_S(S^*(T))$ .

### C.2 Optimal resource tax under a carbon budget without ETS

Under the budget approach applies the private Hotelling rule from (149). The re-arranged socially optimal Hotelling rule (59), however, does not contain a damage term and reads with  $p = f_R$ :

$$r(p - c(S)) = \dot{p} \tag{153}$$

Substituting  $\dot{p}$  from (153) into (149) and solving the ODE for given  $\tau(T)$ , we obtain:

$$\tau(t) = \tau(T)e^{-\rho(T-t)} - (r - \rho) \int_t^T (p^* - c(S^*)) e^{\rho(t-\xi)} d\xi \tag{154}$$

In order to achieve the social transversality condition within the budget approach, we set  $\tau(T) = \mu_T^{CB}$ .

### C.3 Optimal resource tax under a carbon budget with ETS

Under the ETS with banking and borrowing, we have to consider the Hotelling rules (153) and (149) which yields to the same formula for the optimal tax as (154) without ETS. The social transversality condition, however, is already achieved by the limited permit stock, implying  $\tau(T) = 0$ , and thus:

$$\tau(t) = -(r - \rho) \int_t^T (p^* - c(S^*)) e^{\rho(t-\xi)} d\xi \quad (155)$$

## D Exhaustion of the Entire Carbon Budget

*Proof for  $C(T) = 0$  :*

Let us assume, that the permit stock is not exhausted, i.e.  $C(T) > 0$ . From (69) follows that  $\mu(T) = 0$  which implies that (with  $\lambda(T) = 0$  and (65))  $f_R(R(T)) = C(S(T))$ . As in the BAU case  $S^B(T) > 0$  and thus,  $\lambda^B(T) = 0$ , it follows that  $f_R(R^B(T)) = C(S^B(T))$  (where  $x^B$  denotes the corresponding variable in the BAU-scenario without the carbon budget constraint). Thus, we have:

$$f_R(R(T)) = C(S(T)) \quad (156)$$

$$f_R(R^B(T)) = C(S^B(T)) \quad (157)$$

From Assumption 3 follows that

$$S(T) > S^B(T) \quad (158)$$

Equations (156–158) imply together with  $f_{RR} < 0$  and  $c_S < 0$  that:

$$R(T) > R^B(T) \quad (159)$$

i.e. the final resource extraction under the scarce budget constraint is *higher* than without budget constraint.

As  $\int_0^T R dt < \int_0^T R^B dt$  and  $R(T) > R^B(T)$  there must exist a  $t^* : 0 < t^* < T$  with:

$$R(t^*) = R^B(t^*) \quad (160)$$

$$R(t) \geq R^B(t) \quad \text{for } t^* \leq t \leq T \quad (161)$$

In particular, this implies  $\int_0^{t^*} R dt < \int_0^{t^*} R^B dt$  and thus (considering  $c_S < 0$ )

$$c(S(t^*)) < c(S^B(t^*)) \quad (162)$$

The Hotelling rules for the budget and BAU problem read:

$$r = \frac{\dot{f}_R}{f_R - c(S)} = \frac{\dot{f}_R^B}{f_R^B - c(S^B)} \quad (163)$$

Using  $\dot{f}_R = f_{RR}\dot{R}$ , we get by rearranging (163) in  $t = t^*$ :

$$\underbrace{\dot{R}(t^*) - \dot{R}^B(t^*)}_{\geq 0 \text{ from (161)}} = \frac{r}{\underbrace{f_{RR}}_{< 0}} \underbrace{[c(S^B(t^*)) - c(S^B(t^*))]}_{> 0 \text{ from (162)}} \quad (164)$$

which leads to a contradiction as the right hand side is strictly negative while the left hand side is positive (or zero). Thus,  $C(T) > 0$  was a false assumption and it follows that  $C(T) = 0$ .  $\square$

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