Climate Policy Instruments in a Differential Stackelberg Game with Market Imperfections

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Abstract

Several aggregated macroeconomic models of climate change find social optimal economic paths maximizing welfare under climate change. But they widely neglect strategic behaviour of market participants that seek to maximize utility and profit in response to governmental actors' decisions. However, modelling a sectorally disaggregated economy can provide a detailed analysis of policy portfolios and their influences on incomes, profits and their distribution.

In this work we take intertemporal strategic interactions of actors into account by modelling a differential open-loop Nash game for the main economic sectors, including energy production and fossil resource extraction enterprises. Transactions are determined by equilibrium prices. The government acts as Stackelberg leader (of the other actors playing a Nash game) by changing factor prices for supply and demand by charging taxes. It influences the intertemporal paths of the followers in order to achieve the maximum intertemporal welfare subject to a given mitigation goal (that is an upper limit of accumulated emissions).

Analytical and numerical treatment of this model yields some robust results: (1) taxation of fossil resources achieves the mitigation goal in an optimal way, that is the Stackelberg equilibrium equals the social optimum. This is not the case for a pure energy tax. (2) If the government regulates the fossil resource extraction sector not to extract more than the mitigation goal admits, this leads to the social optimum as well. However, profits in the fossil resource sector exceed the revenues of the baseline scenario without any climate protection efforts. (3) A recent paper of Sinn (2007) suggests a constant-rate increasing subsidy for fossil resource extractors to reduce current emissions and an increasing resource tax to acclerate the global warming problem. In contrast, in our model optimal taxation shows an increasing resource tax.

Keywords: Climate Change; Upstream-Downstream Taxation; Prices vs. Quantities; Differential Game; Stackelberg Game.

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1 Introduction

In this paper we compare different climate policy instruments using a multi player differential game of economic sectors, households and government. Climate change has received increasing attention during the last years, in particular with the Nobel Price awarded to Al Gore and the Intergovernmental Panel on Climate Change (IPCC) in 2007. On the other hand, the problem of climate protection has been scientifically analysed for decades. In textbooks of economic theory, preventing climate change serves as a prime example for the provision of a global public good (e.g. Samuelson and Nordhaus, 2006). This entails market failure and incentives for free-riding behaviour. The resulting divergence of Nash equilibrium and Pareto optimum justifies government intervention. The most prominent example for such interventions are Pigouvian taxes (Pigou, 1932). Economic analyses of pollution problems noticed these problems early on (e.g. Coase, 1960; Dales, 1968) and applied game theoretic methods (e.g. Mäler, 1989). For a global public good without a single governement being able to correct market failures, the additional difficulties can be treated within non-cooperative game theory (e.g. Carraro and Siniscalco, 1993).

Integrated assessment models put less emphasis on strategic interactions between nation states in the international arena, but on ways and instruments to achieve optimal climate protection paths. One important subset of these models are dynamic optimization models of economic growth (e.g. Nordhaus and Boyer, 2000; Popp, 2004). Some of them consider damages from climate change and thus determine the optimal mix between mitigation costs and damage costs (e.g. Hope, 2006). Others take an upper limit for (cumulative) emissions, a so called mitigation goal, as given and determine cost-effective strategies to achieve such an objective (e.g. Edenhofer et al., 2005). Both approaches have in common that they take a social planner perspective that chooses an overall development path to optimizes overall welfare. They are capabable of considering very detailed technoligical options, but mostly neglect the strategic interaction of economic actors that may render socially optimal development paths impossible.

Evaluating the feasibility and efficiency of instruments requires an understanding of the strategic interaction of additional actors as economic sectors and households. Each set of instruments requires carefully modelling the implied structure, in particular the control variables and the game theoretical interactions of multiple players (cf. Eisenack et al., 2006).

We consider such interactions within a sectorally disaggregated economy. We introduce an intertemporal growth model of a closed economy featuring an energy sector, a fossil resource extraction industry with non-linear extraction costs, a consumption good sector, and households. As Stackelberg leader, government strives for optimizing household welfare under a mitigation goal, coming as a constraint on cumulated greenhouse gas emissions over the planning interval.[1] We take the mitigation goal as exogeneously given. It can result from an international agreement such as the emissions reduction commitments of the Kyoto protocol. To achieve the goal, different instruments are considered. The government can impose taxes or subsidies on labour, capital, energy, and resource prices. It can also limit the amount of cumulative resource flow utilized by the economy. The government first sets the time paths of taxes, and the different sectors then play a market game resulting in equilibrium quantities and prices. We assess whether the equlibrium of the overall game achieves the climate protection target in an efficient way.

Although often not explicitly stated, Stackelberg games are standard in the economic theory of taxation. One basic result of this theory is that the tax incidence is independent from whether a tax is payed by those who buy or those who sell a good. The market participant with the a less elastic response to price changes bears a larger burden of taxation (Harberger, 1962).

However, classical proposals for pollution control recommend taxation of emissions, i.e. taxing the out-

[1] We simplify the case by equating emissions with extracted fossil resources. This is based on the natural law of mass conservation, i.e. that every carbon atom oxidized by burning coal or oil corresponds to a carbon atom emitted. This neglegts other greenhouse gases and differing characteristics of fuels, but facilitates the comparison of our results with the work on fossil resource taxation.

put of those sectors that actually produce the emissions. The basic results above provide the intuition that taxing the sectors that supply inputs to the polluting sector would be equivalent. While the former are called down-stream taxes, the latter are named up-stream taxes. Sugeta and Matsumoto (2007) compare both in the case of a monopolist causing emissions. They show that a shift from up-stream to down-stream taxation reduces government revenues and increases social welfare. We will validate whether this also holds in our model.

Dasgupta et al. (1981) show that various types of taxes are able to achieve a prescribed path of resource extraction. However, with vanishing extraction costs, not all tax schemes used in practice can achieve that. Sinn (2007) shows that, under the assumption of constant elasticity of resource demand, bounded extraction costs and infinity planning horizon, a resource tax shifts extraction (and therefore greenhouse gas emissions) from the future to the present and vice versa.

Taxes are price instruments. In contrast, quantity instruments limit the amount of emissions for actors, transferable emission certificates being a prime example for the latter (cf. Dales, 1968). Although price and quantity instruments are equivalent in an idealized economy, the debate is yet unsettled. Uncertainties, institutional arrangements, and transaction costs are added to the basic set-up (e.g. Weitzman, 1974; Stavins, 1995; Newell and Pizer, 2003). Our study contributes to this debate by exploring both price and quantity instruments and comparing their effects.

We find that a pure tax on fossil resources, i.e. an up-stream tax, can achieve optimal emission reductions in the game equlibrium. In contrast, a down-stream tax on energy is not sufficient. Even less efficient are taxes on labour or capital. The resource tax leads to substantial government income that is redistributed to households as transfer income. In contrast, when the extraction sector is regulated with a quantity instrument, transfer incomes vanish and additional rents are appropriated by the resource owners. Still, the resulting equilibrium is socially optimal. This shows, contrary to Sinn (2007), that taxes can achive a mitigation goal.

We begin the following section with a description of the model, including the different policy instruments, and show some algebraical properties. We then outline how the mixed Stackelberg-Nash game with mutiple actors is solved numerically using standard optimization software. Based on that we provide a set of numerical experiments to assess the instruments. Main results are confirmed by further algebraic considerations. We conclude by reflecting on these results.

2 Model description

We introduce the assumptions on all actors and the governments problem (as Stackelberg leader) under the assumption that the other actors play a Nash game where prices for supply and demand are in equilibrium (see Fig. 1 for the game structure).

2.1 The Stackelberg game

Households Households are assumed to be homogeneous and to dispose intertemporally over the total capital stock k and labour input L. Current utility $u(C, L)$ is assumed to be strictly concave, increasing in consumption C and decreasing in L. The aggregate capital stock changes with investments I and depreciates at rate δ,

$$\dot{k} = I - \delta k. \tag{1}$$

It can be decomposed into sector specific capital stocks k_P, k_E, and k_R

$$k = k_P + k_E + k_R. \tag{2}$$

Here and in the following, the subscripts P, E and R stand for the production, energy, and resource sector, respectively. Capital generates a gross interest rate r. Due to taxation, it may be reduced to the net interest rate $\bar{r}$. Labor is compensated at the wage rate w. Therefore, households make decisions subject to the budget constraint

$$C = wL + \bar{r}k + \Gamma - I + \pi, \tag{3}$$

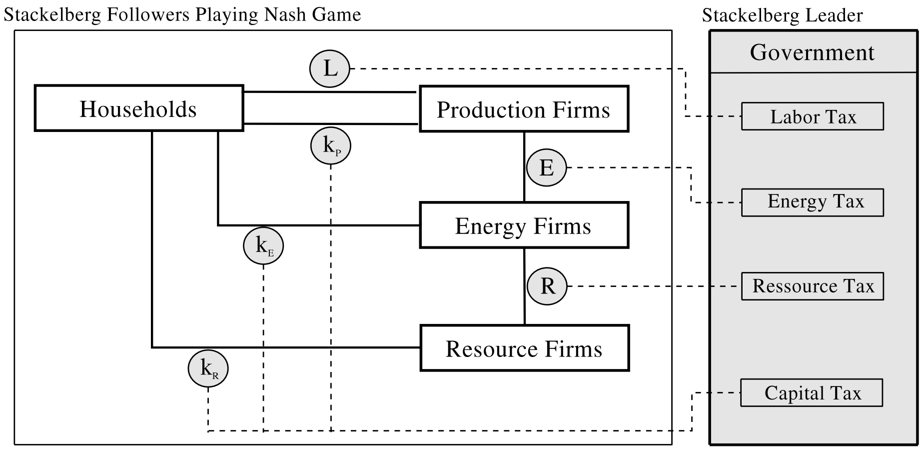

Figure 1: Structure of the differential game. The strategic variables of market participants are labor L, energy E, fossil resources R and the sectoral capital stocks k_P, k_E and k_R. The Stackelberg leader can influence these variables by charging taxes on prices of the specific factors.

where Γ represents potential lump-sum transfers to or from the government. In this formulation, capital is taxed on the income side.[2] Total profits

$$\pi = \pi_P + \pi_E + \pi_R \tag{4}$$

(as sum of sectoral profits) augment the households budget. The objective of households it to select paths $C(\cdot), L(\cdot), I(\cdot)$ in order to maximize the objective functional

$$J_H = \int_0^\infty u(C, L)e^{-\rho t}dt, \tag{5}$$

with the pure rate of time preference ρ for given paths $w(\cdot), \bar{r}(\cdot), \pi(\cdot)$ and $\Gamma(\cdot)$, depending on government policies and the strategies of the other actors. The best open-loop reaction can be determined from the current value Hamiltonian

$$H_H = u(C, L) + \lambda_H(wL + \bar{r}k + \Gamma - C - \delta k + \pi), \tag{6}$$

where λ_H denotes the shadow price of capital for housholds. The first order conditions are

$$u'_C = \lambda_H, \tag{7}$$

$$u'_L = -\lambda_H w, \tag{8}$$

$$\dot{\lambda}_H = \lambda_H(\rho + \delta - \bar{r}). \tag{9}$$

Here and in the following, f'_x, f''_x denote partial derivatives of a function f with respect to its argument x. Eq. $(7-9)$ are sufficient for optimality due to the strict concavity of u and the concavity of Eq. (1), if the transversality condition

$$\lim_{t\to\infty} \lambda_H k e^{-\rho t} = 0. \tag{10}$$

holds. Using Eq. (9), this is equivalent to

$$\lim_{t\to\infty} k e^{\int_0^t (\delta - \bar{r})ds} = 0. \tag{11}$$

That is, the capital stock has to be reduced to zero if the net interest rate $\bar{r}$ is smaller than the depreciation rate δ. Should the net interest rate be higher than depreciation, the transversality condition can be achieved with a moderate growth of the capital stock.

[2] As recalled in the introduction, it makes no difference whether taxes are imposed on the supply side or the demand side.

Production sector Producers of consumption goods select the inputs k_P, L and E (capital, labor and energy) to maximize profits

$$\pi_P = f(k_P, L, E) - rk_p - \bar{w}L - \bar{p}_E E. \tag{12}$$

They are price takers with respect to given factor prices $r, \bar{w}, \bar{p}_E$, the latter denoting the net market prices for labor and energy. The production function f has the common neoclassical convexity properties. Energy and labour costs are payed in gross prices, since they are texed on the demand side, and capital is taxed on the income side. The price for consumption goods is normalized to one as numeraire. Note further that there is no intertemporal decision made in the production sector.[3] The reaction of the sector to given prices yields the standard conditions

$$f'_{k_p} = r, \tag{13}$$
$$f'_L = \bar{w}, \tag{14}$$
$$f'_E = \bar{p}_E. \tag{15}$$

Energy sector The energy sector selects capital k_E and fossil resources R as inputs for energy production to maximize current profits

$$\pi_E = p_E g(k_E, R) - \bar{p}_R R - rk_E, \tag{16}$$

at a net resource price $\bar{p}_R$. Again, we assume a neoclassical production technology g. As no intertemporal allocation decision is made, the optimal response to a given price path is

$$E = g(k_E, R) \tag{17}$$
$$g'_R = \frac{\bar{p}_R}{p_E}, \tag{18}$$
$$g'_{k_E} = \frac{r}{p_E}, \tag{19}$$

Resource sector The resource sector faces an intertemporal decision due to a limited fossil resource stock s, that is reduced depending on extraction R by

$$\dot{s} = -R = -h(k_R, s), \tag{20}$$

with a production function h that is linear in k_R, and convex increasing in the resource stock s. Capital productivity falls with the decreasing resource stock. The resource sector takes the gross resource price p_R as given and determines the input k_R to maximize

$$J_R = \int_0^\infty (p_R h(k_R, s) - rk_R)\, e^{\int_0^t(-\bar{r}+\delta)} ds. \tag{21}$$

Profits at time t are

$$\pi_R = p_R h(k_R, s) - rk_R. \tag{22}$$

The associated current value Hamiltonian is

$$H_R = p_R h(k_R, s) - rk_R + \lambda_R \dot{s}, \tag{23}$$

such that the first order conditions evaluate to

$$r = (p_R - \lambda_R) h'_{k_R}, \tag{24}$$
$$\dot{\lambda}_R = (\bar{r} - \delta)\lambda_R - (p_R - \lambda_R) h'_s, \tag{25}$$

where λ_R is the shadow price of the resource. Note that by substituting Eq. (24) and its derivative with respect to time into Eq. (25), one obtains a modified Hotelling rule

$$\bar{r} - \delta = \frac{\dot{p}_R - \frac{\dot{r}}{h'_{k_R}}}{p_R - \frac{r}{h'_{k_R}}}, \tag{26}$$

which simplifies to the original formulation when extraction costs vanish (i.e. $(h'_{k_R})^{-1} = 0$).

To account for possible quantity instruments from government, we introduce the parameter s_c, defining the constraint

$$s \geq s_c. \tag{27}$$

A minimal amount of the resource that should not be extracted can be introduced by setting $s_c = \underline{s} > 0$. This is an externality that is anticipated by market participants and requires a modified transversality

[3] Applying the Maximum Principle of dynamic optimization to an intertemporal production sector yields the same conditions as the static optimization problem. The same holds for the energy sector.

condition

$$0 = \lim_{t\to\infty} \lambda_R(s - s_c)e^{\int_0^t(-\bar{r}+\delta)ds}. \qquad (28)$$

If $s_c = 0$, the resource sector can extract the entire resource stock if it is profitable.

The Government Together with the initial resource stock s_0, a given set of tax paths and a given quantity constraint s_c, the above conditions completely determine the joint intertemporal market response of all economic sectors. Since a market equilibrium can be described as the solution of a Nash game, this can be seen as a Nash game that is parameterized by exogeneously set government interventions. We now introduce the government as a Stackelberg leader that sets these parameters to optimize its objective functional subject to a policy target (the mitigation goal).

The following options are at the government's disposal. Price instruments are represented by charging ad-valorem taxes $\tau_k, \tau_L, \tau_E, \tau_R$ on capital, labor, energy and resource prices, respectively, such that $\bar{r} = r(1+\tau_K), \bar{w} = w(1+\tau_L), \bar{p}_E = p_E(1+\tau_E)$, and $\bar{p}_R = p_R(1+\tau_R)$. As a quantity instrument the amount of non-extractable fossil resources s_c can be set to a positive value. The mitigation goal is formulated by the constraint

$$s \geq \underline{s}, \qquad (29)$$

for the government, i.e. the climate policy target is an upper bound for cumulative resource extraction over the entire time horizon. The government therefore has to chose instruments such that extracted amount of fossil resources never exceeds $s_0 - \underline{s}$.

Under these conditions, the government seeks to maximize the same objective as housholds, i.e.

$$\int_0^\infty u(C,L)e^{-\rho t}dt, \qquad (30)$$

subject to a balanced government budget

$$-\Gamma = \tau_K rK + \tau_L wL + \tau_E p_E E + \tau_R p_R R. \qquad (31)$$

The Stackelberg leader also takes into account the budget constraints, equations of motion, production technology and implicit reaction functions of followers as listened in Tab. 1.

2.2 Numerical implementation

To perform an analysis of policy instruments in the subsequent section we use, in addition to analytical computations, a parameterized numerical version of the model[4]. The time-continuous differential Stackelberg game is transformed to a discrete one with finite time horizon. The Discrete Maximum Principle is used to determine first-order and transversality conditions of the followers that serve as implicit reaction functions. The optimal strategy of the Stackelberg leader is computed by numerical optimization with the first-order and transversality conditions of the followers as analytical contraints.[5]

Production of consumption goods is expressed by a nested CES-technology

$$f(k_Y, L, E) = (a_1 z^{\sigma_1} + (1-a_1)E^{\sigma_1})^{(1/\sigma_1)} \qquad (32)$$

$$z(k_Y, L) = (a_2 k_Y^{\sigma_2} + (1-a_2)L^{\sigma_2})^{(1/\sigma_2)} \qquad (33)$$

with z being a composite of capital and labor (cf. Kemfert and Welsch, 2000) and $\sigma_1, \sigma_2 < 0$, such that production factors are only substitutable to limited degree. Energy is produced by a CES technology with $\sigma < 0$ (cf. Edenhofer et al., 2005),

$$g(k_E, R) = (ak_E^\sigma + (1-a)R^\sigma)^{(1/\sigma)}, \qquad (34)$$

i.e. it is difficult to substitue capital for fossil resources. Resource extraction uses capital as input with a rising capital intensity at diminishing reserves (cf. Edenhofer et al., 2005),

$$h(k_R, s) = c(s)k_R, \qquad (35)$$

$$c(s) = \frac{\chi_1}{\chi_1 + \chi_2\left(\frac{s_0 - s}{\chi_3}\right)^{\chi_4}}. \qquad (36)$$

[4]Code and model paramters are availble from the authors upon request

[5]We used the NLP solver *conopt3* of the software package GAMS (General Algebra Modelling System, Brooke et al. (2005)).

sector	eq. of motion	budget constraints	technology	reaction function
household	1	2,3,4		7, 8, 9, 10
final products		12		13, 14, 15
energy		16	17	18, 19
resources	20	22	20	24, 25, 28

Table 1: Constraints for the Stackelberg leader, table entries referring to Equation numbers.

Household and government utility are defined as

$$u(C, L) = ln(C) + ln(L_{max} - L), \quad (37)$$

where L_{max} determine total available labor.

3 Evaluation of policy instruments

Keeping the constraint of a mitigation goal requires institutional market interventions. If the goal is justified by externalities due to greenhouse gas emissions, this represents an internalization strategy. This section analyzes the capability of different instruments for that purpose and evaluates them with respect to social optimality and distributional criteria. The following portfolios of instruments are investigated:

1. A quantity instrument restricting the amount of extracted fossil resources (suffix `m_q` is added subsequently in figures).

2. Up-stream taxation where only a resource tax is imposed, while all other taxes vanish (suffix `m_r`).

3. Down-stream taxation, where only an energy tax is imposed (suffix `m_e`).

4. Hybrid taxation with an energy tax and a specific captial tax for the energy sector (suffix `m_e_ke`).

5. Taxation of capital and labour solely (suffix `m_k_l`).

We evaluate them mainly relative to two scenarios: the *business as usual* scenario (BAU) of the social optimal solution, where no mitigation goal is set, i.e. no climate policy efforts are undertaken, and the *reduction* scenario (RED) with a mitigation goal constraining government. We further evaluate distributional effects for different household income sources and the degree of government intervention required in game equilibrium.

Before doing so, we introduce the benchmark model for the socially optimal outcome and the further criteria. We then discuss the basic properties of the instruments and conclude this section with their comparison.

3.1 Social planner

The social planner model serves as benchmark for the decentralized market model. It is a Pareto optimal outcome of the economy without any strategical behaviour of actors. By knowing the optimal intertemporal path for a given mitigation goal, the game equilibrium for a set of instruments can be assessed of whether it achieves or comes close to optimum. The social planner maximizes household utility Eq. (5) by setting optimal levels of consumption and sectoral capital allocation under the macro-economic budget constraint

$$C = f(k_P, L, E) - I, \quad (38)$$

technology constraints and equation of motion as listened in rows two and four in Tab. 1, and the capital constraint Eq. (2). For a detailed view of first-order conditions see appendix A. The social planner solution can be determined for the RED and the BAU scenario, the latter by setting $\underline{s} = 0$, and the former by choosing a positive value for $\underline{s}$.

In the BAU scenario, an almost stable consumption level is achieved after a short initial growth phase. In contrast, the RED scenario shows consumption falling rapidly after a short period of economic growth (see Fig. 2). The substitution effects of mitigation are shown in Fig. 3 and Fig. 4 for the production and the energy sector, respectively.

There are only to ways to deal with the politically constructed scarcity of fossil resources: (1) reduction of consumption and (2) factor substitution. The reduction of consumption affects the whole production chain and also decreases energy and resource demand. This causes high welfare losses although consumption reduction can be compensated partly by higher leisure. In the production sector energy is partly substituted by higher capital and labor input. The energy sector substitutes capital for resource input. The lower the substitutability of resources in the energy sector is, the higher are welfare losses due to higher consumption reduction. The same holds for the elasticity of substitution of energy in the production sector. The considerable reduction of consumption for substitution parameters σ, σ_1 smaller than zero highlights the dependence of the whole economy on fossil resources and the resulting difficulty to deal with a stringent climate policy target.

As considered by Ströbele (1984), there is no chance for economic long term growth if elasticities of substitution of fossil resources and energy are less than one (i.e. $\sigma < 0$ and $\sigma_1 < 0$) and no further technological progress is assumed. Consumption declines to zero in the long run.

3.2 Distributional criteria

The budget constraint of households Eq. (3) can be decomposed to income from labour wL, income from capital $\bar{r}K$, income from the resource sector's profits π_R, and (positive or negative) transfer incomes from government Γ[6]. Depending on taxes and their effects on prices and allocation, the share of these different incomes may differ between policy instruments. Knowing the changes of income types with respect to the BAU scenario may be an indicator for the political feasibility and normative evaluation of policy instruments.

Income shares are calculated as discounted sum over time to account for the household time preference rate. Because most part of income is consumed, it is discounted at households time discount rate ρ instead of the net interest rate $\tilde{r} = (1 - \tau_k)r - \delta$. This furthermore simplifies the comparability of policy instruments, as they lead to different tax rates.

To have an impression of the state's quantitative influence on the economic system to achieve a mitigation goal with a given policy instrument, we define a function $Gov(t)$ that measures the amount of flows controlled by the government. $Gov(t)$ is computed as the sum of the absolute values of all tax flows, divided by output of consumption goods $f(k_P, L, E)$, i.e.

$$Gov(t) = \frac{\sum_i |\tau_i(t) p_i(t) q_i(t)|}{f(k_p(t), L(t), E(t))} , \qquad (39)$$

where p_i and q_i denote factor prices and quantities $i \in \{E, R, K, L\}$[7]. As a sum over time, discounted with ρ, we use this quantity as indicator for government intervention.

3.3 Policy instruments

In the BAU scenario the social planner solution is equivalent to the market solution when all taxes vanish. This is due to the absence of any externalities and the reasonable convexity properties of the model. For the RED scenario, the government faces the problem to make the market actors keep the mitigation goal. The success of a policy instrument lies in its capability to set the "right" price or quantitiy signals to actors and hence to achieve the optimal combination of comsumption reduction and factor reallocation.

Quantity instrument The quantity instrument restricts accumulated resource extraction directly by setting $s_c = \underline{s}$ for the extraction sector in the transversality condition Eq. (28). This means the announce-

[6] As a constant return-to-scale technology is used in production and energy sector Eq. (32–34), profits in both sectors are zero.

[7] This ratio can be greater than one, because the sum of absolute tax flow augments government activity due to subsidies although the latter decrease government net income.

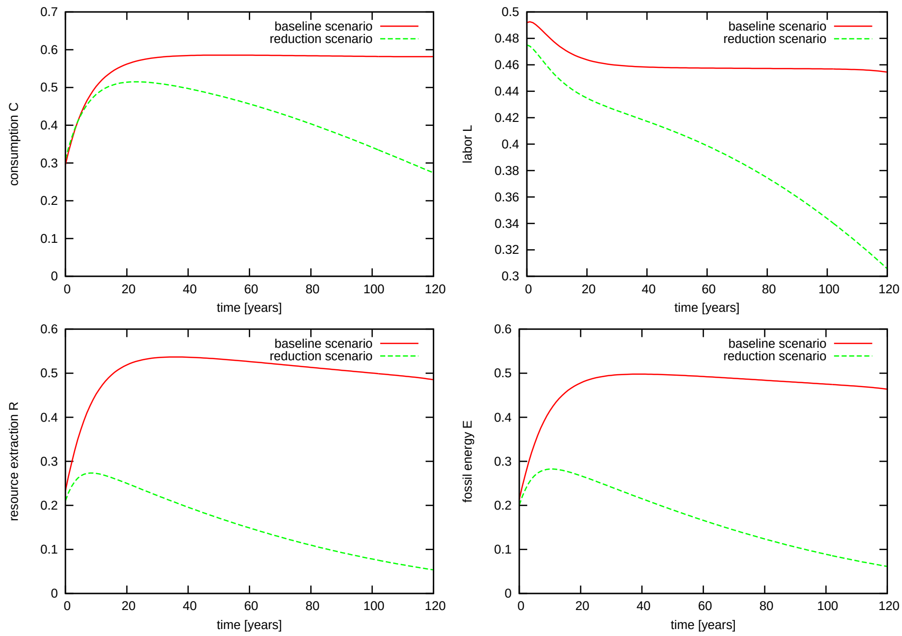

Figure 2: Key flows in the socially optimal BAU and RED scenario.

ment of a credible regulation by government to forbid any extraction below that line. This may be achieved by emitting pollution certificates whose cumulated amount is restricted due to international agreements. The extraction sector then anticipates the mitigation goal. All taxes are set to zero.

Obviously, the resulting extraction path complies with the emissions target, since this is formally required. The numerical experiments show that the quantity instrument also achieves social optimum. Compared to the BAU scenario, the resource price and profits in the resource sector raise dramatically (see Fig. 6).

As shown in appendix B, the game equilibrium is socially optimal if $s_c = \underline{s}$, being a result of the anticipated mitigation goal. The high resource price reflects the mitigation goal and induces factor reallocation in the energy sector. As the high resource price indirectly raises the energy price, the price signal is propagated to the production sector, which reduces its energy input as a consequence (see Fig. 3 and 4). The lower the elasticity of substitution in the energy and production sector, the higher are resource profits. Since the mitigation goal is not achieved by taxes, i.e. without government income, the associated resource scarcity rents are appropriated by the extraction sector.

For later reference, we label the socially optimal values of the quantity instrument with an asterix, p_R^*, p_E^*, s^*, etc.

Resource tax In absence of a quantity restriction, resource extractors do not anticipate the mitigation goal directly. Instead, the ad-valorem tax on the resource price drives a wedge between selling price p_R of extraction sector and purchase price $\bar{p}_R$ of the energy sector. To comply with the mitigation goal in an efficient way, this has to result in the same allocative effect as the quantity instrument.

In the numerical experiments the mitigation goal is achieved without welfare losses compared to the social planner RED scenario. The resource tax increases up to 80. Tax income increases to 60% of total household income. The profits in the resource sector fall slightly compared to the BAU market model.

The taxed purchase price $\bar{p}_R$ equals exactly the resource price p_R^* for the quantity instrument: the resource tax gives the same price signals to the energy sector. The high tax rate is necessary because demand is very inelastic with respect to price changes due to low σ and σ_1 values. The reduction of resource use is primarily demand driven.

We can characterize the social optimal resource tax algebraically as follows. Assuming its existence, all system variables have to equal those of the quantity instrument, except for p_R and λ_R. Substituting the values of the quantity instrument in Eq. (24) yields

$$r^* = (p_R - \lambda_R) h'^*_{k_R}. \quad (40)$$

Since, by Eq. (18), $p_R(1+\tau_R) = \bar{p}_R = p_E^* g'^*_R = p_R^*$, we can transform this to

$$1 + \tau_R = \frac{p_R^*}{\frac{r^*}{h'^*_{K_R}} + \lambda_R}. \quad (41)$$

Given the (unique) solution for $\lambda_R(\cdot)$ from Eq. (25) and Eq. (28), the resource tax is determined explicitly for every instant t. Charging a resource tax as stated in Eq. (41) is sufficient to reach the social optimum of the RED scenario as all other variables and first-order conditions equal those of the quantity policy scheme.

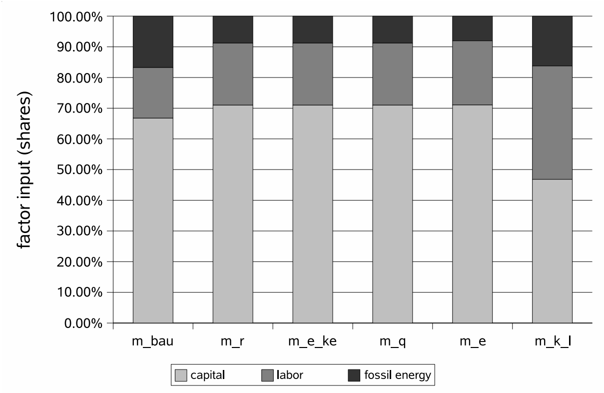

Figure 3: Share of factor inputs in production sector.

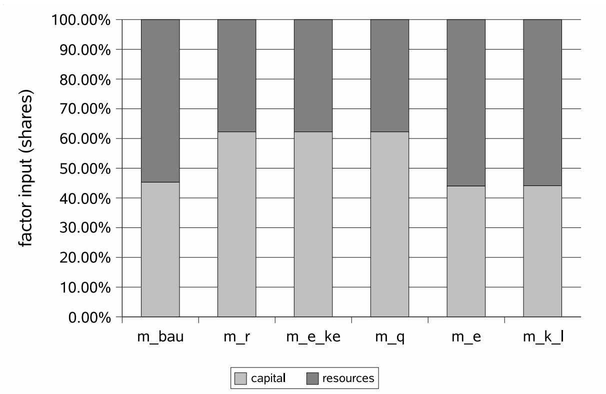

Figure 4: Share of factor inputs in energy sector.

Energy tax The energy tax changes the purchase price $\bar{p}_E$ of total energy for the production sector. This instrument reaches the mitigation goal in the numerical experiments, but allocation of consumption and labour is below the social optimum. The energy tax increases up to a factor of 50, but is always lower than the resource tax above.

The following short argument sketches the problem of the pure energy tax. The demand for energy E depends on energy price p_E and energy tax τ_E. To reach the mitigation goal the resource path R has to be changed by decreasing demand via taxes on p_E or p_R. Because of the CES technology in fossil energy sector, the ratio of factor inputs is known to be characterized by

$$\frac{K_E}{R} = \left(\frac{p_R(1+\tau_R)}{r}\frac{a}{(1-a)}\right)^b$$
$$= (1+\tau_R)^b \left(\frac{p_R}{r}\frac{a}{(1-a)}\right)^b, \qquad (42)$$

with the elasticity of subsitution $b = \frac{1}{1-\sigma}$. That is, the ratio of factor inputs depends only on prices r and $\bar{p}_R$. An energy tax reduces energy demand E and consequently demand for the inputs R and K_E in the energy sector as well. But the ratio of K_E/R remains unchanged because no changes in the prices r and p_R occur.

The incapability of a pure energy tax to reallocate factor inputs in the energy sector in a social optimal way can also be seen in Fig. 4. The ratio of resource and capital input remains the same as in the BAU scenario, while for the (optimal) resource tax and quantity instrument, more resource input is substituted by capital. As the energy tax changes the purchase price of energy in the production sector, the share of factor allocation in this sector is the same as in the optimal case (see Fig. 3). In short, with reference to the Slutsky equation, an energy tax has an income effect, but no substitution effect in the energy sector. It is only capable of achieving a mitigation goal by reducing overall energy consumption, resulting in an inefficient mix of capital and resource inputs in the energy sector.

Eq. (42) leads to the conclusion that with an adequate small elasticity of substitution b the misallocation of the energy tax should diminish to zero, because factor shares become independent from prices if $b = 0$. A numerical parameter study confirms this result by comparing welfare losses of the energy tax with the social optimum in the RED scenario (see Fig. 5).

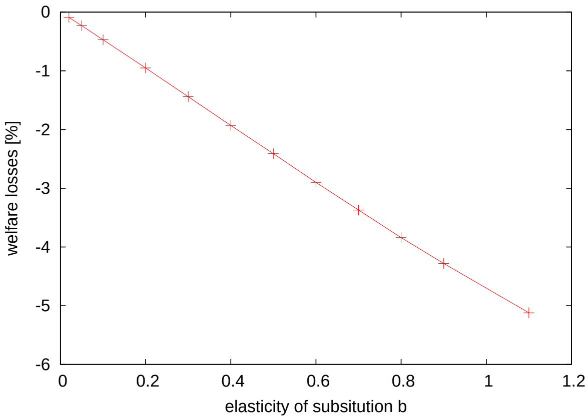

Figure 5: Welfare losses of `m_e` policy relative the socially optimal RED scenario as function of the sector specific elasticity of substitution.

Hybrid tax Seeing the missing substitution effect of a pure energy tax in the energy sector, it is interesting to know whether this can be corrected by augmenting it with a capital tax. As the interest rate r depends on the capital tax rate, a well chosen capital tax would enforce an optimal factor ratio in Eq. (42). But this would have further consequences on the whole economic system caused by the distortions of the capital tax in other sectors. We therefore introduce a sector-specific capital tax τ_{K_E} that affects the energy sector only by charging interest rates for k_E.

This indeed leads to the socially optimal trajectory (see Fig. 3 and 4). While the energy tax is higher than in the case of the pure energy tax, and almost equals the pure resource tax, the sector specific capital tax is negative starting from 20% and reaching a maximum of 99%. That is, capital in the energy sector is strongly subsidized.

Given the optimal pure resource tax τ_R^*, one can calculate the equivalent optimal specific capital tax $\tau_{K_E}^*$ that leads to the same factor share K_E/R. Utilizing the identity of the factor share under both tax approaches ($\tau_R = \tau_R^*, \tau_{K_E} = 0$) and ($\tau_R = 0, \tau_{K_E} = \tau_{K_E}^*$) Eq. (42) leads to

$$\frac{p_R(1+\tau_R^*)}{r} = \frac{p_R}{r(1+\tau_{K_E}^*)} \qquad (43)$$

that can be solved to

$$\tau_{K_E}^* = \frac{1}{1+\tau_R^*} - 1. \qquad (44)$$

If $\tau_R^* > 0$ (which is the case), then $\tau_{K_E}^*$ has to be negative and for high values of τ_R^* the capital tax converges to -1, which coincides with the numerical observation. As side effect the capital subsidy reduces the price of energy and therefore increases energy demand in the production sector. To reach the mitigation goal, the energy tax has to compensate this effect to reach the mitigation goal.

Capital and labor tax If a specific capital tax can have a correcting effect, we may ask whether a general capital tax can achieve this. To keep the answer short, some numerical experiments show that this is generally not the case. Another option is to use the general capital tax combined with labour tax.

The combined tax leads to significant lower consumption and higher leisure. The mitigation goal can only be achieved with strong welfare losses compared to social optimum. They are much higher than for the (suboptimal) pure energy tax (see Fig. 6). Compared to the other instruments, resource extraction is shifted to the present. Both taxes are positive, with capital tax increasing from 10% to 95% an labor tax from 2% to 21%.

The high capital tax leads to lower capital accumulation, allowing more consumption in the present. By anticipating the increasing interest rate, resource extractors shift extraction forwards (cf. Dasgupta et al., 1981; Sinn, 2007). The labor tax has two effects: it compensates some welfare losses by higher leisure, and it reduces production of consumption goods to reach the mitigation goal. However, as Fig. 3 shows, the relative share of labor input is higher than for other policies and in the BAU scenario, because the higher capital tax dominates the labor tax. As with the pure energy tax, only an income effect but not a substitution effect is achieved further down the production chain. Thereby the ratio of inputs in the energy sector is the same as in the BAU scenario (Fig. 4).

3.4 Comparison

Tab. 2 and Fig. 6 summarize the effects of the different policy instruments in comparison to the socially optimal BAU scenario. As shown above, the quantity instrument, a pure resource tax, and a hybrid energy tax with a sector-specific capital subsidy reach the mitigation goal in an optimal way. A pure energy tax and taxation of capital and labor tax fail to reallocate factor shares in an optimal way due their more unspecific nature. They also show a lower total household income.

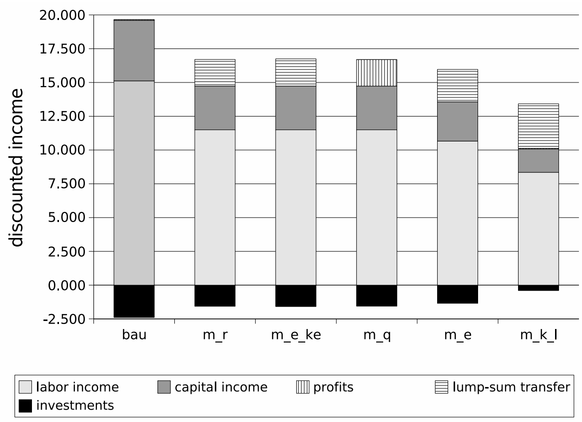

Figure 6: Decomposition of income: BAU and RED scenario for different policy instruments.

The distributional effects of the pure resource tax and the hybrid tax are identical, but the quantity instrument transfers resource scarcity rents – caused by a reduced resource stock available for extraction – from the government to the resource owners. Their volume substantially exceeds the nearly vanishing resource rents in the BAU scenario. With are resource tax, this rent mainly remains with the government such that the resource sector is forced to sell at very low net prices. The energy tax increases the share of transfer incomes, and taxation of capital and labour even more. The latter also has highest ratio between labor and capital income. The mitigation goal is mainly reached by reducing savings and growth, but not by substituting capital for fossil resources.

The degree of governement intervention *Gov* of the resource tax and the hybrid tax differs, although they have the same welfare and distribution effects. Because of the capital subsidy government activity is increased, although tax income lower. The higher *Gov* value for the other policies coincides with the

	m_r	m_e_ke	m_q	m_e	m_k_l
welfare (%)	−5,42	−5,42	−5,42	−7,13	−13,15
consumption (%)	−12,29	−12,29	−12,29	−15,44	−24,56
output (%)	−15,07	−15,07	−15,07	−19,00	−31,72
labor (%)	−8,19	−8,19	−8,19	−10,60	−18,39
energy (%)	−47,35	−47,35	−47,35	−53,38	−50,87
Gov	23,67	28,37	0	30,25	49,05

Table 2: Comparison of discounted key variables and their changes relative to the BAU scenario. The discounted *Gov* value is not relative to BAU scenario as there is no governmental activity.

higher transfer incomes. The quantity instrument works without any taxes, such that *Gov* vanishes.

4 Conclusions

We have investigated five different policy instruments to achieve the same climate protection target. The analysis was based on a combined Stackelberg-Nash differential game of different economic sectors that portrays government as a strategic actor. First order conditions of Stackelberg followers are used for analytical conclusions. In a novel hybrid approach, algebraic reaction functions of the followers were combined with a numerical algorithm for the leader to efficiently compute quantitative experiments.

Restricting the quantity of extracted fossil resources by a quantity instrument is a simple and efficient way to reach the mitigation goal without charging taxes. As stated by Weitzman (1974), a quantity instrument is more robust than a price instrument, since the latter depends on parameters afflicted with great uncertainties.

Differences to other instruments occur in the income distribution. They have to be seen in the light of political power of actors who want to influence decision processes. Resource owners would greatly favour a quantity instrument for taxation, even if the quantity restriction where not credible for them. In constrast, households who mainly rely on labour income would bear higher costs of emission reduction.

Similar to the quantity instrument, a resource tax operates at that position in the economy that is most closely associated with the volume of greenhouse gas emissions. This makes them very efficient compared to a pure energy tax. The latter is not socially optimal if capital and mineral resources are incomplete substitutes for energy production. For typical substitution elasticities $b > 0.2$ (e.g. $b = 0.3$ in Edenhofer et al., 2005), welfare losses are greater than one percent. However, if combined with a specific capital subsidy in the energy sector, a substitution effect can augment an energy tax to achieve an efficient mitigation path. The feasibility of this approach depends on whether a sectoral subsidy can be implemented and whether a stronger government intervention is realistic.

Taxation of capital and labour to achieve greenhouse gas emission reductions is the worst option with respect to welfare since it operates in the most unspecific way. Substitution effects are small and the mitigation goal is achieved mainly by reducing growth. This can only be justified as instrument for climate protection if it can be accepted as price for higher labour income. In addition, this instruments gives a benchmark for the capability of households to reach the mitigation goal by “climate friendly” labor and consumption levels without governmental measures.

In addition to determining the effects of different policies, we can speculate about the number and entry points of instruments that are generally needed to achieve a mitigation goal in an efficient way. This is not trivial when multiple market imperfections (as emission caps or monopoly) are relevant. Intuition may suggest that one instrument is needed per market imperfection, such that government can correct each of them. However, one of our main results is that it is primarily necessary to introduce taxes at the right entry point. Otherwise, even multiple taxes may not be sufficient. The success of a certain policy instrument lies in its capability to set price signals to the market that contain the information about the limited availability of fossil resources. These signals should meet the sector that is closest to the emissions and that has an allocative decision problem: the energy sector. The quantity instrument gives the exact price signals because scarcity is anticipated by resource owners; a resource tax augments the resource price artificially to the same level as the quantity instrument. If price signals only meet higher levels of the production chain, this results in misallocation in lower levels.

Sinn (2007) showed for a similar model of resource extraction, that the entire resource stock is exhausted for an infinite time horizon if elasticities of demand of resources are constant and extraction costs are limited. He concludes that an increasing resource tax would worsen the climate problem (and an increasing resource subsidy may help to overcome it). We cannot validate this statement in our model. Numerical experiments show that such a subsidy encourages the resource sector to extract more resources than in the BAU scenario. Reasons for this discrepancy lie in the different formulation of the mitigation goal and the consideration of a finite time horizon in numerical calculations. Nethertheless, this point shows the high sensitivity of policy recommendations to only few and seemingly negligible model assumptions.

The results so far indicate that the taxes to achieve a mitigation goal tend to be very high, in particular if extraction costs and elasticities of substitution are low and the goal is ambitious. Since capital and labour cannot completely substitute energy, and since also the energy sector cannot completely substitute capital for fossil resources, the mitigation goal can only be achieved if production is reduced. This requires the government to reduce household consumption with high taxes. It is therefore plausible that by introducing renewable energies into the energy sector and endogenous technological change in both production and renewable energy sector, the social costs of the mitigation goal may be substantially reduced. Further work will explore these possibilities and the additional policy instruments they may require.

We think that a game theoretic analysis as presented in this paper would provide further novel insights for climate policy, if further sectoral structures, market imperfections and policy instruments were considered.

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A Analytical properties of the social optimal solution

The Hamiltonian with nested production function

$$\tilde{f}(k_P, k_E, k_R, L, S) := f(k_P, L, g(k_E, h(k_R, S)))$$

reads

$$\begin{aligned} H = & u(\tilde{f}(k_P, k_E, k_R, L, S), L) \\ & + \lambda_P(I_P - \delta k_P) + \lambda_E(I_E - \delta k_E) \\ & + \lambda_R(I_R - \delta k_R) - \lambda_S h(K_R, S) \end{aligned} \quad (45)$$

For an interior solution, the first order conditions and the equations for the costate variables imply

$$u'_C = \lambda_P = \lambda_E = \lambda_R =: \lambda, \quad (46)$$

$$u'_L = -\lambda \tilde{f}'_L, \quad (47)$$

$$\hat{\lambda} = \rho + \delta - \tilde{f}'_{k_P} \quad (48)$$

$$= \rho + \delta - \tilde{f}'_{k_E} \quad (49)$$

$$= \rho + \delta - \tilde{f}'_{k_R} + \frac{h'_{k_R}\lambda_S}{\lambda}, \quad (50)$$

$$\dot{\lambda}_S = (\rho + h'_s)\lambda_S - \lambda \tilde{f}'_s. \quad (51)$$

The mitigation constraint is formulated as constraint

$$s \geq \underline{s}, \tag{52}$$

with $\underline{s} = 0$ in the BAU scenario. The resulting transversality conditions are

$$\lim e^{-\rho t}\lambda k = 0, \tag{53}$$
$$\lim e^{-\rho t}\lambda_S(s - \underline{s}) = 0. \tag{54}$$

B Market and planner solution

In the BAU scenaro, the game equilibrium equals the outcome of the social planner model. This result confirms the economic intuition of the social optimality of markets if no externalities occur. The (more general) Pareto optimality of the market outcome under specific conditions is stated in the fundamental theorems of welfare economics in a more general form (e.g. Debreu, 1954). Here we give a short constructive outline of the transformation of the market constraints into the social planner's equations:

Consider both costate variables in the BAU scenario, λ_H for the capital stock determined by Eq. (7) and Eq. (9), and λ_R for the resource stock determined by Eq. (24) and Eq. (25). By defining

$$\mu := \lambda_H\lambda_R \tag{55}$$

and recalling the definition of $\tilde{f}$ we get

$$\tilde{f}'_s = f'_E g'_R h'_s. \tag{56}$$

Using that $\lambda = \lambda_H$ and the definitions of (shadow) prices Eqn. (7 – 9), Eqn. (13 – 15), Eqn. (18 – 19) and Eqn. (24 – 25), a short calculation shows that μ complies with Eq. (50) if $\lambda_S = \mu$. The derivative of μ respect to time is

$$\dot{\mu} = \dot{\lambda}_H\lambda_R + \lambda_H\dot{\lambda}_R, \tag{57}$$

such that by plugging Eq. (9) and Eq. (25) into Eq. (57), μ also complies with Eq. (51) for $\dot{\lambda}_S = \dot{\mu}$.

By substituting sectoral profit functions Eqn. (12,16,22) into the budget constraint of households, Eq. (3) yields the economic budget constraint of the social planner Eq. (38). Moreover, the transversality condition Eq. (28) and the solution of λ_H from Eq. (9) imply

$$\begin{aligned}\lim \mu(s - s_c)e^{-\rho t} &= \lim \lambda_R\lambda_H(s - s_c)e^{-\rho t}\\ &= \lambda_{H,0}\lim \lambda_R e^{\int_0^t(\rho+\delta-\bar{r})ds}(s - s_c)e^{-\rho t}\\ &= \lambda_{H,0}\lim \lambda_R e^{\int_0^t(\delta-\bar{r})ds}(s - s_c)\\ &= 0.\end{aligned} \tag{58}$$

Thus, the market solution of λ_H and λ_R fulfills the optimality and transversality conditions of the social planner solution with the given transformation $\lambda_S = \lambda_H\lambda_R$ if $s_c = \underline{s}$. That is, the BAU scenario is always social optimal. We get as a corrolary that the solution of a RED scenario is equivalent to the social planner solution if the mitigation goal is anticipated by resource extractors.

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