Methodological and empirical flaws in the design and application of simple climate-economy models

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Abstract. One of the main arguments brought forward in favour of the continued use of simple climate-economy models is their transparency, which should enable researchers to easily interpret the simulation results and adapt the model to their specific research interests. We investigate the degree to which this claim is supported in the case of the DICE model but most of our findings are relevant for other welfare-optimizing climate-economy models as well. Specifically, this paper reviews the handling of time discounting in social welfare functions, the combination of different social welfare functions, the calibration of uncertain climate parameters, the representation of uncertainty about future climate change, and the evolution of carbon abatement costs over time. We find that each of these aspects has been treated inconsistently in the past, and that these inconsistencies can strongly affect the results of several previous studies. We discuss the methodological questions raised by some of these problems and make specific recommendations how to avoid the problems identified here in future analyses.

Keywords: climate change, integrated assessment, DICE model, social welfare function, growth discounting, abatement costs, probabilistic analysis

Abbreviations: DU – discounted utility; GCM – general circulation model; GHG – greenhouse gas; GMT – global mean temperature; GWP – gross world product; PDF – probability density function; PV – present value; PVC – present value of consumption; PVO – present value of output; SWF – social welfare function; THC – thermohaline circulation

1. Introduction

Simple climate-economy models continue to be used for a variety of climate policy analyses. While researchers are well aware of the limitations of these highly aggregated models resulting from their lack of regional and process detail, three advantages are often emphasized (see, *e.g.*, Kolstad, 1998). First, the replacement of complex processes by reduced-form functions allows to easily vary key uncertain parameters in sensitivity analyses. Second, the computational efficiency of simple models allows the performance of sophisticated probabilistic analyses. Third, the *simplicity* of these models is generally equated with *transparency* in the sense that individual researchers can fully understand the model design, adapt it to new research questions, and interpret the simulation results. In this paper, we investigate the degree to which the last argument is supported in the case of welfare-optimizing climate-economy models.

Our analysis focuses on the DICE (Dynamic Integrated model of Climate and the Economy) model, arguably the most widely used global climate-economy model. DICE is a global integrated assessment model of the economics of climate change that was developed by W.D. Nordhaus and collaborators. DICE links a neo-classical optimal-growth model of the world economy in which a central planner maximizes intertemporal welfare subject to certain constraints to a description of anthropogenic climate change with the implied economic impacts. Economic output is described by a constant-returns-to-scale Cobb-Douglas production function with labor and capital as input factors. Applications of DICE in a cost-benefit or cost-effectiveness framework maximize the discounted utility from consumption by determining the optimal division of economic output over time into consumption, investment, and emissions abatement.

Three versions of DICE are distinguished in the literature: DICE (Nordhaus, 1992; Nordhaus, 1993), DICE-94 (Nordhaus, 1994), and DICE-99 (Nordhaus and Boyer, 2000). The main developer of DICE has also developed a probabilistic model version, PRICE (Nordhaus and Popp, 1997), a version that considers induced innovation, R&DICE (Nordhaus, 1999), and a series of regionally disaggregated climate-economy models: RICE-96 (Nordhaus and Yang, 1996), RICE-99 (Nordhaus and Boyer, 2000), and RICE-2001 (Nordhaus, 2001b). The DICE and RICE models have been used for climate policy analysis not only by their original model developers but also by other researchers who have frequently adapted them to investigate a variety of scientific and policy questions (Roughgarden and Schneider, 1999; Keller et al., 2000; Mastrandrea and Schneider, 2001; Azar and Lindgren, 2003; Buonanno et al., 2003; Mastrandrea and Schneider, 2004; Moles et al., 2004; Newell and Pizer, 2004; Popp, 2004; Yohe et al., 2004; Keller et al., 2004; Keller et al., 2005; Fankhauser and Tol, 2005; Schlesinger et al., 2006; Smirnov, 2005; Bosetti and Gilotte, 2005; Yohe et al., 2006). In this paper, we focus on the original DICE-99 model as described by Nordhaus and Boyer (2000) and on a modified version applied by Yohe et al. (2004) and Yohe et al. (2006).

The determination of 'optimal' climate policies by intertemporal welfare maximization has been criticized, among others, for its perfect-market assumption, its assumption of full substitutability between market commodities and environmental goods and services, its neglect of the allocation of rights, its assumption that intergenerational compensation is actually feasible, its assumption of a convex optimization function, its inability to account reliably for deep uncertainty or catastrophic outcomes, and the weak empirical basis of widespread practices such as applying logarithmic utility, exponential time discounting, and assuming representative agents that maximize global intertemporal welfare, (Lind et al., 1982; Taylor, 1982; Lind, 1995; Lind and Schuler, 1998; Howarth, 2001; Spash, 2002; Azar and Lindgren, 2003; DeCanio, 2003; Yohe, 2003; Gowdy, 2005; Hall and Behl, 2005). While acknowledging this important body of critique, we do accept the welfare-maximization framework for the context of the discussion in this paper. Other studies have questioned various parameterizations of the DICE models, such as its assumptions on demographic and economic development, the representation of abatement costs, the formulation of the carbon cycle and the climate models, and the representation of climate change impacts (Chapman et al., 1995; Grubb et al., 1995; Kaufmann, 1997; Schultz and Kasting, 1997; Courtois, 2004; Hall and Behl, 2005).

The present paper focuses on logical, methodological, and empirical inconsistencies in applications of DICE and other welfare-optimizing climate-economy models. Hence, it is particularly relevant for those analysts who see some merit in using these models for climate policy analysis as it helps them steer clear of several important inconsistencies.

The inconsistencies identified here occur in different versions of the DICE model (*e.g.*, the original DICE-99 model *vs.* the version applied by Yohe et al., 2004), and some of them relate to differences between different model implementations (*e.g.*, GAMS *vs.* Excel implementations of DICE-99). For that reason, most of them cannot be addressed by some quick fixes to the model code. We do, however, provide recommendations how the problems uncovered here can be avoided in future climate policy analyses.

This paper is structured as follows. Sect. 2 reviews the application of social welfare functions in welfare optimizing climate-economy models. The discussion addresses different implementations of growth discounting and the link to the index number problem as well as inconsistencies between various welfare functions due to different aggregations across time and states of the world. We then make recommendations for the consistent application of welfare functions in climate policy analyses. For a more extensive treatment of many topics addressed in this section, the reader is referred to Füssel (2006). Sect. 3 identifies several other inconsistencies in applications of DICE, including the extrapolation of uncertain climate parameters beyond their physically plausible range and the undocumented specification of radically different abatement cost curves. While this discussion is more specific to DICE, it provides important lessons for other models as well. Sect. 4 concludes the paper.

2. Social welfare functions in climate-economy models

Climate policy analysis based on the welfare-maximization framework assesses alternative policies according to a predefined social welfare function (SWF). A SWF is an algebraic formulation that assigns numerical social utility to each possible social state. *Ordinal* SWFs only provide a ranking of alternative policies whereas *cardinal* SWFs can also provide a quantitative assessment of the welfare difference between alternative policies.

Ideally, a SWF would be derived from the revealed preferences of the individuals concerned. However, Arrow's Impossibility Theorem (Arrow, 1951) shows that there is no unique method for aggregating individual preferences into social preferences. More exactly, this theorem demonstrates that no aggregated social preference that is based on individual preferences can possibly meet a certain set of reasonable criteria —unrestricted domain, non-imposition, non-dictatorship, monotonicity, and independence of irrelevant alternatives— when there are three or more options to choose from. Even if such an aggregation was theoretically possible, it would not be practical in the context of anthropogenic climate change, which significantly affects future generations who cannot reveal their preferences today. For that reason, the SWFs applied in climate policy analysis are constructed synthetically with the aim of reflecting the implicit or explicit preference structure of current decision-makers.

The global climate-economy models considered here assume that individual utility is determined by a single economic good, and that all individuals can be characterized by the same utility function. Specifically, the three SWFs applied for assessing the costs and benefits associated with alternative climate policies are discounted utility of consumption (DU), present value of consumption (PVC), and present value of economic output (PVO).

In this section, we review the application of SWFs in welfare-optimizing climate-economy models. Even though most examples are drawn from the DICE models, the discussion applies to other optimizing climate-economy models as well. Sect. 2.1 presents the SWFs that have been used in applications of the DICE and FUND models, two of the most widely used welfare-maximizing climate-economy models. Sect. 2.2 and 2.3 investigate the implications of different time discounting schemes for ordinal and cardinal welfare functions, respectively, and Sect. 2.4 examines inconsistencies between several internally consistent SWFs. Based on the discussion in the previous subsections, Sect. 2.5 makes recommendations for the application of SWFs in welfare-maximizing climate policy analysis, and Sect. 2.6 critically reviews selected recent analyses.

2.1. DEFINITIONS

A variety of SWFs have been used for comparing alternative policy strategies in welfare-optimizing climate-economy models. In this section, we present the six SWFs that have been applied by the DICE and FUND models. The following notation is used throughout this section:

$Y \geq 0$	economic output
$C \geq 0$	consumption
$I \geq 0$	investment
$L > 0$	population
$\rho \geq 0$	pure rate of time preference (or 'utility discount rate')
$\theta \geq 0$	intertemporal elasticity of substitution

g	*actual* growth rate of per capita consumption
$\tilde{g}$	*assumed* growth rate of per capita consumption
r	social discount rate

These variables and parameters may be supplemented with a discrete time index t, whereby $t = 0$ refers to the present year and $t = T$ to the final year of a time series. (We neglect the difference between the annual specification of the welfare functions defined here and the decadal time step of the DICE model.) If a time index is missing, the respective variable is assumed to be constant over time. $X_{u \cdots v}$ denotes the stream of variable X from time u to time v (assuming $u \leq v$). For notational convenience, we allow the 'empty product' the value of which is assumed to be unity, *i.e.*, $\prod_{t=1}^{0} X_t = 1$. The change in per capita-consumption at time t is defined by

$$g_t = \frac{C_t / L_t}{C_{t-1} / L_{t-1}} - 1. \tag{1}$$

The definition of SWFs includes two parameters that reflect social value judgments about the distribution of wealth within and across generations: ρ and θ. There is a wide range of literature on the most appropriate values of these parameters (Lind et al., 1982; Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Portney and Weyant, 1999; Toth, 2000; Howarth, 2003; Newell and Pizer, 2004). The standard value for θ in economic models of climate change is unity (Arrow et al., 1996; DeCanio, 2003). The corresponding logarithmic (or Bernoullian) utility function is also applied in the DICE models. However, higher as well as lower values for θ have been suggested by some scholars (*e.g.*, Cline, 1992) and applied in sensitivity analyses of climate-economy models (Gjerde et al., 1998; Azar and Lindgren, 2003). There is more disagreement on appropriate values for ρ, and on the question whether this parameter should be constant over time. The default value in DICE-94 is $\rho = 3\%/yr$ (Nordhaus, 1994, p. 104), DICE-99 model assumes that ρ declines over time from $\rho = 3\%/yr$ in 1995 to $\rho = 1.25\%/yr$ in 2335 (Nordhaus and Boyer, 2000, pp. 15–16), and the adaptation of DICE-99 by Yohe et al. (2004) assumes $\rho = 0\%/yr$. Note that discounted utility does not converge over time for $\rho = 0\%/yr$ even though the *finite horizon* approximations discussed here are defined.

Eq. 2 to Eq. 7 define the six SWFs that have been used for comparing alternative policy strategies in connection with the DICE model. All of them take a finite output or consumption stream (expressed in currency such as dollars) as input and calculate a scalar welfare value (expressed either in currency or in arbitrary 'utils') as output, which is defined as the discounted intertemporal sum of the welfare in each time step.

$$\mathrm{DU}_{\mathrm{DICE}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) = \sum_{t=0}^{T} \frac{L_t \cdot \ln(C_t/L_t)}{\prod_{t'=1}^{t}(1+\rho_{t'})} \tag{2}$$

$$\begin{aligned}\mathrm{PVC}_{\mathrm{DICE}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) &= \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t}(1+\rho_{t'}) \cdot (1+g_{t'})} \\ &= \frac{C_0}{L_0} \cdot \sum_{t=0}^{T} \frac{L_t}{\prod_{t'=1}^{t}(1+\rho_{t'})} \end{aligned} \tag{3}$$

$$\mathrm{PVC}_{\mathrm{end}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}, \theta) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t}(1+\rho_{t'}+\theta \cdot g_{t'})} \tag{4}$$

$$\mathrm{PVC}_{\mathrm{ex}}(C_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}_{1\cdots T}) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t}(1+\rho_{t'}+\theta \cdot \tilde{g}_t)} \tag{5}$$

$$\mathrm{PVO}_{\mathrm{ex}}(Y_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}_{1\cdots T}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t}(1+\rho_{t'}+\theta \cdot \tilde{g}_t)} \tag{6}$$

$$\mathrm{PVO}_{\mathrm{Yohe}}(Y_{0\cdots T}, L_{0\cdots T}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t}(1+\ln(1+g_{t'}))} \tag{7}$$

$\mathrm{DU}_{\mathrm{DICE}}$ describes the logarithmic utility of consumption based on 'classic' utility discounting at the rate of pure time preference. This utility function is used as objective function in the original DICE-99 model (Nordhaus and Boyer, 2000, p. 181).

The other SWFs express welfare in monetary units. They apply some variant of growth discounting, which focusses on the marginal social utility of consumption today compared with consumption in the future and represents the 'classical' approach to time discounting (Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Tol, 1999; Toth, 2000). The conventional formula for social time preference, also known as the 'Ramsey growth discounting rule', is $r = \rho + \theta g$. However, this formula is only an approximate solution of the Ramsey model (Füssel, 2006; DeCanio, 2003, Section 3.3.1).

$\mathrm{PVC}_{\mathrm{DICE}}$ describes the present value of consumption as calculated in the original DICE-99 model, which applies a variant of growth discounting (note the definition of g_t in Eq. 1).

$\mathrm{PVC}_{\mathrm{end}}$ describes the present value of consumption according to the conventional formulation of the Ramsey rule. Analogous to $\mathrm{PVC}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{end}}$ determines the discount rate *endogenously* based on the *actual* growth rate of per capita consumption in each year. This SWF has been widely applied in global economic models of climate change such as FUND (Tol, 1999).

$\mathrm{PVC}_{\mathrm{ex}}$ also describes the present value of consumption according to the Ramsey rule. In contrast to $\mathrm{PVC}_{\mathrm{end}}$, the discount rate is determined *exogenously* based on an *assumed* growth rate of per capita consumption. This welfare measure has been used to determine the total welfare effects of climate policies in DICE-99 (Nordhaus and Boyer, 2000, p. 127).

$\mathrm{PVO}_{\mathrm{ex}}$ describes the present value of economic output according to the Ramsey rule, whereby the discount rate is determined exogenously based on an assumed growth rate. $\mathrm{PVO}_{\mathrm{ex}}$ is identical to $\mathrm{PVC}_{\mathrm{ex}}$, except that economic output is substituted for consumption. A special case of this welfare function (assuming $\theta = \rho = 0$) is applied in Fankhauser and Tol (2005), which apparently uses undiscounted gross world product (GWP) calculated by different versions of DICE-94.

$\mathrm{PVO}_{\mathrm{Yohe}}$ describes the present value of economic output applying yet another variant of growth discounting. This SWF has been applied in a modified version of DICE-99 (Yohe et al., 2004, and Yohe, pers. comm.), which assumes $\theta = 1$ and $\rho = 0$. We note that Yohe et al. (2004, SOM pp. 1–2) states that *"In this Policy Forum, the pure rate of time preference is set equal to zero. With an elasticity of marginal utility equal to unity, the social discount rate is simply the endogenously determined rate of annual growth of per capita consumption."* This statement suggests that monetary values were discounted according to the discounting scheme of $\mathrm{PVC}_{\mathrm{DICE}}$ or $\mathrm{PVC}_{\mathrm{end}}$ (these two SWFs are identical for $\theta = 1$ and $\rho = 0$). However, the model code kindly provided by G. Yohe revealed that $\mathrm{PVO}_{\mathrm{Yohe}}$ was actually used in calculating discounted GWP and determining the costs of alternative policies.

2.2. Growth discounting in ordinal welfare functions

In this subsection, we analyze how the discounting schemes applied in the SWFs defined above rank alternative consumption paths. To this end, we employ the following monotonicity criterion: *If a consumption scenario A has higher consumption levels at all time steps than scenario B (except for the initial time step where consumption levels are identical), the SWF should assign higher welfare to scenario A than to B.* The motivation for this criterion is our firm conviction that the vast majority of climate policy-makers seeking advice from optimal-growth models would clearly prefer a policy scenario with consistently higher consumption over a scenario with lower consumption, everything else being equal. This assumption is also made implicitly in most climate policy analyses with optimal-growth models. For instance, Fankhauser and Tol (2005) apply DICE to compare indirect climate impacts under different assumptions, using future loss in *undiscounted* GDP as the main decision criterion. Their conclusions are, therefore, dependent on the assumption that a high-consumption pathway is always preferred.

For the sake of simplicity, this analysis assumes constant-growth consumption paths of the form

$$\mathrm{C}(t; C_0, g) \;=\; C_0 \cdot (1+g)^t, \tag{8}$$

whereby C_0 denotes initial consumption at $t = 0$ and g the rate of consumption growth. In addition, we assume population, pure rate of time preference, and elasticity of the marginal utility of consumption to be constant.

We find the following behaviour for the SWFs presented above (for proofs, see Füssel, 2006):

$\mathrm{DU}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{ex}}$, $\mathrm{PVO}_{\mathrm{ex}}$ and $\mathrm{PVO}_{\mathrm{Yohe}}$ always prefer the high-growth scenario over the low-growth scenario. In the case of $\mathrm{PVC}_{\mathrm{ex}}$ and $\mathrm{PVO}_{\mathrm{ex}}$, this is true independent of the choice of $\tilde{g}$. Hence, these SWFs do fulfill the monotonicity criterion.

$\mathrm{PVC}_{\mathrm{DICE}}$ is insensitive to the consumption levels after the initial period. Hence, this SWF does *not* fulfill the monotonicity criterion. We note that the "stationarity axiom" proposed by Koopmans (1960) is violated as well. This axiom demands that if two sequences have the same start, then eliminating that common start and bringing the rest forward does not change their ranking.

$\mathrm{PVC}_{\mathrm{end}}$ fulfills the monotonicity criterion for $\theta < 1 + \rho$ but not for $\theta \geq 1+\rho$. Note that the *dimensional* rate parameter ρ is an additive factor of the 'threshold value' for the *dimensionless* parameter θ. As a result, $\mathrm{PVC}_{\mathrm{end}}$ may or may not fulfil the monotonicity criterion depending on the (arbitrary) choice of the time step for its specification. (For instance, the numerical value of ρ is more than ten times larger when it is expressed per decade rather then per year.) The reason for this inconsistency is that growth discounting in $\mathrm{PVC_{end}}$ is based on an approximate solution rather than the exact solution of the Ramsey model.

In summary, even for the extremely simple consumption scenarios considered here, $\mathrm{PVC_{DICE}}$ and $\mathrm{PVC_{end}}$ violate the monotonicity criterion for many plausible parameter choices, including $\theta = 1$ and $\rho = 0$. $\mathrm{PVC_{end}}$ is associated with further inconsistencies due to its lack of a solid theoretical foundation. Consequently, we consider these two SWFs unsuitable for evaluating and comparing alternative climate policies.

2.3. Growth discounting in cardinal welfare functions

The monotonicity criterion discussed in the previous subsection only considers the ranking of alternative policies, *i.e.*, it regards the welfare metrics as *ordinal.* In this subsection, we analyze how the discounting schemes applied in the various *monetary* SWFs value the difference in present value (PV) between alternative policies quantitatively. The key question in this context is as follows: *"Should different or identical* *discount factors be used in the present value calculations for alternative policies?"*

One opinion is expressed by the main developer of the DICE models: *"The present values are computed using the base case discount factors."* (Nordhaus and Boyer, 2000, p. 127) and *"In making welfare comparisons between two different policies, the same relative prices should be used to discount the future consumption streams that result from both policies. Thus, in constructing the comparison measures Total abatement cost of policy [...], we use the base case relative prices to discount both base case consumption and consumption under current policy."* (Nordhaus, 2001a, p. 19). Consequently, the original DICE-99 model determines the monetary welfare associated with different policy alternatives by calculating PVC with the same discount factors (as in $\mathrm{PVC}_{\mathrm{ex}}$).

Other policy analyses with optimizing climate-economy models have applied SWFs that determine the discount factors for each policy option endogenously: $\mathrm{PVC}_{\mathrm{end}}$ is applied in FUND (Tol, 1999; Tol, 2003), and $\mathrm{PVO}_{\mathrm{Yohe}}$ is applied in a variant of DICE-99 (Yohe et al., 2004, and Yohe, pers. comm.). Interestingly, the first authors of these studies are fully aware of the potential problems associated with growth discounting when the discount rates are determined endogenously. They find

Table I. Difference in present value for various discounting schemes between two consumption streams growing at 0% and 3% per year over a 10-year period.

	$\mathrm{PVC}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{end}}$	$\mathrm{PVO}_{\mathrm{Yohe}}$	$\mathrm{PVC}_{\mathrm{ex}}$ $[\tilde{g} = 3\%/yr]$	$\mathrm{PVC}_{\mathrm{ex}}$ $[\tilde{g} = 0\%/yr]$
$g = 0\%/yr$	1000.0	1000.0	878.6	1000.0
$g = 3\%/yr$	1000.0	1001.9	1000.0	1146.4
Difference	0%	0.19%	13.8%	14.6%

that the discounting approach applied in $\mathrm{PVC}_{\mathrm{end}}$ may lead to infinite expected damages from climate change (and thus infinite expected marginal benefits of mitigation) if there is the possibility for a catastrophic outcome (Tol, 2003), or if the analyst assumes an infinite time horizon (Yohe, 2003). As a consequence, Yohe (2003, p. 243) concludes that *"we have added one more element to our list of reasons why it is inappropriate to use the expected value of discounted net benefits to judge mitigation policy"*.

Let us briefly analyze the practical consequences of applying one or the other discounting approach. Table I shows the difference in present value between two finite consumption streams starting at $C_0 = 100$ and growing at $g = 0\%$ or $g = 3\%$ per year over a 10-year period for various discounting schemes. The *undiscounted* PV of these two consumption streams is 1000.0 and 1146.4, respectively (as in the right-most column). For the discounted SWFs, we assume $\theta = 1$ and $\rho = 0\%/yr$

since $\mathrm{PVO}_{\mathrm{Yohe}}$ is only defined for these parameter choices. As noted above, $\mathrm{PVC}_{\mathrm{DICE}}$ and $\mathrm{PVC}_{\mathrm{end}}$ are identical for these parameter values, and they determine the same PV independent of the growth rate of consumption. $\mathrm{PVO}_{\mathrm{Yohe}}$ shows a small difference in PV between the two consumption streams but this difference is about 70 times smaller than the difference in undiscounted consumption. $\mathrm{PVC}_{\mathrm{ex}}$ is the only SWF considered here that adequately reflects the difference in (undiscounted) consumption between the two scenarios, largely independent of the choice of the assumed growth rate, $\tilde{g}$.

The arguments relevant to the discounting question considered here are discussed more extensively in Füssel (2006). One conclusion is that PVC is a valid proxy for DU only when the same discount factors are used in the PV calculations of all policy options. Otherwise, the ranking of policy options (*i.e.*, consumption streams) according to PVC may be inconsistent with the ranking according to DU. Furthermore, the discount factors applied to future welfare losses correspond to the relative prices of different goods in the index number problem, which *"can arise when an attempt is made to compare two [or more] sets of variables at two [or more] points in time using a single number since there are many different ways of aggregating variables into a single measure"* (Pearce, 1986). The index number problem has concerned economists and statisticians since the 19^{th} century at least (Jevons, 1865), and it has long been known that no unique solution exists (see, *e.g.*, Edgeworth, 1888). However, there is unanimous agreement that a single set of prices has to be used for a meaningful comparison of quantities between different periods, providing another strong argument for using the same discount factors in PV calculations across all policy options considered.

In our view, all available evidence indicates that PV calculations need to apply the same discount factors for all policy options under consideration, if the resulting PVs shall be used to compare different policy options quantitatively. Consequently, we consider $\mathrm{PVC}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{end}}$, and $\mathrm{PVO}_{\mathrm{Yohe}}$ as unsuitable for comparing the welfare implications of alternative climate policies. Note that $\mathrm{PVO}_{\mathrm{Yohe}}$ is subject to two other problems as well. First, there is no theoretical basis for the logarithmic relationship between consumption growth rate and discount rate assumed in Eq. 7. Second, $\mathrm{PVO}_{\mathrm{Yohe}}$ is only defined for $\rho = 0$, which is widely regarded as unrealistically low (Arrow et al., 1996).

In conclusion, $\mathrm{PVC}_{\mathrm{ex}}$ and $\mathrm{PVO}_{\mathrm{ex}}$ are the only monetary SWFs from Sect. 2.1 that are not obviously inconsistent. The final question then is how to choose the exogenous discount factors for these SWFs given that the analogy to the index number problem implies that there is no unique method for doing so? The obvious approach is to calculate the discount factors on the basis of the growth rates in a 'baseline' policy scenario, as suggested by Nordhaus (2001a, p. 19). Comparison of the two right-most columns of Table I suggests that the choice of the baseline scenario is relatively unimportant for reasonably small welfare differences between alternative scenarios. The choice of baseline scenario may become more important if alternative scenarios involve very large welfare differences with opposite sign at different points in time. In such a situation, it is recommended to determine PVs based on selected 'extreme' policy scenarios and to consider the range of results. We suppose that the uncertainty related to the choice of discount factors is still not very important given the other uncertainties that are necessarily involved in an analysis that comprises such 'extreme' scenarios.

2.4. INCONSISTENCIES BETWEEN DIFFERENT WELFARE FUNCTIONS

In this subsection, we investigate three inconsistencies between those welfare metrics that have *not* been found obviously inconsistent in Sect. 2.3: $\mathrm{DU}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{ex}}$, and $\mathrm{PVO}_{\mathrm{ex}}$.

First, we look at the difference between the output-based SWF ($\mathrm{PVO_{ex}}$) and the consumption-based SWFs ($\mathrm{DU_{DICE}}$ and $\mathrm{PVC_{ex}}$). The optimal growth models considered here divide (net) economic output into consumption and investment in productive capital: $Y = C + I$. The fraction of net output devoted to investment is denoted as the investment rate: $s = \frac{I}{Y} = 1 - \frac{C}{Y}$. While partial equilibrium models prescribe the investment rate, models following a general equilibrium framework, such as DICE, determine the optimal investment rate endogenously. In this discussion, we neglect the problem of converting investment into consumption equivalents (see, *e.g.*, Lind and Schuler, 1998).

When $\mathrm{PVO_{ex}}$ is substituted as objective function in DICE-99 for $\mathrm{DU_{DICE}}$, the optimal (*i.e.*, PVO-maximizing) policy is characterized by an investment rate of 100% over the full time horizon. All economic output is used for investment, none remains for consumption, and utility from consumption becomes minus infinity. Thus the very policy that maximizes $\mathrm{PVO_{ex}}$ minimizes $\mathrm{PVC_{ex}}$ and $\mathrm{DU_{DICE}}$. While this example is obviously unrealistic, Füssel (2006) shows that inconsistent rankings between $\mathrm{PVO_{ex}}$ on the one hand and $\mathrm{DU_{DICE}}$ and $\mathrm{PVC_{ex}}$ on the other hand may also occur for policy strategies determined by DICE-99 when maximizing its original objective function, $\mathrm{DU_{DICE}}$. Since the decision strategy that maximizes $\mathrm{PVO_{ex}}$ is obviously unrealistic, we further conclude that $\mathrm{PVO}_{\mathrm{ex}}$ is generally not an appropriate SWF for comparing alternative climate policies.

Second, we investigate the difference between discounting monetary values (*e.g.*, consumption expressed in dollars or other currency) and discounting utility expressed in 'utils'. This distinction is often blurred, because the two are almost identical for marginal differences between policies.

Let us assume an agent that lives for two periods, and that has a baseline consumption of \$1 in each period. This agent is offered a choice between an additional consumption a now or $a \cdot (1 + \delta)$ in the next period. The relationship between the equivalent discount rates for utility, r, and for consumption, δ is as follows (Füssel, 2006):

$$r = \frac{\ln\left(1 + a \cdot (1 + \delta)\right)}{\ln\left(1 + a\right)} - 1 \tag{9}$$

$$\delta = \frac{\left(1 + a\right)^{1+r} - 1}{a} - 1 \tag{10}$$

The two discount rates are very similar for marginal changes in baseline consumption (*i.e.*, $a \ll 1$). For non-marginal consumption differences, however, the utility discount rate is significantly smaller than the equivalent discount rate for consumption.

Table II. Consumption and logarithmic utility for two policies and two equally likely states of the world (see text). The preferred policy is indicated by bold face.

State of the world	SOW 1		SOW 2		Aggregated measures		
Welfare measure	C	U(C)	C	U(C)	E(C)	E(U(C))	C^*
Policy A	**1.5**	**0.405**	0.5	-0.693	**1.0**	-0.144	0.866
Policy B	1.0	0.000	**0.8**	**-0.223**	0.9	**-0.112**	**0.894**

The finding that discounting (logarithmic) utility is inconsistent with discounting consumption has potential implications for the ranking of alternative consumption paths by $\mathrm{DU}_{\mathrm{DICE}}$ and $\mathrm{PVC}_{\mathrm{ex}}$. These two SWFs produce identical rankings for the constant-growth consumption paths considered in Sect. 2.2. They may, however, rank alternative consumption trajectories inconsistently if these trajectories involve significant welfare deviations at different points in time (for an example, see Füssel, 2006).

Third, we investigate the difference between (expected) consumption and utility in probabilistic analyses. Table II provides a simple example showing that expected consumption and expected logarithmic utility may produce inconsistent rankings of policies since they aggregate differently across possible states of the world. The four left columns show consumption (C) and logarithmic utility ($\mathrm{U}(C) = \ln C$) for two equally likely states of the world (SOW 1 and 2) and for two different policies (A and B). Policy A is associated with higher consumption (and utility) under SOW 1 whereas policy B leads to higher consumption (and utility) under SOW 2. The three right columns show three welfare measures aggregated over the two SOWs. E$(\cdot)$ denotes the expected value, and C^* denotes the certainty equivalent. A certainty equivalent is the certain monetary value that would make an individual with a given utility function indifferent between it and the uncertain outcome, *i.e.*, $\mathrm{U}(C^*) = \mathrm{E}(\mathrm{U}(C))$. We find that policy A has higher expected consumption but policy B has higher expected utility, and thus a higher certainty equivalent. The importance of this inconsistency for a particular analysis requires careful examination by the analyst.

According to Arrow et al. (1996, p. 130), *"Most economists believe that considerations of risk can be treated by converting outcomes into certainty equivalents, [. . .] and discounting these certainty equivalents"*. While it is straightforward to compute certainty equivalents for individual time steps (as in Table II), Füssel (2006) shows that there exists no discounting scheme such that the present value of a series of certainty equivalents is consistent with the expected value of discounted logarithmic utility.

2.5. RECOMMENDATIONS

Based on the findings in the previous subsections, we make the following recommendations for the application of social welfare functions in welfare-optimizing climate policy analyses:

1. The choice of SWFs should be based on the (supposed) preferences of target decision-makers. This choice should be made explicit.
2. For the results of monetary SWFs to be comparable across different policy options, the same discounting factors have to be applied to all policy options.
3. The same SWF should preferably be used in all optimizations as well as to report the relative 'desirability' of alternative policies. If different SWFs are combined in an analysis, the analyst needs to demonstrate that the conclusions are not affected by the inconsistencies between them.
4. In a probabilistic analysis with non-marginal welfare differences between different states of the world, the monetary value of a policy option should be determined as its certainty equivalent.
5. Social welfare cannot be objectively compared across different population scenarios (for details, see Füssel, 2006).

What do these recommendations mean for the choice of SWFs for climate policy analysis involving global welfare maximization? $\mathrm{PVC}_{\mathrm{ex}}$ is an appropriate SWF if (and only if!) an analyst assumes risk-neutral decision-makers. However, economists find very little empirical support for risk-neutral behaviour in individuals (Arrow et al., 1996). While the Arrow-Lind theorem (Arrow and Lind, 1970) holds that if risk can be pooled or spread in such a way that aggregate risk is negligible, governments can be considered risk-neutral these conditions are generally not met in climate policy analysis. $\mathrm{DU}_{\mathrm{DICE}}$ is an appropriate SWF if an analyst believes that the degree of risk aversion and other preferences of target decision-makers are adequately reflected by the logarithmic utility function. This SWF is indeed very commonly used in economic models of climate change (DeCanio, 2003, Table 2.4), and it can in principle be modified to accommodate different degrees of risk aversion.

A potential disadvantage associated with non-monetary SWFs, such as $\mathrm{DU}_{\mathrm{DICE}}$, is that they are expressed in arbitrary utility units. As a result, *"Any economist doing this work will obviously feel a strong urge to discount the difference in the consumption streams to a present value"* (Lind and Schuler, 1998, p. 80). Following this "strong urge", some analysts have attempted to convert utility differences into monetary costs, defined as the difference in present value between alternative policies (see Sect. 2.6). However, the combination of different welfare metrics in an analysis is likely to introduce inconsistencies because different welfare metrics aggregate differently across components of economic output, time, possible states of the world (Sect. 2.4), and regions or population groups (not discussed in this paper). Analysts who nevertheless combine different welfare metrics (*e.g.*, by maximizing expected utility and reporting the present value of the certainty equivalents of consumption over time) need to demonstrate that the inconsistencies between these metrics do not affect the policy conclusions of the analysis. Otherwise, the analysis must be regarded as potentially inconsistent.

An anonymous reviewer has suggested the following method for determining the monetary value (*i.e.*, the net benefits) of adding either a specific policy option or a specific constraint in a global model (see Sect. 2.6 for two examples): First, determine the maximum utility of any policy scenario *with* the option or constraint, U_{option}, and the maximum utility in the base case *without* that option or constraint, U_{base}. Second, determine the change in present consumption, V_{option}, so that the maximum utility in the base case changes from U_{base} to U_{option}. In our view, this method has two major limitations. First, it is only defined in the case of one region (or population group). Additional assumptions on the distribution of costs or benefits have to be made to determine the equivalent total change in present consumption if more than one region is considered. Second, and more importantly, this method may produce counterintuitive and/or arbitrary results for constraints with non-marginal welfare effects. For instance, adding a very stringent constraint that reduces consumption to just above subsistence levels in *all* time periods is equivalent in terms of discounted (logarithmic) utility to reducing consumption to just above subsistence levels in the *first* model period. Consequently, the costs of such a constraint, determined as V_{option}, would be approximately equal to world consumption in the first model period. As a consequence, these costs are directly proportional to the (arbitrary) length of a time step in the model. For instance, if we assume world consumption to be about 40 trillion US$ per year, a model with an annual time step would determine the costs (defined as the equivalent first-period consumption difference) of this stringent constraint as close to 40 trillion US$ (absolute, not per year) whereas a model with a 10-year time step would determine the costs as close to 400 trillion US$ (*i.e.*, factor 10 higher).

2.6. REVIEW OF RECENT CLIMATE POLICY ANALYSES

In this subsection, we review the use of welfare metrics in two recent applications of DICE that have not followed the recommendations from Sect. 2.5: Yohe et al. (2004) and Fankhauser and Tol (2005)

Yohe et al. (2004) presents a hedging analysis that aims to identify the optimum short-term policy under uncertainty about climate change and the long-term stabilization target. This uncertainty is described by discrete "policy cases", which are characterized by a specific value for the climate sensitivity and an upper bound for the greenhouse gas (GHG) concentration level. Each policy case is assigned a probability based on an empirical probability density function (PDF) for climate sensitivity, assuming that all considered GHG stabilization levels are equally likely. It is further assumed that the 'true' policy case will be revealed in 2035. A modified version of DICE-99 is used to determine the optimal decision strategy for each policy case by maximizing $\mathrm{DU}_{\mathrm{DICE}}$ for different initial levels of the carbon tax (until 2035) and without such a constraint. For each of those utility-maximizing strategies, the discounted gross world product (GWP) is calculated according to $\mathrm{PVO}_{\mathrm{Yohe}}$. The "discounted adjustment costs" for each policy case and initial carbon tax level are then defined as the difference in discounted GWP between the utility-optimal strategies with and without prescribing the initial carbon tax level. Finally, the “optimal” initial carbon tax level is determined by minimizing the *expected* discounted adjustment costs for each tax level, considering the probability of the various policy cases.

We argue that the use of welfare metrics in Yohe et al. (2004) involves several inconsistencies, with important implications for the results presented. First, Sect. 2.3 finds that $\mathrm{PVO}_{\mathrm{Yohe}}$ is internally inconsistent and underestimates the undiscounted welfare differences between alternative policies by about two orders of magnitude. We find similar problems in Yohe et al. (2004). According to Yohe et al. (2004, Fig. S3.B), the discounted GWP difference between a 400 ppm and a 900 ppm CO_2 concentration target is less than 0.025% (14241.1 and 14244.6 trillion US$, respectively), which is about two orders of magnitude smaller than the cost estimates from most other studies (Metz et al., 2001). According to Yohe et al. (2004, Fig. S7), variation in expected discounted GWP across all considered initial carbon tax levels (0 to 30 US$ per ton of carbon) is a mere 0.0004% (from 14243,70 to 14243.76 trillion US$). We argue that the internal inconsistencies associated with $\mathrm{PVO}_{\mathrm{Yohe}}$, which violates the second recommendation from Sect. 2.5, are the main reason for the surprisingly low variation in discounted GWP in Yohe et al. (2004), Second, Sect. 2.4 finds that DU and PVO may produce inconsistent policy rankings. In Yohe et al. (2004), $\mathrm{DU}_{\mathrm{DICE}}$ is initially maximized but $\mathrm{PVO}_{\mathrm{Yohe}}$ is later used as the basis for selecting the 'optimal' policy for each policy case. In violation of the third recommendation, there is no discussion whether these two SWFs produce similar rankings for the policies considered in that analysis, or what the potential implications of the inconsistent rankings could be. Third, Sect. 2.4 finds that expected consumption and expected logarithmic utility may produce inconsistent policy rankings due to different degrees of risk aversion. The same arguments hold in relation to Yohe et al. (2004), where expected output is maximized within a limited set of policy strategies that were initially determined by utility maximization in a deterministic context.

What are the implications of these flaws for the results of Yohe et al. (2004)? The study concludes that *"An initial \$10 tax policy is remarkably robust across the remaining possibilities"*, noting further that it is *"surprising that climate insurance over the near term can be so inexpensive and that an economically efficient near-term hedging policy can be so robust across a wide range of futures in comparison with doing nothing"*. We have argued above that the reported costs of policies depicted in Yohe et al. (2004) are incorrect, most likely by about two orders of magnitude. Furthermore, even if these values were correct, the tiny GWP variation across different policies would hardly support such a strong conclusion. Determining the correct optimal carbon tax level (subject to the other assumptions of this particular analysis) would require a rerun of the whole modelling exercise in accordance with the recommendations from Sect. 2.5. Since such a reanalysis is beyond the scope of this paper, we cannot say for sure whether its results would still support the conclusions cited above.

Fankhauser and Tol (2005) apply DICE-94 to compare indirect climate impacts under different assumptions. This analysis defines several modifications to the production function of DICE-94, determines the optimal decision strategy for each model variant by maximizing the standard discounted utility function of DICE-94, $\mathrm{DU}_{\mathrm{DICE}}$, and presents the time paths and growth rates of undiscounted GDP (corresponding to $\mathrm{PVO}_{\mathrm{ex}}$ for $\theta = \rho = 0$) for these decision strategies. Given the findings from Sect. 2.4 that $\mathrm{DU}_{\mathrm{DICE}}$ and $\mathrm{PVO}_{\mathrm{ex}}$ may produce inconsistent policy rankings, we have to consider the results of Fankhauser and Tol (2005) as potentially flawed.

3. Other flaws in DICE and its application

In Sect. 2, we discussed the use of social welfare functions in climate policy analysis. In this section, we examine other problems related to DICE and/or its application. Sect. 3.1 examines the flawed representation of uncertain climate parameters in two probabilistic model analyses; Sect. 3.2 identifies contradicting assumptions about the development of carbon abatement costs over time in the Excel and GAMS implementations of DICE-99; and Sect. 3.3 focuses on numerical errors in the GAMS version of DICE-99. Our motivation in this section is not to dwell upon these particular problems but to discuss the lessons to be learnt for model-based climate policy analysis.

3.1. Calibration of uncertain climate parameters

This subsection focusses on the calibration of uncertain climate parameters in DICE. In particular, we show that the inappropriate specification of the uncertainty about future climate change in two probabilistic analyses with DICE-99 (Yohe et al., 2004; Yohe et al., 2006) results in a significant underestimation of the likelihood of large transient climate change, which has important implications for the conclusions drawn from one of these studies. We also provide recommendations for avoiding the various problems identified here. For a discussion of other weaknesses of the DICE climate model, see Hall and Behl (2005).

DICE-94 and DICE-99 apply the same climate model, except for various changes in the names of model variables and parameters (Nordhaus and Boyer, 2000, p. 62–67). This model is an adaption of the two-box climate model by Schneider and Thompson (1981):

$$\dot{T}_{up} = \frac{1}{R_1} \cdot \left(F - \frac{F_{2\times}}{T_{2\times}} \cdot T_{up} - \frac{R_2}{\tau_{12}} \cdot (T_{up} - T_{lo}) \right) \tag{11}$$

$$\dot{T}_{lo} = \frac{1}{\tau_{12}} \cdot (T_{up} - T_{lo}) \tag{12}$$

The three time-dependent variables and five parameters of this model are:

$T_{up}(t)$	[K]	temperature of the atmosphere and upper ocean
$T_{lo}(t)$	[K]	temperature of the deep ocean
$F(t)$	$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$	net change in radiative forcing
$T_{2\times}$	[K]	equilibrium temperature increase from a CO_2 doubling
$F_{2\times}$	$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$	increase in radiative forcing from a CO_2 doubling
R_1	$\left[\frac{\mathrm{W\,yr}}{\mathrm{K\,m}^2}\right]$	thermal capacity of the atmosphere and upper ocean
R_2	$\left[\frac{\mathrm{W\,yr}}{\mathrm{K\,m}^2}\right]$	thermal capacity of the deep ocean

τ_{12}	[yr]	time-scale of heat transfer from upper to deep ocean

The two most uncertain parameters are $T_{2\times}$, which determines the equilibrium change in global mean temperature (GMT), and τ_{12}, which determines the speed of adjustment (Allen et al., 2000; Wigley and Raper, 2001; Knutti et al., 2005). The values of the other three parameters are relatively well known. In particular, R_1 can be easily determined from mixed layer depth. Recent research estimates average mixed layer depth at 70–100 m, with considerable global and seasonal variation (de Boyer Montégut et al., 2004). The reduced–form model applied in one of the studies reanalyzed here was calibrated to a general circulation model (GCM) with a 60 m deep mixed-layer ocean model (Yohe et al., 2006, p. 62). The heat content of the atmosphere is approximately equivalent to a 2 m ocean layer and can thus be neglected in this context. Assuming a ratio of ocean surface to total Earth surface of $f_o = 71\%$ (Coble et al., 1987), a mixed layer depth of $h_m = 70\,\mathrm{m}$, a density of ocean surface water of $\rho_o = 1026\ \mathrm{kg}\cdot\mathrm{m}^{-3}$, and a heat capacity of ocean water of $c_{po} = 3996\ \mathrm{J}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$, we determine the heat capacity of the ocean mixed layer per area of the Earth's surface as

$$R_1 = f_o \cdot h_m \cdot \rho_o \cdot c_{po} = 2.04 \cdot 10^8\,\frac{\mathrm{J}}{\mathrm{K\,m^2}} = 6.46\,\frac{\mathrm{W\,yr}}{\mathrm{K\,m^2}}. \qquad (13)$$

Table III. Correspondence of uncertain parameters in different versions of the DICE climate model with those of the climate model by Schneider and Thompson (1981). Abbreviations: No94=Nordhaus (1994); NB00=Nordhaus and Boyer (2000); ST81=Schneider and Thompson (1981).

DICE-94 [No94]			**DICE-99** [NB00]		[ST81]
App.	Table 3.2	Eq. (3.7')	App. E	Ch. 2	Eq. 11–12
$C1$	$c(1)$	α_1	$C1$	σ_1	$1/R_1$
LAM	λ	α_2	$4.1/CS$	λ	$F_{2\times}/T_{2\times}$
$C3$	$c(3)$	α_3	$C3$	σ_2	R_2/τ_{12}
$C4$	—	α_4	$C4$	σ_3	$1/\tau_{12}$

As shown in Table III, the DICE climate model combines the five physical parameters from the model described in Eq. 11–12 (right-most column) into four parameters, most of which can no longer be interpreted physically. In the model formulation of DICE-94 (left-most column), LAM, $C3$, and $C4$ are associated with large uncertainty (since they depend on the highly uncertain parameters $T_{2\times}$ or τ_{12}), whereas $C1$ is much less uncertain (since it depends only on the relatively well known parameter R_1).

Even though $C1$ is much less uncertain than $C3$ and $C4$, Nordhaus (1994, Chapter 3) sets out to calibrate $T_{2\times}$ and $C1$, using historical data as well as the results from GCM experiments, and prescribes $C3$ and $C4$. Hall and Behl (2005, p. 17) correctly points out that $C3$ and $C4$, described as "the two least important parameters" by Nordhaus (1994, p. 40), "are important for understanding even short-term climate change". The joint PDF for $T_{2\times}$ and $C1$ constrained by historical forcing and temperature data shows a negative correlation between $T_{2\times}$ (varied from 1–5 K) and $C1$ (varied from 0.01–0.1 $\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$), but the conditional PDF for $C1$ given $T_{2\times}$ is often rather flat (Nordhaus, 1994, p. 43). As shown above, the upper and lower bounds for R_1 (and thus $C1$) differ by a factor 2 at best. Hence, the variation of $C1$ by a factor 10 is inconsistent with the physically plausible range of R_1.

Upon finding that simulations by GCMs and historical data disagree in constraining the uncertain parameters in the DICE climate model (most likely due to the lack of consideration of the cooling effect of aerosols in the GCM experiments considered), Nordhaus (1994, pp. 46–47) asserts that *"For the DICE model, we employ the parameter pair (Sc2) drawn from the results of the SJ (*Schlesinger and Jiang, 1990*) model"*. However, the value $C1 = 0.0226\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ used in DICE (Nordhaus, 1994, Table 2.4) does not agree with the corresponding value $C1 = 0.048\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ from the SJ model (Nordhaus, 1994, Table 3.2.B), nor with the value $C1 = \frac{1}{R_1} = 0.155\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ determined according to Eq. 13.

There are several problems associated with the calibration of the DICE climate model described above. First, the reformulation of the climate model by Schneider and Thompson (1981) in such a way that most parameters can no longer be physically interpreted made it more difficult to focus on the main sources of uncertainty, and to identify all available data for constraining the uncertainty of individual parameters. Second, the calibration of a parameter with low uncertainty while holding fixed more uncertain parameters contributed further to the calibration of this model parameter outside its physically plausible range. Third, the final choice of parameter values is not well documented.

A fourth problem, which is partly caused by the first and second one, occurs in Yohe et al. (2004) and Yohe et al. (2006). These probabilistic analyses with DICE-99 represent the uncertainty about future climate change by a single uncertain parameter. Analogous to the approach in Nordhaus (1994, Chapter 3), $T_{2\times}$ and $C1$ are calibrated using a large ensemble of climate projections. A single value for $C1$ is then assigned to each value of $T_{2\times}$ (Yohe et al., 2004, Table S1) even though there is clear evidence (including from Nordhaus, 1994, Table 3.5) that $C1$ and $T_{2\times}$ are *not* perfectly correlated.

Fig. 1 depicts GMT trajectories for the DICE-99 baseline emissions scenario calculated with the modified DICE-99 model that assumes perfect correlation between $C1$ and $T_{2\times}$. This figure from Yohe et al. (2004, Fig. S1) is essentially equivalent with Yohe et al. (2006, Fig. 2). The calculations represent a wide range of climate sensitivities from 1.5 to 9 K, which covers more than the 5–95% range of most published climate sensitivity PDFs (Meinshausen, 2006). Nevertheless, there is no discernible uncertainty in temperature change before 2050, and only moderate uncertainty (about 0.6 K) in 2100.

In contrast to Fig. 1, detailed probabilistic analyses find a large uncertainty range for 21st-century climate change. The width of the 5–95% range of GMT increase is estimated at 1.0 K by the 2020s independent of the emissions scenario (Stott and Kettleborough, 2002), at 1.5 K by the 2040s for the medium IS92a scenario (Allen et al., 2000), at 2.1 K (Stott and Kettleborough, 2002) and 2.2 K (Knutti et al., 2002) by 2100 for the low SRES B1 scenario, and at 3.9 K by 2100 for the high SRES A1FI scenario (Stott and Kettleborough, 2002). Similar results have been found by Wigley and Raper (2001), Webster et al. (2003), and Knutti et al. (2005).

Fig. 2 (Cubasch et al., 2001, Fig. 9.15) depicts global mean temperature projections determined by the simple climate model MAGICC tuned to several GCMs for the six illustrative SRES emissions scenarios (Nakicenovic and Swart, 2000). The GHG emissions in the unmitigated reference scenario of DICE-99 are most closely resembled by the medium-low SRES B2 scenario. Fig. 2 projects a GMT increase for the SRES B2 scenario of approximately 0.7–1.1 K from 1990 to 2030 and 1.9–3.4 K from 1990 to 2100. The corresponding projections in

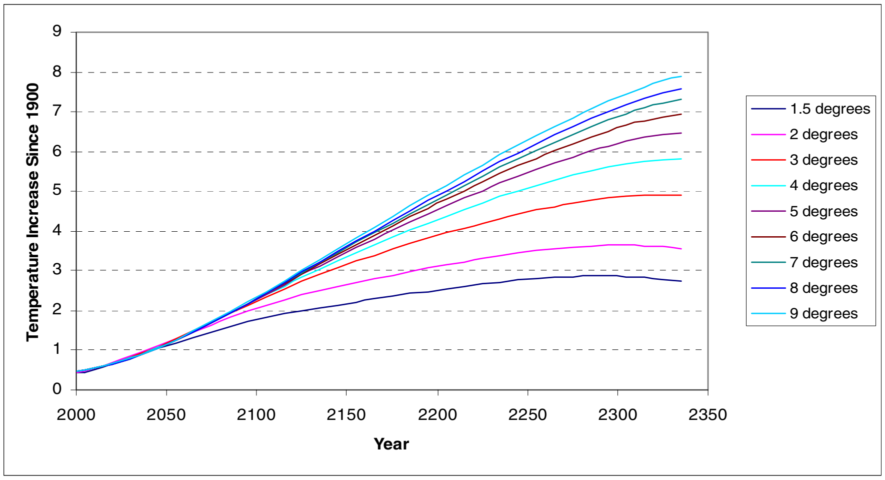

Figure 1. Global mean temperature trajectories for the DICE-99 baseline emissions scenario determined for alternative climate sensitivities from 1.5–9 K and associated calibrations of the heat capacity of the atmosphere and the upper ocean layer (reprinted from Yohe et al., 2004, Fig. S1).

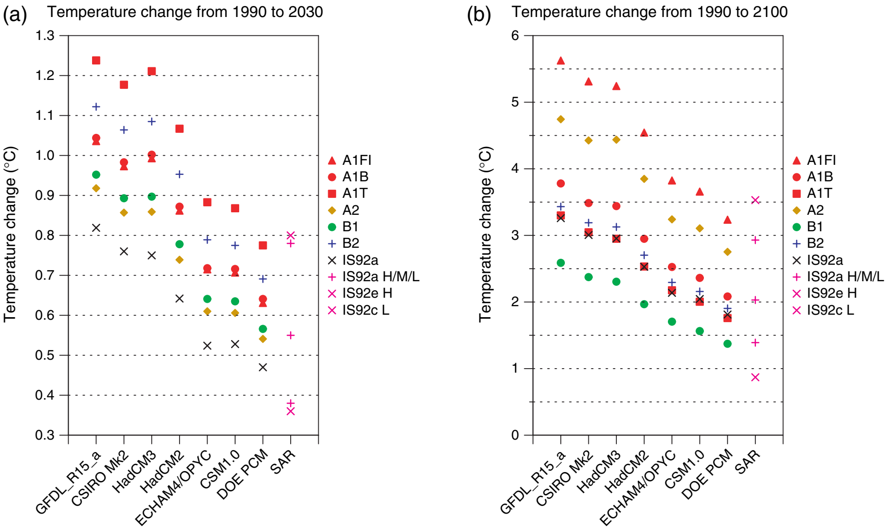

Figure 2. Global mean temperature projections determined by the simple climate model MAGICC tuned to several GCMs from 1990 to 2030 (left plate) and 2100 (right plate) for the six illustrative SRES emissions scenarios (reprinted from Cubasch et al., 2001, Fig. 9.15).

Fig. 1 are only 0.5 K and 1.4–2.0 K, respectively, even though the underlying range of climate sensitivity considered is much wider. We conclude that the assumption of perfect correlation between the two uncertain climate parameters $C1$ and $T_{2\times}$ in Yohe et al. (2004) and Yohe et al. (2006) as well as the non-consideration of uncertainties in other factors such as aerosol forcing in the DICE climate model causes significant overconfidence in probabilistic projections of 21st-century climate change for a given emissions scenario, whereby there is a strong bias toward low estimates of transient climate change.

What are the implications of the overconfident climate projections for the policy conclusions drawn in Yohe et al. (2004) and Yohe et al. (2006)? The cost-effectiveness analysis presented in Yohe et al. (2004) applies constraints on GHG concentrations rather than temperature. Since the only effect of climate change on the choice of cost-effective policies is through the smooth damage function of DICE-99, the underestimation of the uncertainty about transient climate change is expected to have only a small impact on the results of this study. (Note, however, the discussion of the inconsistent use of SWFs in this analysis in Sect. 2.6.)

The risk analysis presented in Yohe et al. (2006) estimates the likelihood of a collapse of the thermohaline ocean circulation (THC) before

Table IV. Sensitivity of the maximum likelihoods of THC collapse through 2105 and 2205, respectively, to the ranges of four uncertain climate parameters in the absence of a carbon tax (adapted from Yohe et al., 2006, Table 3 and 4).

THC collapse before	2105	2205
Kappa (K)	1–97%	30–100%
Alpha (α)	14–70%	25– 88%
Climate sensitivity (ΔT_{2x})	**38–50%**	**44– 78%**
Critical temperature (ΔT_c)	38–49%	45– 65%

2100 or 2200, respectively, under different levels of climate mitigation policy (expressed as carbon tax levels), subject to an empirical PDF for climate sensitivity and uniform distributions for three uncertain parameters of the THC model. Table IV shows one of the key findings of this study. According to this table, the uncertainty about climate sensitivity is much less important for estimating the risk of a THC collapse than the uncertainty about two other uncertain climate parameters, K and α. It is further suggested that even for the highest value of climate sensitivity, the risk of a THC collapse before 2105 is 'only' 50%. As argued above, the flawed probabilistic representation of future climate change in Yohe et al. (2006) significantly underestimates the uncertainty of transient climate change, primarily by falsely excluding high values. Consequently, a more accurate probabilistic representation would show a higher importance of the uncertainty about climate sensitivity, and it would produce higher estimates of the likelihood of a THC collapse.

While we do not have access to the collection of models applied in Yohe et al. (2006) to replicate their analysis with an improved probabilistic representation, we can still provide a rough estimate of the magnitude of the effect. The range of GMT change projected in Yohe et al. (2006) for 2200 is reached in the detailed probabilistic analyses cited above already around 2100. Hence, we suspect that the high estimates for the likelihood of a THC collapse before 2205 reported in the bold-faced row in Table IV (*i.e.*, 78% for a climate sensitivity of 9 K) are more indicative of the risk up to 2105 (which is estimated at a maximum of 50%).

The discussion above has concentrated on a specific weakness of probabilistic analyses with the DICE model. It should be noted that the modelling framework applied in Yohe et al. (2006) is probably inadequate for calculating the likelihood of a THC collapse since coupled GCMs do not show the abrupt collapse simulated by simple box models (Gregory et al., 2005).

How can the four problems identified here be addressed? If the dynamic equations of the DICE climate model shall not be changed, the single most important measure would be to retain the original model formulation by Schneider and Thompson (1981) and to focus the calibration on the most uncertain physical parameters in that model ($T_{2\times}$ and τ_{12}), using all available data for constraining their PDFs. A more general problem is that the DICE climate model has not been updated since its original formulation and calibration despite the tremendous improvement in climate modelling during the last 15 years. For instance, we now have a much better understanding of the role of aerosols in the climate system, and recent analyses have estimated joint PDFs for several uncertain climate parameters (Forest et al., 2002; Knutti et al., 2002). Uncertainty in the DICE climate model should preferably have been represented by the joint PDF of the key uncertain model parameters. In this particular case, the uncertainty range for transient climate change determined in the detailed studies cited above is much better reproduced when only $T_{2\times}$ is varied and $C1$ is held fixed at its default value (as in Keller et al., 2004; Mastrandrea and Schneider, 2004; Keller et al., 2005) than when $T_{2\times}$ and $C1$ are varied assuming a deterministic relationship (as in Yohe et al., 2004; Yohe et al., 2006).

We summarize the lessons from this subsection in the following recommendations for the calibration and probabilistic representation of uncertain model parameters:

1. As far as possible, a model should be specified so that uncertain model parameters correspond to observable properties in the real world.

2. The calibration of a model should focus on those parameters whose uncertainty is most important for the analysis results.

3. As far as possible, uncertain model parameters should be calibrated using all available information for constraining them.

4. Probabilistic analyses should carefully consider the intercorrelation between different uncertain parameters. While it may sometimes be justified to treat uncertain parameters as either independent or perfectly correlated, it will often be necessary to apply their joint PDF.

5. An existing model should be updated to reflect recent knowledge improvements before it is applied in a policy context.

In the present example of the DICE climate model, Nordhaus (1994) violates the first three recommendations; the probabilistic analyses by Yohe et al. (2004) and Yohe et al. (2006) also violate the fourth and fifth recommendation.

3.2. Contradicting assumptions on the evolution of abatement costs

DICE identifies the optimal climate policy by solving an intertemporal optimization problem. One of the most important assumptions affecting the simulation results is the development of carbon abatement costs over time. The costs of emissions abatement in DICE-99 depend on the deviation of actual emissions from the unabated baseline emissions scenario at a specific point in time:

$$\mathrm{Cost}(\mu_t, t) \;=\; \mathrm{b}_1(t) \cdot \mu_t^{\mathrm{b}_2} \cdot \mathrm{Y}^*(t)$$

$\mathrm{b}_1(t) \in [0,1]$ is a time-dependent abatement cost factor that denotes the fraction of gross world product (GWP) lost if carbon emissions were reduced to zero in year t, $b_2 = 2.15$ is the exponent of the cost function, $\mu_t \in [0,1]$ is the emission control rate ($\mu_t = 0$ refers to baseline emissions and $\mu_t = 1$ refers to zero emissions), and $\mathrm{Y}^*(t)$ is the GWP in the unabated business-as-usual scenario in year t. Several authors have criticized the representation of abatement costs in DICE (Grubb et al., 1995) or have developed alternative formulations that consider the effects of induced technological change (Nordhaus, 1999; Buonanno et al., 2003; Popp, 2004). The discussion here, in contrast, focusses on the *temporal development* of abatement costs in the DICE-99 model.

DICE-99 is available for download from the main developer's homepage `http://www.econ.yale.edu/~nordhaus/homepage/web/%20table%20of%20contents%20102599.htm` as Excel spreadsheet (`dice99.xls`) and as GAMS program (`Dice020899.gms`). The model developers assert that *"The Excel and GAMS versions are identical for the DICE-99 model"* (Nordhaus and Boyer, 2000, p. 107). The Excel spreadsheet, which is largely equivalent to the GAMS code in Nordhaus and Boyer (2000, Appendix E), assumes the abatement cost factor $b_1(t)$ to *increase* over time: from 3.0% in 1995 to 11.1% in 2335. Since the emissions intensity of the world economy is assumed to decrease 57-fold during this period, the increase is even stronger in absolute terms: from 110 US$/tC in 1995 to 23,000 US$ in 2335 (in constant 1995 dollars). The downloadable GAMS program, in contrast, assumes the abatement cost factor $b_1(t)$ to *decrease* over time: from 4.5% in 1995 to 1.5% in 2335. The GAMS program (`Dice020899.gms`) is identical to the GAMS code in Nordhaus and Boyer (1999, Appendix C), which is titled *"Computer Code for DICE-98 Model"*. Hence, we assume that it refers to an earlier model version denoted as DICE-98, despite the contradicting file name. The qualitatively different assumptions about the evolution of

Table V. Time development of carbon abatement costs in two versions of DICE.

	DICE-99	DICE-98
File name	`dice99.xls`	`Dice020899.gms`
Platform	Microsoft Excel	GAMS/MINOS5
Reference	Nordhaus and Boyer (2000, App. E)	Nordhaus and Boyer (1999, App. C)
$dmiufunc$	-8	0.26
$b_1(1995)$	3.0%	4.5%
$b_1(2335)$	11.1%	1.5%

abatement costs in DICE-99 *vs.* DICE-98 are caused by opposing signs of the parameter $dmiufunc$ in the two models. For comparison, b_1 was assumed to be constant in DICE-94, where $b_1 = 6.86\%$ and $b_2 = 2.887$ (Nordhaus, 1994, p. 193). These findings are summarized in Table V.

Nordhaus and Boyer (2000, p. 104) explain the determination of the coefficients b_1 and b_2 as follows: *"The coefficients $b_1(t)$ and b_2 [...] were set so that the optimal carbon tax and emissions control rates in DICE-99 matched the projections of these variables in the optimal run of RICE-99."* In contrast to the DICE models, RICE-99 determines the costs of carbon abatement by including carbon-based energy as a separate factor in its production function. For that reason, it is not possible to directly compare the specification of RICE-99 with the abatement cost curves of DICE-99/98. Furthermore, there is also a significant discrepancy between the downloadable and the published version of the

RICE-99 model. The RICE-99 GAMS code available for download at `http://www.econ.yale.edu/~nordhaus/homepage/Rice020899.gms` assumes the elasticity of GDP with respect to carbon —denoted as ALPHA(T,N)— to decrease over time whereas the code in Nordhaus and Boyer (2000) assumes the elasticity —denoted as OW("ALPHA",N)— to be constant. The model developers do not provide an explanation why the assumptions in the earlier versions of RICE and DICE have later been radically changed.

The fact that the two significantly different versions of the DICE-99 model are not distinguished in the literature is unsatisfying by itself. In fact, the column titled "DICE-99" in Table V refers to two slightly different models, since the GAMS program described in Nordhaus and Boyer (2000, Appendix E) contains some numerical errors that are not present in the Excel spreadsheet `dice99.xls` (see Sect. 3.3). Consequently, there are actually three different models that are commonly referred to as DICE-99. Furthermore, the choice of one or the other abatement cost function has wide-ranging implications for the optimal policy strategy determined by DICE-99/98. The DICE-99 Excel spreadsheet (as well as the GAMS program presented in Nordhaus and Boyer, 2000, App. E but *not* available for download) recommends a strategy with modest emission controls peaking at 11%. The DICE-

98 GAMS program available for download recommends much stronger emission controls peaking at 56%. The DICE-99 model modified to apply the DICE-98 abatement cost function recommends emission controls peaking at 49%, indicating that the different assumptions regarding abatement costs are indeed the main cause of the large differences in maximum abatement rates between the two model versions. In agreement with earlier studies (*e.g.*, Kaufmann, 1997), we find little variation in the optimal savings rate between the original DICE-99 model and the DICE-99 model applying the DICE-98 abatement cost function.

Two aspects of the representation of abatement costs in DICE-99/98 (as well as in RICE-99/98) are particularly disturbing. First, at least one of the model versions grossly misrepresents the model developers' knowledge (or expectations) about the development of abatement costs over time. The model developers neither point to the existence of these different models nor do they provide an explanation how the widely diverging parameterizations were determined. Second, most scholars are not aware of the differences between the two implementations of DICE-99 available for download, which were described as "identical" by their developers. Hence, different analysts may unknowingly arrive at very different results, depending on whether they use the Excel version of the model denoted as DICE-99 (such as Yohe et al., 2004) or the GAMS implementation (such as Bruckner and Zickfeld, 2006).

The inconsistent assumptions regarding abatement cost curves are obviously specific to DICE-99. However, we see this problem as an example of the more general challenge of ensuring the consistency of aggregated models where important parameters have to be calibrated to observational data or to the results of more complex models. In this context, we reiterate our first recommendation from Sect. 3.1, *i.e.*, to specify a model in such a way as to minimize the number of parameters that cannot be checked against observations. In addition, we emphasize the importance of providing a "traceable account" (Moss and Schneider, 2000) explaining how the values of the remaining parameters were determined, including the results of alternative assumptions. Neither of these recommendations was followed in the case of DICE-99.

3.3. Numerical errors

The GAMS versions of DICE-98 (Nordhaus and Boyer, 1999, Appendix C) and DICE-99 (Nordhaus and Boyer, 2000, Appendix E) are distinguished not only by changes in the values of many empirical parameters but also by changes in the units in which these parameters are expressed. Most (but not all) flow parameters are expressed in *fractions per decade* in DICE-98, whereas most (but not all) of them are expressed in *percent per year* in DICE-99. For instance, the value of $GA0$ changed from 0.055 (fraction per decade) to 3.8 (percent per decade), and the value of $ET0$ changed from 11.28 (GtC per decade) to 1.128 (GtC per year); at the same time $ET0$ was renamed to $LU0$. Apparently some errors have been introduced to the GAMS version of DICE-99 during this conversion:

E, $ETREE$, $LU0$: Equation EE, which defines the total carbon emissions per decade (E), applies a factor 10 to annual industrial emissions but not to annual land-use emissions ($ETREE$). Comparison with the Excel version of DICE-99 confirms that the definition of E incorrectly lacks the factor 10 for $ETREE$. Hence, land-use change emissions are underestimated by a factor 10 in the GAMS version of DICE-99. This error has a small effect in cost-benefit analyses with DICE-99, where land-use emissions are soon marginalized by the growing industrial emissions, but it can have significant effects in cost-effectiveness analyses. For instance, optimal carbon taxes for GMT stabilization at 2 °C above preindustrial levels are about 20% higher when land-use emissions are accounted for correctly.

$DELA$, GA: $DELA$ is defined as *(percent?) change per decade* but the definition of GA requires that $DELA$ is specified as *percent change per year*. Since the numerical value of $DELA$ is very small, this error has a negligible effect on the model results.

$DESIG$, $DESIG2$: $DESIG$ is defined as percent per *decade* but the definition of $GSIG$ requires that $DESIG$ is specified as percent per *year*. There is also confusion about $DESIG2$, for which no units are specified. The definition of $GSIG$ requires that $DESIG2$ is defined as fraction per year per decade, which would be a rather unusual choice of units.

MIU: Equation EE suggests that MIU is specified as percentage but its upper bound is set to 1.0, which only makes sense for pure numbers.

Once again, our main motivation for this discussion is to draw lessons for future analyses. The obvious lesson to be learnt from the errors identified in this subsection is that the units of all variables in a simulation model should be explicitly specified, either as documenting text or (preferably) using specialized simulation software that can check the consistency of model equations in terms of the units involved.

4. Summary and conclusions

This paper investigates whether the simplicity of global climate-economy models actually leads to transparency and consistency in their application. The goals of this analysis are to point out problem areas in earlier studies, to discuss the theoretical questions raised by these problems, to investigate the implications for the study results, and to provide recommendations for avoiding the problems identified here in future climate policy analyses. Our analysis focusses on the DICE model but most of the findings are relevant to other climate-economy models as well.

Sect. 2 reviews the application of social welfare functions in welfare-optimizing climate policy analyses. We find that several SWFs applied in climate-economy models are internally inconsistent. In particular, different methods for calculating the present values of alternative policy options lead to vastly different cost estimates. The close link with the index number problem implies that there is no single 'correct' method for comparing the present values of alternative climate policies with large welfare differences. We further show that the various internally consistent welfare metrics are generally inconsistent with each other since they aggregate differently across regions, time, states of the world, and components of economic output. Based on these findings, we present several recommendations for the consistent application of social welfare functions in climate-economy models. A reanalysis of two climate policy analyses with DICE that violate these recommendations reveals, among others, cost estimates of climate policies that are about two orders of magnitude smaller than those of other studies, questioning the validity of the policy conclusions drawn from the model results.

Sect. 3 identifies further empirical and numerical flaws in applications of DICE. These flaws include the calibration of uncertain climate parameters beyond their physically plausible range, the inappropriate characterization of uncertainty about transient climate change, the undocumented specification of radically different abatement cost curves in two implementations of DICE-99, and several numerical errors. We estimate the importance of these problems for model-based climate policy analysis, and we provide recommendations for preventing them in future analyses.

Our reanalysis of several published climate policy analyses demonstrates that the logical and empirical flaws identified in this paper are not only of theoretical interest. Most of them can strongly affect the policy recommendations drawn from the simulation results (*e.g.*, by significantly underestimating the range of 21-st century climate change).

The existence of these flaws is particularly disturbing given that DICE has been publicly available for many years, and that this model has been used and adapted by many different scholars.

Our findings indicate that much more caution is needed in the development, application, and modification of simple climate-economy models, and in the interpretation of their results. The combined efforts of original model developers, of analysts adopting existing models, and of peer reviewers are required to ensure that model applications are scientifically sound, and that the policy conclusions drawn from a particular model experiment are actually supported by the simulation results. Specific recommendations how to prevent the various problems identified in this paper are provided in the text.

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References

Allen, M. R., P. A. Stott, J. F. B. Mitchell, R. Schnur, and T. L. Delworth: 2000, 'Quantfifying the uncertainty in forecasts of anthropogenic climate change'. *Nature* **407**, 617–620.

Arrow, K. J.: 1951, *Social Choice and Individual Values*. New Haven, CT: Yale University Press.

Arrow, K. J., W. R. Cline, K.-G. Mäler, M. Munasinghe, R. Squitieri, and J. E. Stiglitz: 1996, 'Intertemporal equity, discounting, and economic efficiency'. In: J. P. Bruce, H. Lee, and E. F. Haites (eds.): *Climate Change 1995: Economic and Social Dimensions of Climate Change*. Cambridge, UK: Cambridge University Press, Chapt. 4.

Arrow, K. J. and R. C. Lind: 1970, 'Uncertainty and the Evaluation of Public Investment Decisions'. *American Economic Review* **60**, 364–378.

Azar, C. and K. Lindgren: 2003, 'Catastrophic Events and Stochastic Cost–benefit Analysis of Climate Change'. *Climatic Change* **56**, 245–255.

Bosetti, V. and L. Gilotte: 2005, 'Carbon Capture and Sequestration: How Much Does this Uncertain Option Affect Near-Term Policy Choices?'. Nota di Lavoro 86.2005, Fondazione Eni Enrico Mattei, Milano, Italy.

Bruckner, T. and K. Zickfeld: 2006, 'Low Risk Emissions Corridors for Safeguarding the Atlantic Thermohaline Circulation'. *Mitigation and Adaptation Strategies for Global Change.* (forthcoming).

Buonanno, P., C. Carraro, and M. Galeotti: 2003, 'Endogenous technological change and the costs of Kyoto'. *Resource and Energy Economics* **25**, 11–34.

Chapman, D., V. Suri, and S. G. Hall: 1995, 'Rolling DICE for the Future of the Planet'. *Contemporary Economic Policy* **13**, 1–9.

Cline, W. R.: 1992, *The Economics of Global Warming.* Washington, CD: Institute for International Economics.

Coble, C. R., E. G. Murray, and D. R. Rice: 1987, *Earth Science.* Englewood Cliffs, NJ: Prentice Hall.

Courtois, P.: 2004, 'The status of integrated assessment in climate policy making: An overview of inconsistencies underlying response functions'. *Environmental Science & Policy* **7**, 69–75.

Cubasch, U., G. A. Meehl, G. J. Boer, R. J. Stouffer, M. Dix, A. Noda, C. A. Senior, S. Raper, and K. S. Yap: 2001, 'Projections of Future Climate Change'. In: *Climate Change 2001. The Scientific Basis.* Cambridge: Cambridge University Press, Chapt. 9.

de Boyer Montégut, C., G. Madec, A. S. Fischer, A. Lazar, and D. Iudicone: 2004, 'Mixed layer depth over the global ocean: An examinaton of profile data and a profile-based climatology'. *Journal of Geophysical Research* **109**, C12003.

DeCanio, S. J.: 2003, *Economic Models of Climate Change*. New York, NY: Palgrave Macmillan.

Edgeworth, F. Y.: 1888, 'Some new methods of measuring variations in general prices'. *Journal of the Royal Statistical Society* **51**, 346–368.

Fankhauser, S. and R. S. J. Tol: 2005, 'On climate change and economic growth'. *Resource and Energy Economics* **27**, 1–17.

Forest, C. E., P. H. Stone, A. P. Sokolov, M. R. Allen, and M. D. Webster: 2002, 'Quantifying Uncertainties in Climate System Properties with the Use of Recent Climate Observations'. *Science* **295**, 113–117.

Füssel, H.-M.: 2006, 'Social welfare functions in global climate-economy models: Methodological inconsistencies and their policy implications'. Paper accepted for presentation at the Third World Congress of Environmental and Resource Economists, Kyoto (Japan), 3–7 July 2006. Available at `http://ssrn.com/abstract=900023`.

Gjerde, J., S. Grepperud, and S. Kverndokk: 1998, 'Optimal Climate Policy under the Possibility of a Catastrophe'. Discussion Paper 209, Research Department of Statistics Norway. available at `http://ideas.repec.org/p/ssb/dispap/209.html`.

Gowdy, J.: 2005, 'Toward a new welfare economics for sustainability'. *Ecological Economics* **53**, 211–222.

Gregory, J. M., K. W. Dixon, R. J. Stouffer, A. J. Weaver, E. Driesschaert, M. Eby, T. Fichefet, H. Hasumi, A. Hu, J. H. Jungclaus, I. V. Kamenkovich, A. Levermann, M. Montoya, S. Murakami, S. Nawrath, A. Oka, A. P. Sokolov, and R. B. Thorpe: 2005, 'A model intercomparison of changes in the Atlantic thermohaline circulation in response to increasing atmospheric CO_2 concentration'. *Geophysical Research Letters* **32**, L12703.

Grubb, N., T. Chapuis, and M. Ha-Duong: 1995, 'The economics of changing course: Implications of adaptability and inertia for optimal climate policy'. *Energy Policy* **23**, 417–431.

Hall, D. C. and R. J. Behl: 2005, 'Integrating economic analysis and the science of climate instability'. *Ecological Economics* [**in press**].

Heal, G.: 1997, 'Discounting and Climate Change'. *Climatic Change* **37**, 335–343.

Howarth, R. B.: 2001, 'Intertemporal social choice and climate stabilization'. *International Journal of of Environment and Pollution* **15**, 386–405.

Howarth, R. B.: 2003, 'Discounting and Uncertainty in Climate Change Policy Analysis'. *Land Economics* **79**, 369–381.

Jevons, W. S.: 1865, 'Variations of Prices and the Value of Currency since 1762'. *Journal of the Royal Statistical Society* **28**, 294–325.

Kaufmann, R. K.: 1997, 'Assessing the DICE Model: Uncertainty Associated with the Emission and Retention of Greenhouse Gases'. *Climatic Change* **35**, 435–448.

Keller, K., B. M. Bolker, and D. F. Bradford: 2004, 'Uncertain climate thresholds and optimal economic growth'. *Journal of Environmental Economics and Management* **48**, 723–741.

Keller, K., M. Hall, S.-R. Kim, D. F. Bradford, and M. Oppenheimer: 2005, 'Avoiding dangerous anthropogenic interference with the climate system'. *Climatic Change* **73**, 227–238.

Keller, K., K. Tan, F. M. M. Morel, and D. F. Bradford: 2000, 'Preserving the Ocean Circulation: Implications for Climate Policy'. *Climatic Change* **47**, 17–43.

Knutti, R., F. Joos, S. A. Müller, G.-K. Plattner, and T. F. Stocker: 2005, 'Probabilistic climate change projections for CO_2 stabilization profiles'. *Geophysical Research Letters* **32**, L20707.

Knutti, R., T. F. Stocker, F. Joos, and G.-K. Plattner: 2002, 'Constraints on radiative forcing and future climate change from observations and climate model ensembles'. *Nature* **416**, 719–723.

Kolstad, C. D.: 1998, 'Integrated Assessment Modeling of Climate Change'. In: W. D. Nordhaus (ed.): *Economics and Policy Issues in Climate Change*. Washington, DC: Resources for the Future, Chapt. 9, pp. 263–304. Includes comments by J. P. Weyant and J. Edmonds.

Koopmans, T.: 1960, 'Stational Ordinal Utility and Impatience'. *Econometrica* **28**, 287–309.

Lind, R. C.: 1995, 'Intergenerational equity, discounting, and the role of cost-benefit analysis in evaluating global climate policy'. *Energy Policy* **23**, 379–389.

Lind, R. C., K. C. Arrow, G. R. Corey, P. Dasgupta, A. K. Sen, T. Stauffer, J. E. Stiglitz, J. A. Stockfisch, and R. Wilson: 1982, *Discounting for Time and Risk in Energy Policy*. Washington, DC: Resources for the Future.

Lind, R. C. and R. E. Schuler: 1998, 'Equity and Discounting in Climate-Change Decisions'. In: W. D. Nordhaus (ed.): *Economics and Policy Issues in Climate Change*. Washington, DC: Resources for the Future, Chapt. 3, pp. 59–96.

Mastrandrea, M. D. and S. H. Schneider: 2001, 'Integrated Assessment of Abrupt Climatic Changes'. *Climate Policy* **1**, 433–449.

Mastrandrea, M. D. and S. H. Schneider: 2004, 'Probabilistic Integrated Assessment of "Dangerous" Climate Change'. *Science* **304**, 571–575.

Meinshausen, M.: 2006, 'What Does a 2C Target Mean for Greenhouse Gas Concentrations? A Brief Analysis Based on Multi-Gas Emission Pathways and Several Climate Sensitivity Uncertainty Estimates'. In: H. J. Schellnhuber, W. Cramer, N. Nakicenovic, T. Wigley, and G. Yohe (eds.): *Avoiding Dangerous Climate Change*. Cambridge, UK: Cambridge University Press, Chapt. 28, pp. 265–279.

Metz, B., O. Davidson, R. Swart, and J. Pan (eds.): 2001, *Climate Change 2001: Mitigation*. Cambridge: Cambridge University Press.

Moles, C. G., J. R. Banga, and K. Keller: 2004, 'Solving nonconvex climate control problems: pitfalls and algorithm performances'. *Applied Soft Computing* **5**, 35–44.

Moss, R. H. and S. H. Schneider: 2000, 'Uncertainties in the IPCC TAR: Recommendations to Lead Authors for More Consistent Assessment and Reporting'. In: R. Pachauri, T. Taniguchi, and K. Tanaka (eds.): *Guidance Papers on the Cross-cutting Issues of the Third Assessment Report of the IPCC*. Geneva: World Meteorological Organization, pp. 33–51.

Nakicenovic, N. and R. Swart (eds.): 2000, *Emissions Scenarios*. Cambridge: Cambridge University Press.

Newell, R. G. and W. A. Pizer: 2004, 'Uncertain discount rates in climate policy analysis'. *Energy Policy* **32**, 519–529.

Nordhaus, W. and J. Boyer: 1999, 'Appendix C: Computer Code for DICE-98 Model'. In: *Warming the World: Economics Models of Global Warming (Internet Edition)*. Available on `http://www.econ.yale.edu/~nordhaus/homepage/appendix%20c%20121898.PDF` (accessed on 11 Dec 2005).

Nordhaus, W. D.: 1992, 'Rolling the 'DICE': An Optimal Transition Path for Controlling Greenhouse Gases'. *Science* **258**, 1315–1319.

Nordhaus, W. D.: 1993, 'Optimal Greenhouse-Gas Reductions and Tax Policy in the "DICE" Model'. *American Economic Review* **83**, 313–317.

Nordhaus, W. D.: 1994, *Managing the Global Commons*. Cambridge, MA: MIT Press.

Nordhaus, W. D.: 1997, ‘Discounting in Economics and Climate Change’. *Climatic Change* **37**, 315–328.

Nordhaus, W. D.: 1999, ‘Modeling induced innovation in climate-change policy’. In: A. Grübler, N. Nakicenovic, and W. D. Nordhaus (eds.): *Technological Change and the Environment*. Washington, DC: Resources for the Future, Chapt. 8.

Nordhaus, W. D.: 2001a, ‘Documentation of the DICE-99 Excel spreadsheet’. Available at `http://www.econ.yale.edu/~nordhaus/homepage/dice99doc.doc`.

Nordhaus, W. D.: 2001b, ‘Global Warming Economics’. *Science* **294**, 1283–1284.

Nordhaus, W. D. and J. Boyer: 2000, *Warming the World: Economic Models of Global Warming*. Cambridge, MA: MIT Press.

Nordhaus, W. D. and D. Popp: 1997, ‘What is the Value of Scientific Knowledge? An Application to Global Warming Using the PRICE Model’. *The Energy Journal* **18**, 1–45.

Nordhaus, W. D. and Z. Yang: 1996, ‘Optimal Greenhouse-Gas Reductions and Tax Policy in the “DICE” Model’. *American Economic Review* **86**, 741–765.

Pearce, D. W. (ed.): 1986, *The MIT Dictionary of Modern Economics*. Cambridge, MA: MIT Press, third edition.

Popp, D.: 2004, ‘ENTICE: endogenous technological change in the DICE model of global warming’. *Journal of Environmental Economics and Management* **48**, 742–768.

Portney, P. R. and J. P. Weyant (eds.): 1999, *Discounting and Intergenerational Equity*. Washington, DC: Resources for the Future.

Roughgarden, T. and S. H. Schneider: 1999, ‘Climate change policy: quantifying uncertainties for damages and optimal carbon taxes’. *Energy Policy* **27**, 415–429.

Schlesinger, M. E. and X. Jiang: 1990, 'Simple model representation of atmosphere-ocean GCMs and estimation of the timescale of CO_2-induced climate change'. *Journal of Climate* **3**, 1297–1315.

Schlesinger, M. E., J. Yin, G. Yohe, N. G. Andronova, S. Malyshev, and B. Li: 2006, 'Assessing the Risk of a Collapse of the Atlantic Thermohaline Circulation'. In: H. J. Schellnhuber, W. Cramer, N. Nakicenovic, T. Wigley, and G. Yohe (eds.): *Avoiding Dangerous Climate Change*. Cambridge, UK, pp. 37–47, Cambridge University Press.

Schneider, S. H. and S. L. Thompson: 1981, 'Atmospheric CO_2 and climate: Importance of the transient response'. *Journal of Geophysical Research* **86**(C4), 3135–3147.

Schultz, P. A. and J. F. Kasting: 1997, 'Optimal reductions in CO_2 emissions'. *Energy Policy* **25**, 491–500.

Smirnov, A.: 2005, 'Attainability analysis of the DICE model'. Interim Report IR-05-000, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Spash, C. L. (ed.): 2002, *Greenhouse Economics: Values and Ethics*. New York: Routledge.

Stott, P. A. and J. A. Kettleborough: 2002, 'Origins and estimates of uncertainty in predictions of twenty-first century temperature rise'. *Nature* **416**, 723–726.

Taylor, C.: 1982, 'The diversity of goods'. In: A. Sen and B. Williams (eds.): *Utilitarianism and Beyond*. Cambridge, UK: Cambridge University Press, pp. 129–144.

Tol, R. S. J.: 1999, 'Time discounting and optimal emission reduction: an application of FUND'. *Climatic Change* **41**, 351–362.

Tol, R. S. J.: 2003, 'Is the Uncertainty about Climate Change Too Large for Expected Cost-Benefit Analysis?'. *Climatic Change* **56**, 265–289.

Toth, F. L.: 2000, ‘Intergenerational equity and discounting’. *Integrated Assessment* **1**, 127–136.

Webster, M., C. Forest, J. Reilly, M. Babiker, D. Kicklighter, M. Mayer, R. Prinn, M. Sarofim, A. Sokolov, P. Stone, and C. Wang: 2003, ‘Uncertainty analysis of climate change and policy response’. *Climatic Change* **61**, 295–320.

Wigley, T. M. L. and S. C. B. Raper: 2001, ‘Interpretation of High Projections for Global-Mean Warming’. *Science* **293**, 451–454.

Yohe, G., N. Andronova, and M. Schlesinger: 2004, ‘To Hedge or Not Against an Uncertain Climate Future’. *Science* **306**, 416–417. (Includes Supporting Online Material.

Yohe, G., M. E. Schlesinger, and N. G. Andronova: 2006, ‘Reducing the risk of a collapse of the Atlantic thermohaline circulation’. *The Integrated Assessment Journal* **6**, 57–73.

Yohe, G. W.: 2003, ‘More Trouble for Cost-benefit Analysis. An Editorial Comment’. *Climatic Change* **56**, 235–244.