Abrupt Events and the Global Supply Network: 
A Network Measure for Cascading Production Losses

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Abstract:

Local production losses caused by unanticipated catastrophic events such as extreme weather, earthquakes and terror attacks entail a high risk of being spread through the globalised supply network thereby affecting many economies. As a contribution to the assessment of the supply network vulnerability to these events, we provide a novel measure for the propagation of production losses. It is in particular developed to identify supply dependencies that can become critical in the short period following extreme events. This measure generalizes the concept of import and export dependencies by taking into account higher-order supply relationships and structural properties of the network. Using information on trade connections and trade volumes as the only input, it tracks the dissemination of the production losses from one supply chain layer to the next in a recursive routine. An extended version of this measure reflects the producers’ efforts to compensate the input shortages. Applying the measure to recent input-output data, we demonstrate that the network is structured in a way that some regional sectors’ production losses have the potential to affect the global economy significantly. For these cases we identify rapid cascade effects of considerable size. We also find that other network measures do not fully reflect this new measure, as a linear combination of them does not sufficiently reproduce the result on the maximum production loss. The measure introduced in this paper can also be used to gain insights into the production loss dissemination across countries. We find that many countries can be affected substantially through indirect trade relationships with regions that are highly exposed to natural catastrophes.

Keywords: supply network, extreme events, loss cascades, input-output linkages

JEL-Classification: C67, Q54

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1 Introduction

Local production losses can be passed on to other economies via a globalised network of supply dependencies. In particular unanticipated and sudden shocks from extreme events such as weather extremes, earthquakes and terror attacks entail a high risk of propagating through the supply network, as production is difficult to reorganise at short notice (Levermann (2014)). Micro-shocks on the supply side like these thus threaten to become aggregate shocks that bear the risk to snowball into economic crises (e.g. Acemoglu et al. (2013, 2015a), Gabaix (2011)). With regard to an enhanced future risk of climatic extremes (IPCC (2012), Schellnhuber et al. (2012)), it is of utmost importance to understand and quantify the potential for shock propagation. In this spirit, this paper’s contribution is to introduce a new measure that can be used to identify critical interdependencies arising through the structural properties of the network.

An emerging literature employs network theory to explain how micro-shocks are passed on and possibly spread to the aggregate level. It aims to explore whether the risk of cascading failures is increased by structural properties of the network and is increasingly applied to the financial system network (e.g. Acemoglu et al. (2015b), Chinazzi et al. (2013), Elliott et al. (2014), Roukny et al. (2013), Stiglitz (2010)) and to supply chains (Battiston et al. (2007), Bierkandt et al. (2014), Weisbuch and Battiston (2007), Wenz et al. (2014)). A recent theoretical contribution on supply networks by Acemoglu et al. (2012) shows that the structure of the network determines the possibility and the duration of a production loss cascade, where the losses are not only passed on to the immediate customer but to further supply chain layers. In particular, they prove the existence of significant asymmetries between sectors as direct or indirect suppliers to other producers to be key to the potential of shock propagation. This result adds to a contribution from the non-network literature made by Gabaix (2011), stating that firm-level idiosyncratic shocks can lead to fluctuations at the aggregate level, if the largest firms have a disproportionally large share of the output. Another cause for an increased vulnerability of the global supply chain is suggested to be a higher level of interconnectedness as demonstrated for heat-stress induced production losses (Wenz and Levermann (2016)).

To estimate the higher-order effects of extreme events and shocks, various other approaches such as input-output frameworks, computable general equilibrium models and empirical studies have been employed. Introduced by Leontief (1936), input-output frameworks are based on input-output tables that reflect the economic interdependencies within a regional economy (or in case of multi-regional input-output tables those of several regional economies) in a detailed way. An extension of this framework is the inoperability input-output model, of which the static version aims to quantify how the outage of one system affects the overall economy and the dynamic version is developed to assess the recovery times after the shock (Haines and

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1Beyond that, this approach is also applied to examine contagion effects in various other networks. Among them are traffic networks (Su et al. (2014)), power grids (Brummitt et al. (2012), Pahwa et al. (2014)), the internet (Newman et al. (2002)) and combinations of these infrastructures (Buldyrev et al. (2010)) and social networks to explore the dissemination of information and opinions (e.g. Gale and Kariv (2003), Acemoglu et al. (2010)).
Jiang (2001)). This model, however, rests on the equilibrium assumption that the industries’ production is in balance with the final demand. Yet, after a perturbation on the supply side, in particular if it cascades into a shock on the aggregate level, non-equilibrium conditions dominate (Haimes et al. (2005), Santos and Haimes (2004)). Other applications thus concentrate on indirect effects that propagate through modified demand (e.g. Rose et al. (1997)). Input-output accounts are also part of and parcel to many computable general equilibrium models. In contrast to the input-output frameworks, they are capable of accommodating input substitutions and demonstrating responses to price changes and their effects (e.g. Rose and Liao (2005), Tsuchiya et al. (2007)). Recently, Roson and Sartori (2016) investigated the link between the economic structure and the aggregate sensitivity to sectoral productivity shocks using stochastic simulations with a global computable general equilibrium model. Gourio (2012) explores a real business cycle model with a small risk of an economic disaster. With respect to the application to catastrophes, the general equilibrium models have been criticized for relying on the market-clearing hypothesis. In the disaster aftermath, shortages are more likely to be passed on due to rationing than through price increases and substitution (Hallegatte (2008)). Another type of model using input-output tables is the adaptive-regional input-output model developed by Hallegatte (2008). This model accounts for interactions between a number of different industries and households and for adaptive behavior, and thus resembles the modelling philosophy of the network approach described above. Using empirical methods, Carvalho and Gabaix (2013) examine the role of the microeconomic composition of the economy for macroeconomic volatility, while Foerster et al. (2011) and Atalay (2014) aim to understand the importance of aggregate and micro-shocks for output-volatility. An empirical study on the French economy by di Giovanni et al. (2014) shows that a significant share of the aggregate volatility stems from firm-level shocks being spread through the input-output linkages. Furthermore, Carvalho et al. (2014) find significantly negative effects for firms directly and indirectly linked to suppliers hit by the Great East Japan Earthquake in 2011. Boehm et al. (2015) reveal that the U.S. output of Japanese affiliates declined considerably following that earthquake. As the size of this decline is shown to correspond roughly to the decline in imported inputs, the elasticity of substitution between imported and domestic inputs is very low, which almost implies a Leontief production function.

The measure we propose is also related to the theoretical literature on network centrality. One of the most crucial problems in this field is to determine the importance (the centrality) of a particular vertex (or an edge) for the network to be functional. The developed measures only rely on information on structural network properties. Most of them assign a vertex a great importance, if it is well connected with many other vertices (degree centrality) or if the neighbours are linked with many others as well (eigenvector centrality and variants, e.g. Katz centrality and PageRank; Bonacich (1987), Katz (1953)). A different concept of centrality is betweenness centrality (Freeman (1977)), which assesses the extent to which a vertex lies on the shortest paths between other vertices, thus limiting its applicability to specific classes of networks. Betweenness centrality, however, was modified to be suitable for other
networks as well, in which the centrality of vertices is associated with being on paths with other characteristics, e.g. the highest capacity (Borgatti (2005)). For supply networks, Acemoglu et al. (2012) introduced three more concepts: the influence vector describing the equilibrium share of a producer’s sales in all economy-wide sales; the coefficient of variation measuring the relative variation in amount of direct connections producers have; and the second-order interconnectivity coefficient, quantifying the extent to which major suppliers (with high out-degrees) are linked with each other through common suppliers.

To the best of our knowledge, the literature does not provide a centrality measure that assesses the importance of a producers’ output for any other producer, who could be located anywhere in the network and does not have to be a direct customer. Likewise, there are no measures that describe the economy-wide or global dependance on some sector. The merit of such a measure is the identification of interdependencies, that might be critical in the sense that the suppliers are at risk of suffering from a production failure. Such a measure could quantify the strength of these dependencies, which is important to understand why some production shortages have the potential to be passed on and to eventually snowball into economic crises. In other words, building on the findings by Acemoglu et al. (2012), this measure would be a tool to assess the potential of shock propagation arising through the structural properties of the network.

The measure we develop is referred to as the Production Shortage Interdependency (PSI, \(\Psi\)), henceforth. It is in particular developed to identify supply dependencies that might become critical in the short period following production loss inducing events. It extends the concept of export and import dependencies by incorporating higher-order supply relationships. The idea is to track the production shortage dependencies from one supply chain layer to the next by using a recursive definition.\(^2\) The measure accounts for the existing trade connections and the potential input shortages that can be passed on. We assume perfect complementarity of the production factors to express the limitations / difficulties in substituting the lacking input factors in the short term. Dynamic responses or precautionary measures to compensate for input shortages (e.g. inventory holding and the possibility to readdress demand at short notice) are abstracted from in this study. PSI thus gives an appraisal of the cause-and-effect relationship between the network structure and the dissemination of supply shortages. In addition, we extend this measure by incorporating a factor reflecting that the downstream producers succeed in compensating the inherited input losses to some extent.

It must be stressed that this measure is not designed to replace any of the elaborate dynamic models mentioned above. Its usefulness is given through its simplicity, which allows PSI to complement other models that lack the network assumption and it allows analytical considerations of cascades in networks. It also contributes in easily determining the dependencies between regional sectors that are not directly linked by trade relationships. The production shortage interdependence on some selected sectors in the Philippines, Japan and the Nether-

\(^2\)A global average of direct supply dependencies on tropical countries (i.e. PSI\(^{(1)}\), referred to as SPC) has been analyzed in Wenz and Levermann (2016).
lands, which belong to the 12 countries most exposed to natural catastrophes according to the World Risk Index (Alliance Development Work and UNU-EHS (2015)), is also demonstrated in this paper.

The remainder of the paper is as follows. Section 2 gives a detailed mathematical explanation of PSI, an illustration for some specific simple network structures and it demonstrates how to extend PSI to incorporate loss compensation. In Section 3 the measure is applied to recent input-output data with a particular focus on countries that are frequently exposed to natural catastrophes. Section 4 concludes.

2 The Production Shortage Interdependence Measure

In the following, the mathematical description of PSI is developed. Afterwards, PSI is illustrated for some specific network structures. In the subsequent subsection we expand this measure by incorporating a factor reflecting that the downstream producers succeed in dampening the inherited input losses to some extent.

2.1 Definition

PSI is aimed to reflect the short-term propagation effects of an unanticipated shock affecting some production entity, which could be a production facility or a production sector. In the following we explain the measure for the direct trade connections and afterwards for the higher-order or indirect supply relationships.

2.1.1 First-order PSI

Assume there is a set of regions \( R \), where each region accommodates a set of sectors \( I \). Each sector in \( I \) consists of firms producing the same product \( i \in I \) according to a Leontief production function. Let the producers be referred to as by some index-pair \( ir \), where the first index specifies a sector \( i \) out of set \( I \) and the second index denotes a region \( r \) in \( R \). The input flow from some producer \( ir \) to any other \( js \) is represented by \( Z_{ir \rightarrow js} \) and in the undisturbed state, in which \( ir \) supplies the regular quantity, by \( Z_{ir \rightarrow js}^* \). Accordingly, \( js \)’s production function is given by

\[
Y_{js} = \min_{i \in I} \left( \alpha_{js,i} \sum_{r \in R} Z_{ir \rightarrow js} \right)
\]

and in the undisturbed state by

\[
Y_{js}^* = \min_{i \in I} \left( \alpha_{js,i} \sum_{r \in R} Z_{ir \rightarrow js}^* \right).
\]

The coefficients \( \alpha_{js,i} \), \( i \in I \), are the technological coefficients determining how much of the respective inputs \( i \) are required to produce one unit of the output in \( js \)’s production facility. Assuming that the production \( Y_{js}^* \) and the supply \( Z_{ir \rightarrow js}^* \) in the undisturbed state are known, we can derive the values of the coefficients by claiming the relationship

\[
Y_{js}^* = \alpha_{js,i} \sum_{r \in R} Z_{ir \rightarrow js}^*
\]

(1)
for all $i \in I$.

Now assume that $ir$ suffers from some production loss and that $ir$ cuts down the supply to each customer equally, i.e. she does not grant any customer a preferential treatment. That is, the supplies to $js$ evolve as $Z_{ir \rightarrow js} = \frac{Y_{ir}}{Y_{ir}^*} Z_{ir \rightarrow js}^*$. If $ir$’s production fails completely, i.e. $Y_{ir} = 0$, and all other suppliers of $js$ remain unaffected by $ir$’s production outage, then $js$’s production function evolves as

$$Y_{js} = \min \left( \alpha_{js,i} \left( \sum_{r' \in R} Z_{ir' \rightarrow js}^* - Z_{ir \rightarrow js}^* \right) \right), \min_{i' \in I \setminus \{i\}} \left( \alpha_{js,i'} \sum_{r' \in R} Z_{ir' \rightarrow js}^* \right)$$

$$= \alpha_{js,i} \left( \sum_{r' \in R} Z_{ir' \rightarrow js}^* - Z_{ir \rightarrow js}^* \right)$$

$$= Y_{js}^* \left( 1 - \frac{Z_{ir \rightarrow js}^*}{\sum_{r' \in R} Z_{ir' \rightarrow js}^*} \right),$$

where in the first equation the supply by $ir$ is subtracted from all supplies of good $i$ to $js$. As a result of Equation (1), the minimum of all values in the first equation is given by the first term in the outer bracket, as stated in the second equation. This expression is then transformed by employing again Equation (1) in the third equation.

The production is reduced to $\left( 1 - \frac{Z_{ir \rightarrow js}^*}{\sum_{r' \in R} Z_{ir' \rightarrow js}^*} \right)$ of its original level. The fraction, $\sum_{r' \in R} Z_{ir' \rightarrow js}^*$, can thus be perceived as the production dependency of direct customer $js$ from producer $ir$:

$$\Psi_{ir \rightarrow js}^{(1)} = \Psi_{ir \rightarrow js} := \begin{cases} \frac{Z_{ir \rightarrow js}^*}{\sum_{r' \in R} Z_{ir' \rightarrow js}^*} & \text{for } ir \neq js \\ 1 & \text{for } ir = js. \end{cases}$$

The superscript (1) indicates the direct, first-order, trade relationship. For $r \neq s$ PSI expresses the dependency of regional sector $js$ on the imports from $ir$. As PSI is static, $\Psi_{ir \rightarrow js} = 1$ simply reflects $ir$’s total production outage for $ir = js$.

PSI expresses the production shortage dependencies that materialize in the short-term. We therefore assume that each supply link within the network can transmit production shortages not more than once. This is implemented by a set of all trade links that have been already accounted for in the computation of PSI. This set describes which trade links are not allowed to be traversed by PSI in the following. For PSI referring to a direct connection, as given by Equation (3), the set is simply given by

$$L_{ir \rightarrow js}^{(1)} := \begin{cases} \{(ir, js)\} & \text{for } ir \neq js \\ \{\} & \text{for } ir = js. \end{cases}$$

where the tuple $(ir, js)$ represents the supply link from $ir$ to $js$. 
2.1.2 Higher-order PSI

Now assume that ir is not a direct supplier of js. Let ir supply some producer ku who then supplies js, forming a supply chain denoted by ir → ku → js, henceforth. For the moment, let us assume that this is the only supply chain through which js could inherit production losses. We know that a complete production outage in ir leads to a production loss of 

\[
1 - \sum_{r' \in R} Z_{ir' \rightarrow ku} \cdot Z_{ku \rightarrow js}
\]

of ku’s usual production. As ku is assumed to reduce all of the supplies to its customers equally and ku’s share of supply of input k to js is 

\[
Z_{ku \rightarrow js} \cdot \sum_{u' \in R} Z_{ku' \rightarrow js}
\]

the supply of k to js is reduced by 

\[
\sum_{r' \in R} Z_{ir' \rightarrow ku} \cdot Z_{ku \rightarrow js} \cdot \sum_{u' \in R} Z_{ku' \rightarrow js}
\]

The production interdependency is thus given by the product of the corresponding PSIs, i.e. by \(\Psi_{ir \rightarrow js} = \Psi_{ir \rightarrow ku} \Psi_{ku \rightarrow js}\).

Now assume that ir and js are indirectly connected through more than one intermediate producers as illustrated by Figure 1. If they all produce the same intermediate good k, we can simply add up the corresponding products of the PSIs. For instance, in Figure 1 js experiences a production shortage of 

\[
\Psi_{ir \rightarrow k1u1} \Psi_{k1u1 \rightarrow js} + \Psi_{ir \rightarrow k1u2} \Psi_{k1u2 \rightarrow js}
\]

of product k1. If js is short of more than one input, it is easy to see from Equation (2) that the output is determined by the input lacking most. In Figure 1 js’s production is thus determined by the shortage of k1 or of k2 that is greater. In summary, the production interdependency evolves as 

\[
\Psi_{ir \rightarrow js} = \max_{k \in I} \sum_{u \in R} \Psi_{ir \rightarrow ku} \Psi_{ku \rightarrow js}
\]

Assuming the argument of the maximum, i.e. the input k causing the greatest shortage, is k2, then the set of the supply links taken into account is 

\[
L_{ir \rightarrow js}^{(2)} = L_{ir \rightarrow k2u2}^{(1)} \cup \{(k2u2, js)\}
\]

which is the same as 

\[
L_{ir \rightarrow js}^{(2)} = \{(ir, k2u2), (k2u2, js)\}
\]

If the greatest shortage occurs for k1, then the set evolves as 

\[
L_{ir \rightarrow js}^{(2)} = \bigcup_{u \in R} L_{ir \rightarrow k1u}^{(1)} \cup \{(k1u, js)\}
\]

which is the same as 

\[
L_{ir \rightarrow js}^{(2)} = \{(ir, k1u1), (ir, k1u2), (k1u1, js), (k1u2, js)\}
\]

Continuing this line of argument of how PSI evolves for longer supply chains, it becomes clear that the sector’s production shortages depend on the production losses the direct suppliers suffer from and the direct suppliers’ share of supply to this sector. The direct suppliers’ production losses are also given by PSI. These considerations give rise to a recursively defined
measure PSI that allows computing $\Psi^{(m)}_{ir\rightarrow js}$ for all layers $m$ by tracing back the production shortage interdependencies layer by layer to $ir$. As illustrated by Figure 2, to compute $\Psi^{(3)}_{ir\rightarrow js}$ one considers first the PSI-value corresponding to the second layer, $\Psi^{(2)}_{ir\rightarrow ku}$, and the PSI-value corresponding to the direct connection, $\Psi_{ku\rightarrow js}$. To derive $\Psi^{(2)}_{ir\rightarrow ku}$, one incorporates the PSI associated with the layer above, $\Psi_{ir\rightarrow lv}$, and the PSI of the direct links, which is the same in this case.

Figure 2: Illustration of the layers of a supply network and the associated PSI, which build on each other.

Expressed in formalistic terms, the production interdependency $\Psi^{(m)}_{ir\rightarrow js}$ for $m - 1$, $m \geq 2$ evolves as

$$
\Psi^{(m)}_{ir\rightarrow js} = \begin{cases} 
\max_{k \in I} \sum_{u \in R} \Psi^{(m-1)}_{ir\rightarrow ku} \Psi_{ku\rightarrow js} & \text{for } ir \neq js \\
1 & \text{for } ir = js.
\end{cases}
$$

(5)

After having computed $\Psi^{(m)}_{ir\rightarrow js}$, one has to account for the links traversed in the $m$th layer. If the argument of the maximum of the sum in Equation (5) is $\bar{k}$, the set is given by

$$
L^{(m)}_{ir\rightarrow js} = \bigcup_{u \in R} L^{(m-1)}_{ir\rightarrow ku} \cup \{(\bar{k}u, js)\}.
$$

(6)

As soon as the set $L^{(m)}_{ir\rightarrow js}$ contains all links that can possibly be traversed by PSI, we set

$$
\Psi^{(m+1)}_{ir\rightarrow js} = \Psi^{(m)}_{ir\rightarrow js}.
$$

Equation (5) states that for computing the production shortage interdependency for two sectors that are separated by (at most) $m$ tiers, one needs to consider $js$’s direct suppliers $ku$ first, being in the $(m - 1)$th tier. As pointed out above, the direct suppliers’ production
shortages that are inherited from \(ir\), \(\Psi^{(m-1)}_{ir \rightarrow ku}\), are to be multiplied with their share of supply to \(js\), \(\Psi_{ku \rightarrow js}\). The thus computed shortages have to be added up for all regions supplying the same good. The assumption of perfect complementarity now implies that the production shortages are determined by the input \(k\) that induces the most severe scarcity. Note that if this sector happens to be the one from which the shock originates, then it must be accounted for that customer \(js\) might be directly linked with \(ir\) as well. In that case one of the summands \(\Psi^{(m-1)}_{ir \rightarrow ir} \Psi_{ir \rightarrow js}\) equals \(\Psi_{ir \rightarrow js}\), which describes the direct dependency. By claiming that \(\Psi^{(m)}_{ir \rightarrow ir} = 1\) for all \(m \in \mathbb{N}\), the measure \(\Psi_{ir \rightarrow ir}\) thus covers all supply dependencies that are of length \(m\) at most. Equation (5) also expresses that a supply link \(ku \rightarrow js\) already traversed by \(\Psi^{(m-1)}_{ir \rightarrow ku}\) is ignored in the calculation of \(\Psi^{(m)}_{ir \rightarrow js}\). Note that a producer may influence another producer through multiple links (as \(ir\) does in Figure 1 and in Figure 2), but loop structures as illustrated by Figure 3 are to be accounted for only once by PSI. Furthermore, we exclude the case of self-supply by claiming \(ku \neq js\) in Equation (5). However, in input-output tables self-supply is a common feature, as many different commodities, which may serve as inputs for one another, are grouped together. Note that the restriction of \(ku \neq js\) can be dropped without limiting the applicability of Equation (5). Just as well, one might allow PSI to traverse all supply loops as often as this is possible within \(m\) steps. In that case the major suppliers’ production shortages may exert a repeated influence on the dissemination of the losses. PSI would thus crucially depend on the number of times these suppliers are accounted for. Another feasible option would be to incorporate the asymptotic contribution of the loop to the production losses. This choice would rather correspond to situations in which the production losses persist because the disruption lasts a long time and the customers cannot adapt meanwhile. Note that adopting the first alternative choice to deal with loops only requires to drop Equation (6).

Equations (5) - (6) provide the production supply interdependencies between two producers \(ir\) and \(js\). The production supply dependence of a sector/country/region on some producer/sector/country/region are easily derived by calculating a (weighted) average of the corresponding PSIs. For instance, the dependence of the global economy on some producer can
be expressed as

$$
\Psi_{ir}^{(m)} = \sum_{j \in I} \sum_{s \in R} \sum_{l \in I} \sum_{v \in R} Z^{s \to l}_{j \to ir} \Psi_{ir \to js}^{(m)}
$$

with

$$
Z^{s \to l}_{j \to ir} = \sum_{l \in I} \sum_{v \in R} Z^{s \to l v}_{j \to ir}
$$

Equation (8) simply adds up all (direct) supply flows from one producer $js$. To assess the impact of $ir$’s production outage on the global economy, each $\Psi_{ir \to js}^{(m)}$ is assigned a weight that describes $ir$’s relative importance as a direct supplier. This gives credit to the fact that a small production failure of a major supplier is presumed to cause greater economic losses than a minor one. In principle, other weights are feasible as well. For instance, one might take into account the relative share of $ir$’s output in its sector $i$. Alternatively, producers providing goods or services that satisfy basic needs can be equipped with greater weights.

### 2.2 Some Specific Network Structures and PSI

For some specific network structures, the evolution of PSI for large $m$ can be easily assessed. A star-shaped network centred around $ir$, as illustrated by Figure 4a, generates the maximum possible production shortage interdependence of 1 for all producers. Star-shaped structures are for instance given for economies that depend on monopolies.

A fundamentally different constellation is given for a producer $ir$ occupying a remote position within the network in the sense that she supplies only a few customers. Figure 4b illustrates a network in which $ir$ and $js$ are connected through only one supply chain (blue) to which other producers (grey) also contribute. All horizontal vertices represent producers from the same sector. It is easy to see that there is no production shortage interdependence between $ir$ and the grey producers. By contrast, the grey producers reduce the production shortage interdependence between $ir$ and $js$. If there are $|R|$ producers in each layer and if every producer divides her orders equally among the suppliers, i.e. every supplier provides $\frac{1}{|R|}$ of the input, we obtain $\Psi_{ir \to js}^{(m)} = \frac{1}{|R|^m}$. Hence, the production shortage interdependence converges towards zero the longer the considered production chain and/or the more regions and thus the more unaffected producers contribute to the supply of the inputs. Under supply asymmetry, the convergence is faster the smaller the share of supply from $ir$ and from the downstream producers. In contrast to the star-shaped network, this network thus features a structure that allows a shock to dissipate.

Next, let us consider a fully connected network, in which all producers share a customer and supplier relationship with each other. For simplicity we assume that only one good $i$ is produced. In the following, we show that within 3 layers all supply links are affected, give a closed-form solution for PSI and demonstrate that for a higher number of integrated regions in the network the production shortage interdependence converges towards zero.

We start our considerations with a fully connected network of only one sector and four regions (Figure 4c). For this example, we compute $\Psi_{ir_1 \to ir_2}^{(m)}$ for all $m \leq 3$. While doing so, we
must keep track of all links that have already been accounted for. In total, there are nine links that come into consideration, as we ignore all links pointing back towards the origin of the shock, say \( ir_1 \). Assuming symmetric supply as before, it is clear that \( \Psi_{ir_1 \rightarrow ir_\mu} = \frac{1}{3} \) for all \( \mu \in \{ 2, 3, 4 \} \). For \( m = 2 \), it is \( \Psi_{ir_1 \rightarrow ir_2}^{(2)} = \Psi_{ir_1 \rightarrow ir_2} + \frac{1}{3} \Psi_{ir_2 \rightarrow ir_2} + \frac{1}{4} \Psi_{ir_3 \rightarrow ir_2} \), where the first summand corresponds to the direct trade connection and where we used the former result on \( \Psi_{ir_1 \rightarrow ir_\mu} \) for all \( \mu \in \{ 2, 3, 4 \} \). The values of \( \Psi_{ir_3 \rightarrow ir_2} \) and of \( \Psi_{ir_4 \rightarrow ir_2} \) are also \( \frac{1}{3} \), hence \( \Psi_{ir_1 \rightarrow ir_2}^{(2)} = \frac{5}{9} \). The set \( L_{ir_1 \rightarrow ir_2}^{(2)} = \{(ir_1, ir_2), (ir_1, ir_3), (ir_3, ir_2), (ir_1, ir_4), (ir_4, ir_2)\} \) contains already five of the nine possible paths. For \( m = 3 \), it is \( \Psi_{ir_1 \rightarrow ir_2}^{(3)} = \Psi_{ir_1 \rightarrow ir_2} + \Psi_{ir_2 \rightarrow ir_3} \Psi_{ir_3 \rightarrow ir_2} + \Psi_{ir_2 \rightarrow ir_4} \Psi_{ir_4 \rightarrow ir_2} \). Now, we have to compute \( \Psi_{ir_1 \rightarrow ir_2}^{(3)} \) and \( \Psi_{ir_1 \rightarrow ir_3}^{(3)} \), and have to check whether the tuple \( (ir_3, ir_2) \) is in \( L_{ir_1 \rightarrow ir_3}^{(2)} \) and the tuple \( (ir_4, ir_2) \) in \( L_{ir_1 \rightarrow ir_4}^{(2)} \). Doing so, we see that these PSI-values also are \( \frac{5}{9} \) and the tuples are not in these sets, respectively. Thus, \( \Psi_{ir_1 \rightarrow ir_2}^{(3)} = \frac{19}{27} \). Now, the union of the sets \( L_{ir_1 \rightarrow ir_3}^{(2)} \) and \( L_{ir_1 \rightarrow ir_4}^{(2)} \) and the tuples \( (ir_3, ir_2) \) and \( (ir_4, ir_2) \) gives \( L_{ir_1 \rightarrow ir_2}^{(3)} \), which already contains all possible paths.

Now, we examine an arbitrarily large network with \( |R| \geq 3 \) regions that is fully connected. Ignoring all self-supply links, the total number of links is \( |R| \cdot (|R| - 1) \). In addition, we exclude all links pointing back to the producer \( ir_1 \) who suffers from the outage, hence there are \((|R| - 1)^2\) links PSI can traverse. Assuming symmetric supply as before, it is \( \Psi_{ir_1 \rightarrow ir_\mu} = \frac{1}{|R|-1} \) for all \( \mu \in \{ 2, ..., |R| \} \). Just as in the case described above, PSI for all other direct connections

\(^3\)It is \( L_{ir_1 \rightarrow ir_3}^{(2)} = \{(ir_1, ir_3), (ir_1, ir_2), (ir_2, ir_3), (ir_1, ir_3), (ir_1, ir_4)\} \) and \( L_{ir_1 \rightarrow ir_4}^{(2)} = \{(ir_1, ir_4), (ir_1, ir_2), (ir_2, ir_4), (ir_1, ir_3), (ir_3, ir_4)\} \).
possible is of the same value. For \( m = 2 \), we obtain \( \Psi_{ir_1 \rightarrow ir_2}^{(2)} = \frac{1}{|R|-1} + (|R| - 2) \left( \frac{1}{|R|-1} \right)^2 \).

The first summand stems from the direct connection \( ir_1 \rightarrow ir_2 \). Excluding the direct link and the self loop \( ir_2 \rightarrow ir_2 \), there are \((|R| - 2)\) connections left for which we simply multiply the corresponding PSI values with \( m = 1 \). Instead of writing down all sets to check which links have been accounted for so far, one might derive this information by sketching trees.

Each branch represents a link that has not been traversed so far and is newly added to the respective set \( L \). All branches in the same column correspond to the same layer. Figure 5 relates to \( L_{ir_1 \rightarrow ir_2}^{(2)} \). In the first layer there are all links connecting \( ir_1 \) directly with the other \(|R| - 1\) producers. These connections are represented by the branches on the left-hand side. Apart from \( ir_2 \), all these customers are linked with \( ir_2 \) in the second layer. These are \(|R| - 2\) connections as illustrated by the branches on the right-hand side in Figure 5. Thus, in \( L_{ir_1 \rightarrow ir_2}^{(2)} \) there are already \((|R| - 1) + (|R| - 2)\) supply links contained.

\[ \text{Figure 5: A tree representing all links contained in } L_{ir_1 \rightarrow ir_2}^{(2)}. \text{ All branches in the same column correspond to the same layer. Each branch depicts a link that has not been traversed so far and is newly added to the set of traversed links. Underneath the tree, the number of the newly added links for each layer is given.} \]

Next, we find \( \Psi_{ir_1 \rightarrow ir_2}^{(3)} = \frac{1}{|R|-1} + (|R| - 2) \left( \frac{1}{|R|-1} + (|R| - 2) \left( \frac{1}{|R|-1} \right)^2 \right) \). The first summand again represents the direct connection from \( ir_1 \) to \( ir_2 \). Ignoring the direct link and the self-supply link \( ir_2 \rightarrow ir_2 \), there are \((|R| - 2)\) connections for which we multiply the corresponding PSI with \( m = 2 \) with the respective PSI with \( m = 1 \). This is indeed possible, as in the calculation of the terms \( \Psi_{ir_1 \rightarrow ir_\nu}^{(2)} \Psi_{ir_\nu \rightarrow ir_2}, \nu \in \{3, \ldots, |R|\} \) the links of the kind \((ir_\nu, ir_2)\) are not in \( L_{ir_1 \rightarrow ir_\nu}^{(2)} \). Interestingly, \( L_{ir_1 \rightarrow ir_2}^{(3)} \) contains all links. To see this, we refer to the tree that corresponds to \( L_{ir_1 \rightarrow ir_2}^{(3)} \) shown by Figure 6. Again, in the first layer there are \(|R| - 1\) links. In the second layer, \( ir_2 \) is connected with \(|R| - 2\) other producers, precluding \( ir_1 \) and itself. All other \(|R| - 2\) producers are linked with \(|R| - 3\) producers. The three producers which are not supplied are herself to rule out self-supply, \( ir_2 \) to prevent self-supply in the third layer and \( ir_1 \). In the third layer all producers apart from \( ir_1 \) and \( ir_2 \) supply \( ir_2 \). Adding all links up,
i.e. $|R| - 1 + |R| - 2 + (|R| - 2) \cdot (|R| - 3) + |R| - 2$, we obtain the total number, $(|R| - 1)^2$, of possible links.

Using these trees it is also straightforward to check whether a link has been already accounted for. For instance, consider the link $ir_3 \rightarrow ir_2$ in the third layer. In the computation of $\Psi_{ir_1 \rightarrow ir_2}^{(3)}$ we must check whether the corresponding tuple $(ir_3, ir_2)$ is contained in $L_{ir_1 \rightarrow ir_3}^{(2)}$. In Figure 6 this set is given by all link combinations pointing to $ir_3$ in the second layer. Looking at these branches, we quickly see that $(ir_3, ir_2)$ is not in $L_{ir_1 \rightarrow ir_3}^{(2)}$.

Figure 6: A tree representing all links contained in $L_{ir_1 \rightarrow ir_2}^{(3)}$. It shows that all supply links are traversed within three layers, regardless of the size of the network.

Now in $m = 3$, the total network is affected by the production outage of $ir_1$. In other words, regardless of the network size, within three layers all supply links are affected. Writing the expression given above more nicely as $\Psi_{ir_1 \rightarrow ir_2}^{(3)} = \frac{1}{|R|-1} + \left(\frac{|R| - 2}{|R|-1}\right)^2 + \left(\frac{1}{|R|-1}\right)^3$, we see that $\Psi_{ir_1 \rightarrow ir_2}^{(3)}$ converges towards zero for $R \to \infty$. Accordingly, the impact on the other producers declines with a growing number of regions, as $\Psi_{ir_1 \rightarrow ir_2}^{(3)}$ converges towards zero for $R \to \infty$.

By analogy, these considerations hold for all trade connections in a fully connected network.

### 2.3 PSI with Adaptive Damping

The analysis described above is carried out under the assumption that dynamic response mechanisms to avoid production losses are not applied. The dependencies are certainly weaker if the producers are capable to counteract the supply shortages. Then, the production losses
can dissipate along the supply chain even if the network is of a structure that does not favour dissipation. Supply losses that propagate to very distant layers are accordingly rather unlikely.

While it is clearly necessary to build a fully-fledged model in which the producers form decisions on adaptation measures to reduce or prevent input shortages, we opt here for introducing a simple discounting factor to represent the effects of adaptation. This factor shall reflect that each layer in the supply network succeeds in dampening the shock inherited to some extent. Such adaptation may include storage use, production capacity expansions, finding new suppliers on short notice or the like.

That is to say, we introduce a factor $e^{-\frac{1}{M_{ir\rightarrow js}}}$ for each $\Psi_{ir\rightarrow js}$, $ir \neq js$, in Equation (5). For $ir = js$, we keep $\Psi^{(m)}_{ir\rightarrow ir} = 1$, i.e. $\Psi^{(m)}_{ir\rightarrow ir}$ is not discounted. More specifically, Equation (5) is modified to

$$\Psi^{(m)}_{ir\rightarrow js} = \begin{cases} \max_{k \in I} \sum_{u \in R} \Psi_{ir\rightarrow ku}^{(m-1)} \Psi_{ku\rightarrow js}^{(m)} e^{-\frac{1}{M_{ku\rightarrow js}}} & \text{for } ir \neq js \\ 1 & \text{for } ir = js. \end{cases}$$  

(9)

The parameter $M_{ir\rightarrow js}$ incorporates information on the possibilities to compensate input shortages. For example, when assuming full visibility of the network, one might presume that producers in more distant supply chain layers are granted more time for an effective response. The distance in the network might be specified by more information about actual transport times. One might also include information on possibilities of substituting or storing input goods and of switching to other suppliers. The parameter could be calibrated by employing supply chain simulation models or by elicitation studies that ask for different regional sector’s capabilities to counteract production shortages.

In the following section, in which PSI and this extension is applied to input-output data, we simply assume the same factor $e^{-\frac{1}{M}}$ for each for each $\Psi_{ir\rightarrow js}$, $ir \neq js$. Then Equation (9) exhibits the discount factor $e^{-\frac{1}{M}}$ and corresponds to

$$\Psi^{(m)}_{ir\rightarrow js} = \begin{cases} \max_{k \in I} \sum_{u \in R} \Psi_{ir\rightarrow ku}^{(m-1)} \Psi_{ku\rightarrow js}^{(m)} e^{-\frac{1}{M}} & \text{for } ir \neq js \\ 1 & \text{for } ir = js. \end{cases}$$  

(10)

If $\Psi^{(m)}_{ir\rightarrow js}$ relates to a supply relationship of the exact length $m$, then this measure is even further reduced to Equation (5) multiplied with $e^{-\frac{1}{M}}$. Hence, Equation (5) is scaled down corresponding to the distance in the supply chain. As in general $\Psi^{(m)}_{ir\rightarrow js}$ is associated with a relationship of the length $m$ at most, the discount factor cannot be easily pulled out of the sum in Equation (10) and is thus not a scaled version of Equation (5).
3 Application to Input-Output Data

Now, we apply Equation (7) to an actual supply network specified by input-output data. It allows us to assess which sector $ir$ might play a critical role in spreading supply losses by comparing $\Psi_{ir}^{(m)}$ for different $m$.

For this purpose, we use the Eora multi-region input-output table (MRIO) database, which provides high resolution annual data for 186 countries and 26 producing sector categories (Lenzen et al. (2012a), Lenzen et al. (2013b)). Launched in 2012, Eora has been widely used to study various issues concerning global trade, as for example the consequences for biodiversity (Lenzen et al. (2012b)), for distribution of scarce water (Lenzen et al. (2013a)), for the ecological and other footprints (Moran et al. (2013), Oita et al. (2016)), for material requirements (Wiedmann et al. (2015)) and for income inequality (Alsamawi et al. (2014)). We use Eora to calibrate the supply network for which we compute PSI. In view of the typically low reliability of the data describing small flows (Lenzen et al. (2013b)), we neglect flows below 1 million USD/year. The remaining database comprises trade volumes of about 500,000 connections, which are used to calibrate the input flows $Z_{ir \rightarrow js}^*$ in Equation (3). Thus, $\Psi^{(1)}$ can be computed and, applying the recursively defined Equation (5), $\Psi^{(m)}$ for all $m > 1$. The resulting numbers are then used to determine the production dependence of the global economy on each regional regional sector, as given by Equation (7). It shall be kept in mind that the results obtained in the following pertain to impacts caused by the structural properties of the network.

In Figure 7a, the values of $\Psi_{ir}^{(m)}$ for $m \leq 10$ are plotted for each of the 4836 regional sectors $ir$. The curves grant three fundamental insights. Firstly, most of them are ‘S’-shaped featuring an inflection point, i.e. a point in which the loss increase is the highest, and a point of saturation. This reflects that the production losses are not dampened or absorbed when being spread over a higher number of layers, as it is the case for the exemplified network structure in Figure 4b. On the contrary, they cascade. Secondly, some of the curves reach a saturation level of one. Accordingly, the production of the entire global economy is influenced by these regional sectors through higher-order dependencies. This indicates that these regional sectors are very well connected with the global economy. Most of all other curves saturate in values lower than 1 (Figure 7b), implying that their (direct and indirect) connections are not spread over the entire global economy. Thirdly, the number of downstream layers over which the accumulative supply dependencies mount up is an important characteristic. In Figure 7c, all curves that attain their midpoint, being half of the saturation point, in the same $m$ are given the same colour. The gradation of colour provides information on the number of layers over the losses increase and affect increasingly many regional sectors. Loosely spoken, this indicates the potential speed of propagation. For instance, the two red curves on the left hand side reach their midpoint already in the first layer, indicating a very fast propagation of losses along the layers. Most of the red and orange curves saturating are associated with the

An overview of all 26 sectors is provided in the appendix.
sectors in China (e.g. manufacturing and construction). Quickly propagating losses hint at a limited leeway for downstream production facilities to respond to looming input shortages, which indicates an increased potential vulnerability of the global supply network to production losses in these crucial regional sectors. Figure 7d shows the number of regional sectors that attain their midpoint in the respective values of $m$, where the colour coding corresponds to the colourbar in Figure 7c. The number given by the red bar is rather high, because there are, in fact, many regional sectors saturating in a small PSI value and the associated midpoint is thus reached in the first layer. The number of regional sector saturating in one and affecting half of the economy in the first layer is, as also indicated by Figure 7c, two. All in all, we observe that there is a significant number of sectors with the potential to affect large parts of the economy within a few layers.

Next, we want to gain some insights into whether an important characteristic of PSI, the saturation level, features a statistical relationship with other network measures. It can be suspected that the regional sector’s connectivity might determine the potential impact on the global economy. The connectivity measures we consider are the outdegree (counting the direct customers) the Katz centrality (counting the direct and indirect customers) and the distance of the shortest paths to the other vertices. Another surmise is that the regional sector’s market power might be related with the saturation levels. This information can be obtained by applying the influence vector by Acemoglu et al. (2012), calculated here as the regional sector’s value of supply divided by the economy-wide value of supply. The market power is also reflected by the regional sector’s importance within the global sector, computed as the share of the regional sector’s output in the sector’s output. Furthermore, the value-added of the region is considered, since production losses in a very productive local economy might cause great immediate losses. The correlation of these measures with the saturation level turns out to be either weak or moderate. Furthermore, we find that none of the linear ordinary least square (OLS) regressions using different samples of these measures as explanatory variables are associated with a $R^2$ higher than 0.31, although most of the measures are highly significant.\footnote{More detailed information on the regression and the correlations can be found in Appendix B.} In summary, we may conclude that the relationship of this important characteristic obtained by PSI with other network measures is not strong and this characteristic cannot be sufficiently explained by a linear combination of these network measures.

As propagation of production losses along global supply chains might be induced, for instance, by natural and man-made disasters, those regions most exposed to natural catastrophes are of particular interest. According to the World Risk Index (Alliance Development Work and UNU-EHS (2015)), the Philippines, Japan and the Netherlands belong to the 12 countries most exposed. This measure refers to the number of people and to the value of economic assets concentrated in hazard areas subject to one or more natural catastrophes such as earthquakes, cyclones, droughts, floods and sea level rise (Birkmann et al. (2011)). Direct economic damage costs on the Philippines are primarily caused by tropical cyclones accounting for an average of estimated direct costs of about one billion US$ per year (Guha-Sapir et al. (2015)). Japan is
Figure 7: Global production shortage propagation: a) The triggered global production losses caused by each regional sector, respectively. The colouring of the curves correspond with value PSI attains in $m = 10$, i.e. red for $\Psi_{ir}^{(10)} \in (0.9,1]$, orange for $\Psi_{ir}^{(10)} \in (0.8,0.9]$, yellow for $\Psi_{ir}^{(10)} \in (0.7,0.8]$, light green for $\Psi_{ir}^{(10)} \in (0.6,0.7]$, green for $\Psi_{ir}^{(10)} \in (0.5,0.6]$, light blue for $\Psi_{ir}^{(10)} \in (0.4,0.5]$, blue for $\Psi_{ir}^{(10)} \in (0.3,0.4]$, indigo for $\Psi_{ir}^{(10)} \in (0.2,0.3]$, violet for $\Psi_{ir}^{(10)} \in (0.1,0.2]$ and pink for $\Psi_{ir}^{(10)} \in [0,0.1]$; b) Histogram of the PSI values attained $m = 1$ to $m = 10$; c) The same curves as given by a), but the coloured according to the value of $m$ in which the midpoint (i.e. half of the saturation) is approximately reached, which indicates the ‘speed’ of the propagation. The colourbar assigns the colours to the $m$-values of the midpoint; d) Histogram associated with c) in the same colour coding.
exposed to a multitude of extremes including earthquakes, floods and volcanic eruptions. The average annual direct costs are gauged to be 24 billion US$ (Guha-Sapir et al. (2015)). The Netherlands are mostly affected by storms and floods, which are assessed to cause relatively low average annual direct costs of about 69 million US$ (Guha-Sapir et al. (2015)). Figure 8 gives an isolated view of $\Psi_{ir}^{(m)}$ for these countries’ sectors. It shows that natural catastrophes affecting regional sectors in Japan can trigger great losses throughout the global supply network. Apart from the gastronomy sector, all sectors in Japan belong to the regional sectors aboved mentioned that can potentially influence the entire global economy. The damage propagation impacts triggered by losses in the Netherlands and the Philippines are of a lower magnitude, but are still of significance.

By way of example we now investigate the loss propagation triggered by one sector in each of these three countries. More specifically, we consider the financial intermediation sector in Japan, which generates the highest output value in Japan according to the input-output table we employ. Moreover, Figure 8 indicates a great production dependency of the global economy on in this regional sector. We also inspect the propagation dynamics triggered by the petroleum, chemical and non-metallic mineral products sector in the Netherlands and the agriculture sector in the Philippines. With respect to the output values in 2011, the former sector is the second most important in the Netherlands and agriculture is the third most important in the Philippines. Although the PSI values associated with these regional sectors can be expected to be significantly lower than the PSI value relating to the financial intermediation sector in Japan, it is interesting to investigate the geographical reach of the production losses induced.

The production dependence of the continents on these three sectors is provided by Figure 9. Figure 9a shows that the Japanese financial intermediation sector is conducting business primarily with Asian countries. Therefore, production losses are spread mainly in Asia for $m = 1$. PSI extends the concept of import/export dependencies. For $m = 2$, the propagated

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6The average costs are calculated as a 10-year moving average between 2005-2014 (http://www.preventionweb.net/english/).

7The loss dissemination after a cessation of all exports from the Philippines is discussed in Levermann (2014).
losses affect North American production significantly and for $m = 3$ the European economy bears considerable production impacts as well. Figure 9b and Figure 9c provide qualitatively similar insights: while the direct trade relationships are mainly established intracontinentally, the losses caused further downstream affect the global economy as well. Interestingly, the production dependence of some other continent might be even stronger than the dependence of the domestic continent’s economy for higher values of $m$. For instance, Asia’s production is significantly dependent on the petroleum, chemical and non-metallic mineral products sector in the Netherlands. In the past years, the processing of these and other raw materials has been driving the Asian countries’ economic growth, generating an significant share of the worldwide demand for these resources (International Monetary Fund (2011)). These raw materials thus serve as inputs for processed goods that generate a higher value added in Asia. An outage in the agricultural sector on the Philippines, as it happened in the aftermath of Typhoon Haiyan, causes the greatest production losses in North America for higher $m$. A possible reason might be that the US food industry imports already processed agricultural products, leaving increasingly more processing steps to trade partners in foreign countries.\textsuperscript{8} It is thus, that a major commodity imported from the Philippines is not coconuts of which the Philippines are the world’s largest producer but of coconut oil (Philippine Statistics Authority (2012)). This kind of outsourcing have been augmented the complexity of the supply network.

Figures 10 - 12 grant a geographically detailed inspection of the propagation dynamics. For this, the regional sector-to-country production shortage interdependence is computed according to

$$\Psi^{(m)}_{ir \rightarrow s} = \sum_{j \in I} \sum_{l \in I} Z_{js}^{*} Z_{ls}^{*} \Psi^{(m)}_{ir \rightarrow js}$$

with $m = 1, \ldots, 4$ for all countries $s$.

Figure 10 shows the world map of the production shortage dependencies on the financial intermediation in Japan. As shown by Figure 10a the immediate production shortages incurred in each country through direct trade relationships ($m = 1$) are of negligible size. The largest export of production losses are illustrated by the arrows indicating a loss of more than 1%. Interestingly, the economic losses for $m = 2$ in some countries soar to significant levels. And this is especially the case for countries that have not been affected in the first instance, i.e. for $m = 1$, as for example Russia, India and Bolivia. The financial intermediation sector thus inflicts losses in other countries’ sectors whose supplies are crucial for production in the countries mentioned above. Consequently, extreme events in foreign countries might have a significant impact on the domestic economy through higher-order supply chain connections. In view of the significant losses in the second instance the vast economic damage in some countries for $m = 3$ is not surprising, but highlights the potential impact of loss propagation.

Figure 11 shows that the petroleum, chemical and non-metallic mineral products sector in the Netherlands is first influences the production in the other European countries, since the

EU is a highly connected economic region, and in some countries in South East Asia, Africa and the Middle East. The American continent is less affected in the first instance. In contrast, for $m = 2$ the production losses might be spread globally, possibly due to having affected many economically strong sectors in Europe before.

Compared to the impacts depicted above by Figure 10 and 11, the losses illustrated by Figure 12 appear to be negligible for $m = 1$ and $m = 2$. The only PSI value higher than 0.01 is associated with the supply to Hong Kong. Then, in $m = 3$ the production shortages make some geographical leaps: some countries in Europe, the USA and Argentina are most affected. Hence, the production losses do not necessarily spread evenly across countries as in Figure 11.

Next, we turn to applying the modified measure that incorporates the adaptive damping factor, given by the Equations (9) and (10) in Section 2.3, to the Eora 2011 data. For simplicity, we opt here to apply Equation (10) setting $M_{ir\rightarrow js} = M$ and explore the results for different values of $M$. Note that this idea is not tantamount to simply multiplying $\Psi^{(m)}_{ir\rightarrow js}$ from above with $e^{-\frac{M}{m}}$. As $\Psi^{(m)}_{ir\rightarrow js}$ covers all supply dependencies that are of length $m$ at most, the PSI-value is determined by the layers incurring the greatest input shortages. While before the greatest losses can be caused by the more distant suppliers in the network just as well, it can be expected now that the greatest input shortages are incurred from the producers close to the source of the production losses. Put differently, the argument of the maximum $k$ in Equation
Figure 10: A global map of production shortage dependencies on the financial intermediation sector in Japan: a) PSI for the direct trade relationships with the arrows indicating the largest production shortages caused ($\text{PSI} > 0.01$); b) PSI for $m=2$ and c) PSI for $m=3$. 

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Figure 11: A global map of production shortage dependencies on the petroleum, chemical and non-metallic mineral products sector in the Netherlands: a) PSI for the direct trade relationships with the arrows indicating the largest production shortages caused (PSI>0.01); b) PSI for m=2 and c) PSI for m=3.
Figure 12: A global map of production shortage dependencies on the agricultural sector in the Philippines. In a), the arrow indicates the only PSI-value >0.01, caused for the trade relationship with Hong Kong.
(9) might belong sectors in the first layers rather than in the layers further downstream.

Figure 13 shows the results for $M = 1, 2, \ldots, 6$ and compares them with PSI as defined in Equation (5) (blue curve), which does not incorporate a loss reduction factor. We observe two important points here. Firstly, the loss reduction factor can have a significant impact on PSI and in particular on its level of saturation. Secondly, even for $M = 1$, which is tantamount to discounting the production shortages significantly, PSI saturates in a value greater zero. The losses from the first layers apparently persist, confirming the above made considerations about PSI being determined by the losses in the first layers.

![Figure 13: The influence of different loss reduction factors on the global production shortage propagation for the financial intermediation sector in Japan. The blue curve displays $\Psi_{ir}$ as defined in in Equation (5), which does not feature any reduction factor. The other curves correspond to Equation (9) with $M_{ir \rightarrow js} = M$ for all $js$.](image)

Finally, the production shortage dependencies on the Japanese financial intermediation sector computed with the expanded version of PSI shall be demonstrated. More specifically, Figure 14a and Figure 14b demonstrate the results for $M = 1$ and $M = 3$, respectively. These figures can be contrasted with Figure 10c that illustrates the results in which the losses are not discounted. With respect to the most affected countries, we can draw the same conclusions. Of course, the level of the loss differ significantly across the figures. This insight emphasizes the importance of finding a sophisticated calibration of the loss reduction factor.
Figure 14: A global map of the dampened production shortage dependencies on the financial intermediation sector in Japan: a) PSI with a loss reduction of $e^{-1}$; b) PSI with a loss reduction of $e^{-\frac{1}{3}}$. 
4 Summary and Conclusion

Unexpected events such as weather extremes or earthquakes entail a great risk of leading to production losses in the downstream supply chain layers. The growing literature on this issue finds the possibility and duration of a production loss cascade to be determined by the structure of the supply network (Acemoglu et al. (2012)). Here we propose a new measure, PSI, that generalizes the concept of import and export dependencies by taking into account higher-order supply relationships. It incorporates information on the network structure and tracks production losses from supply chain layer to supply chain layer. This measure thus facilitates the identification of critical supply dependencies, in particular of those that occur for indirect supply connections spanning a number of intermediate producers.

We illustrate the measure for some simple network structures, demonstrating that some structures cause the losses to dissipate while others imply production shortages greater zero. Applying this measure to a network calibrated by a multi-region input-output table, we find that the production losses are not dampened or absorbed when being spread over a higher number of supply chain layers. On the contrary, they cascade - for some sectors in a very rapid manner. An important characteristic of PSI is the saturation level, which describes the maximum value of global production losses. To get an impression of whether this characteristic can be explained by other network measures, we conduct a linear regression and find that they cannot sufficiently reproduce the saturation level.

As propagation of production losses along global supply chains might be induced, for instance, by natural and man-made disasters, those regions most exposed to natural catastrophes are of particular interest. Focusing on these sectors, we find that in particular production losses in Japan have the potential to induce a global loss propagation. By way of example we present PSI for three sectors in Japan, the Netherlands and the Philippines, all of them being of great importance for the respective domestic economy. While the production losses in the first supply chain layer primarily affect the domestic economy, they are already passed on to other continents in the second layer. Some of the production losses are even higher for these other continents. A country-level analysis reveals that in some distant countries which are not directly supplied by the sector suffering from a production outage the production losses soar in the second layer. Importantly, we find asymmetries in the dissemination of the losses with respect to the magnitude and the geographical dispersion. Therefore it is of utmost importance for the producers to gain an overview of the supply network they are part of. This overview shall enable to anticipate potential input shortages caused by suppliers further upstream. However, in view of the increasing opaqueness of the globalised trade network gaining this overview is challenging.

The measure as it is introduced in Section 2.1 gives an appraisal of the cause-and-effect relationship between the structure of the network and the dissemination of the input shortages. We also introduce a version of PSI that incorporates a discounting factor to represent the effects of producers succeeding in dampening the shock inherited to some extent. While we set the
discounting factors to the same value and investigate the effects for different choices of this value, it is clearly necessary to find a sophisticated calibration of this parameter by employing supply network models or by conducting elicitation studies.

Further research may use PSI to complement models that account for dynamic responses or proactive adaptation measures the producers can decide on. In particular, models that do not explicitly incorporate input-output linkages can be augmented by PSI to represent contagion effects.

Appendix

A Overview of sectors

Overview of all 26 sectors in EORA:

- Agriculture
- Fishing
- Mining and Quarrying
- Food and Beverages
- Textiles and Wearing Apparel
- Wood and Paper
- Petroleum, Chemical and Non-Metallic Mineral Products
- Metal Products
- Electrical and Machinery
- Transport Equipment
- Other Manufacturing
- Recycling
- Electricity, Gas and Water
- Construction
- Maintenance and Repair
- Wholesale Trade
- Retail Trade
- Hotels and Restaurants
- Transport
- Post and Telecommunications
- Financial Intermediation and Business Activities
- Public Administration
- Education, Health and Other Services
- Private Households
- Others
- Re-export and Re-import

B Regression Results

For $X_1 = \text{outdegree}$, $X_2 = \text{Katz centrality}$, $X_3 = \text{mean shortest distance}$, $X_4 = \text{influence vector}$, $X_5 = \text{market power}$ in the respective sector, $X_6 = \text{value-added of the regional sector}$ and $X_7 = \text{value-added of the region}$, we obtain the following correlations with the saturation level

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>$X_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>saturation</strong></td>
<td>0.45</td>
<td>0.5</td>
<td>-0.19</td>
<td>0.25</td>
<td>0.38</td>
<td>0.01</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The correlation matrix of the measures $X_1, ..., X_7$ is:
Some linear regressions on different samples of $X_1, ..., X_7$:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>$X_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>1.0000</td>
<td>0.9750</td>
<td>-0.1308</td>
<td>0.6756</td>
<td>0.5094</td>
<td>0.2496</td>
</tr>
<tr>
<td>$X_2$</td>
<td>0.9750</td>
<td>1.0000</td>
<td>-0.1477</td>
<td>0.5667</td>
<td>0.4682</td>
<td>0.1824</td>
</tr>
<tr>
<td>$X_3$</td>
<td>-0.1308</td>
<td>-0.1477</td>
<td>1.0000</td>
<td>-0.0466</td>
<td>-0.0684</td>
<td>-0.0008</td>
</tr>
<tr>
<td>$X_4$</td>
<td>0.6756</td>
<td>0.5667</td>
<td>-0.0466</td>
<td>1.0000</td>
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<td>0.13263***</td>
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<td>-8.82E-04***</td>
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<td>(8.71E-05)</td>
<td>(8.71E-05)</td>
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<td>-41.21730***</td>
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<td>2.56050***</td>
<td>3.85181***</td>
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** significant at 1%, *** at 0.1%

The results on the other regressions can be obtained upon request.

References


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