Estimating Confidence Intervals for Flood Quantiles: A Bootstrap Approach

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Motivation

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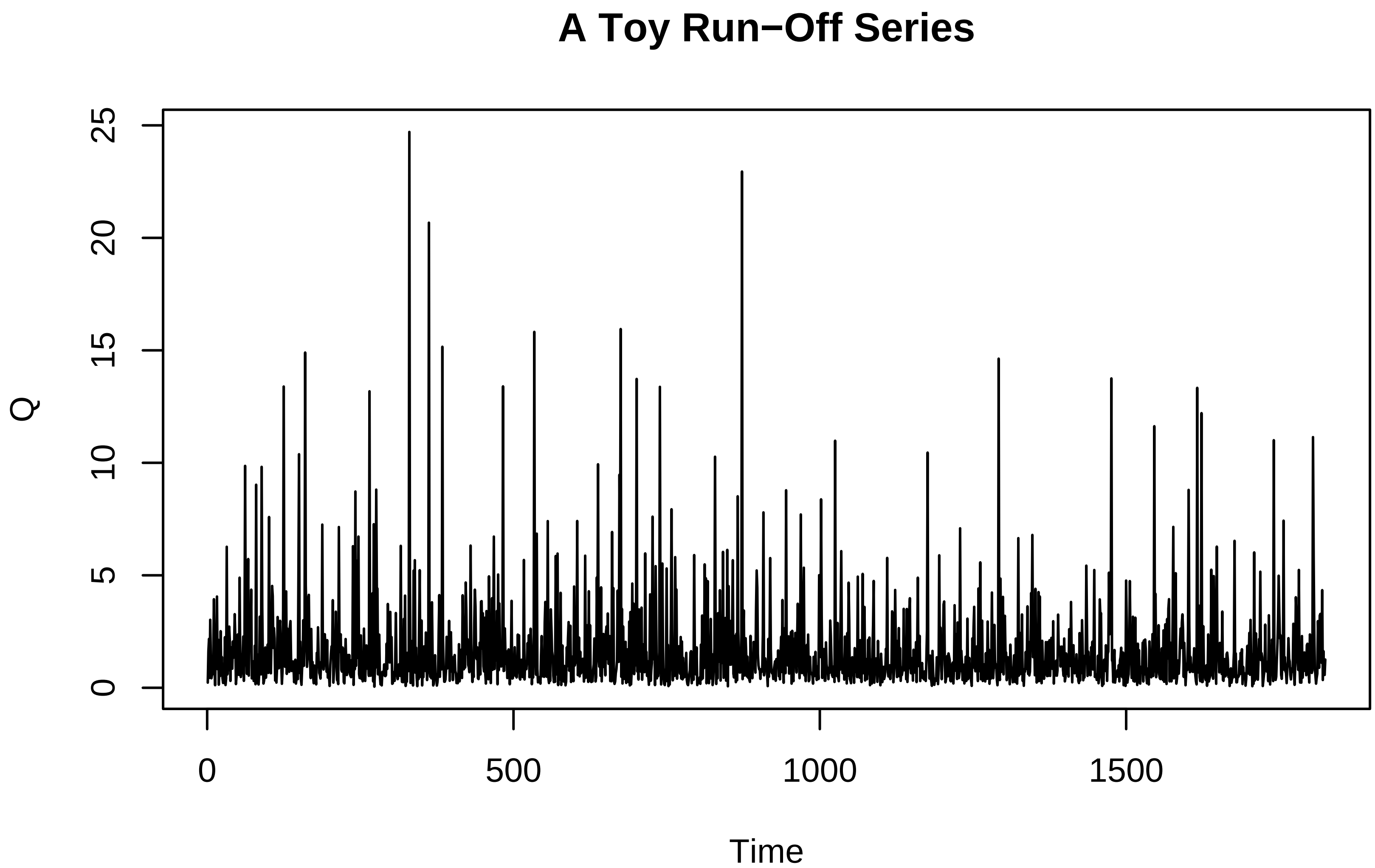

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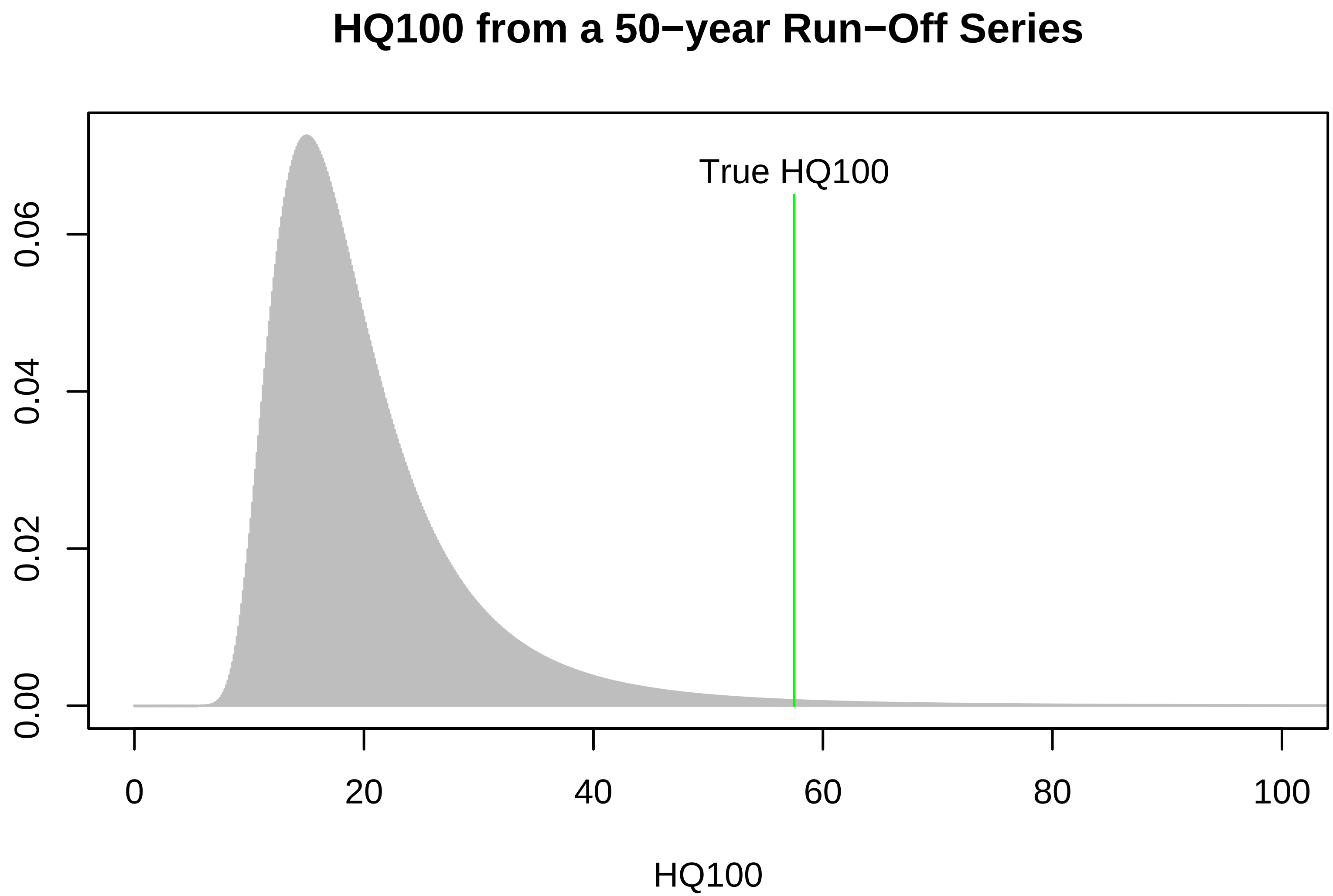

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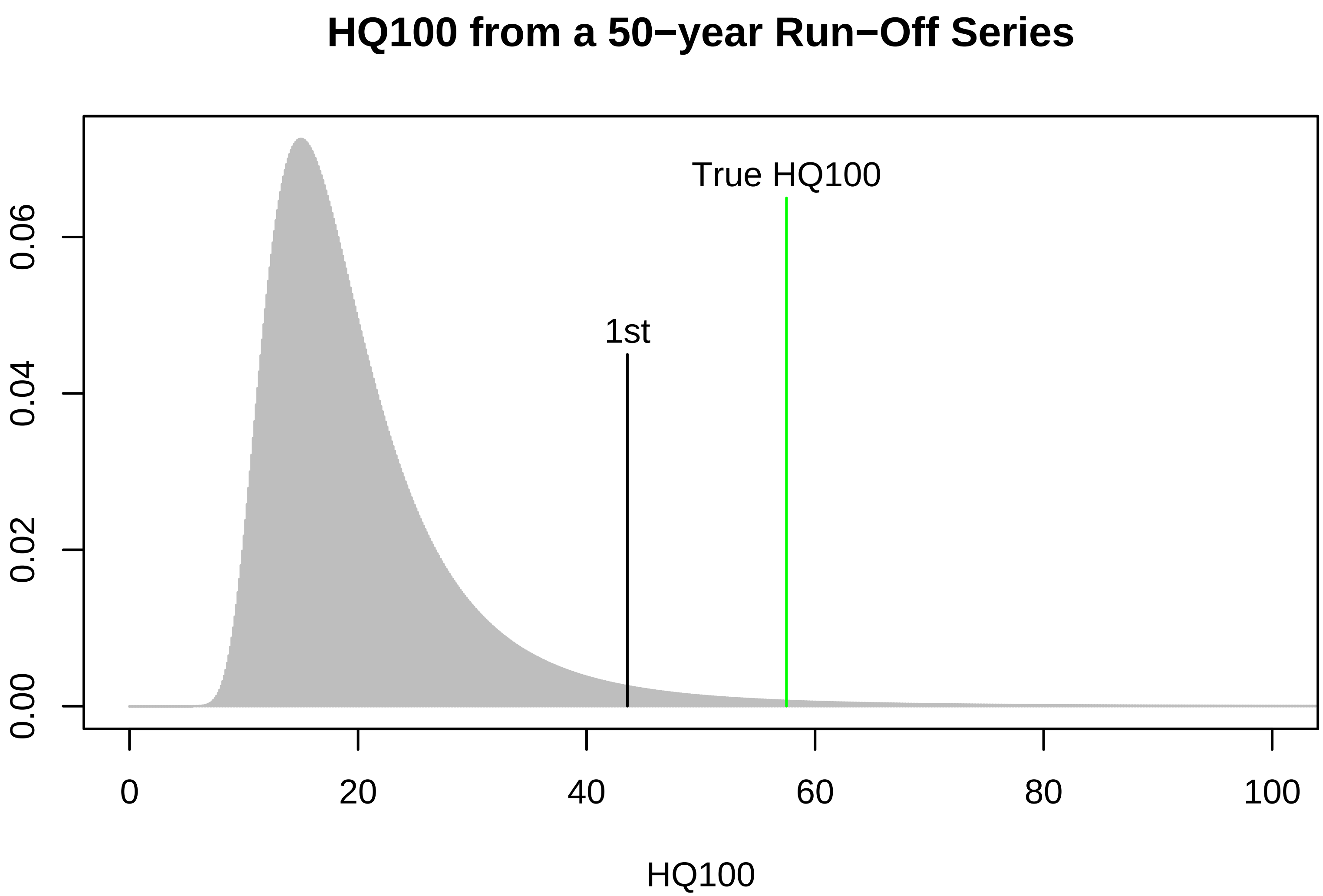

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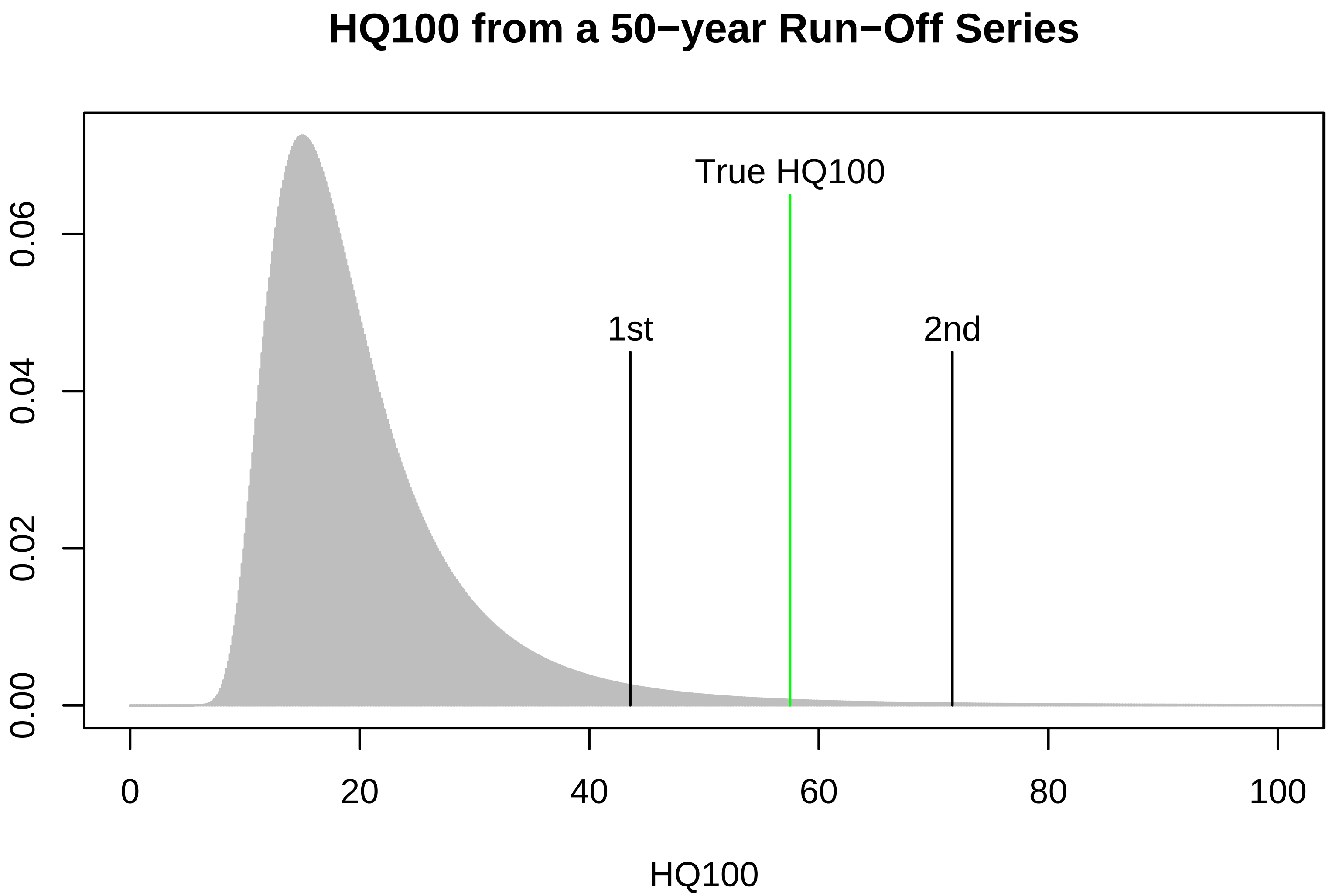

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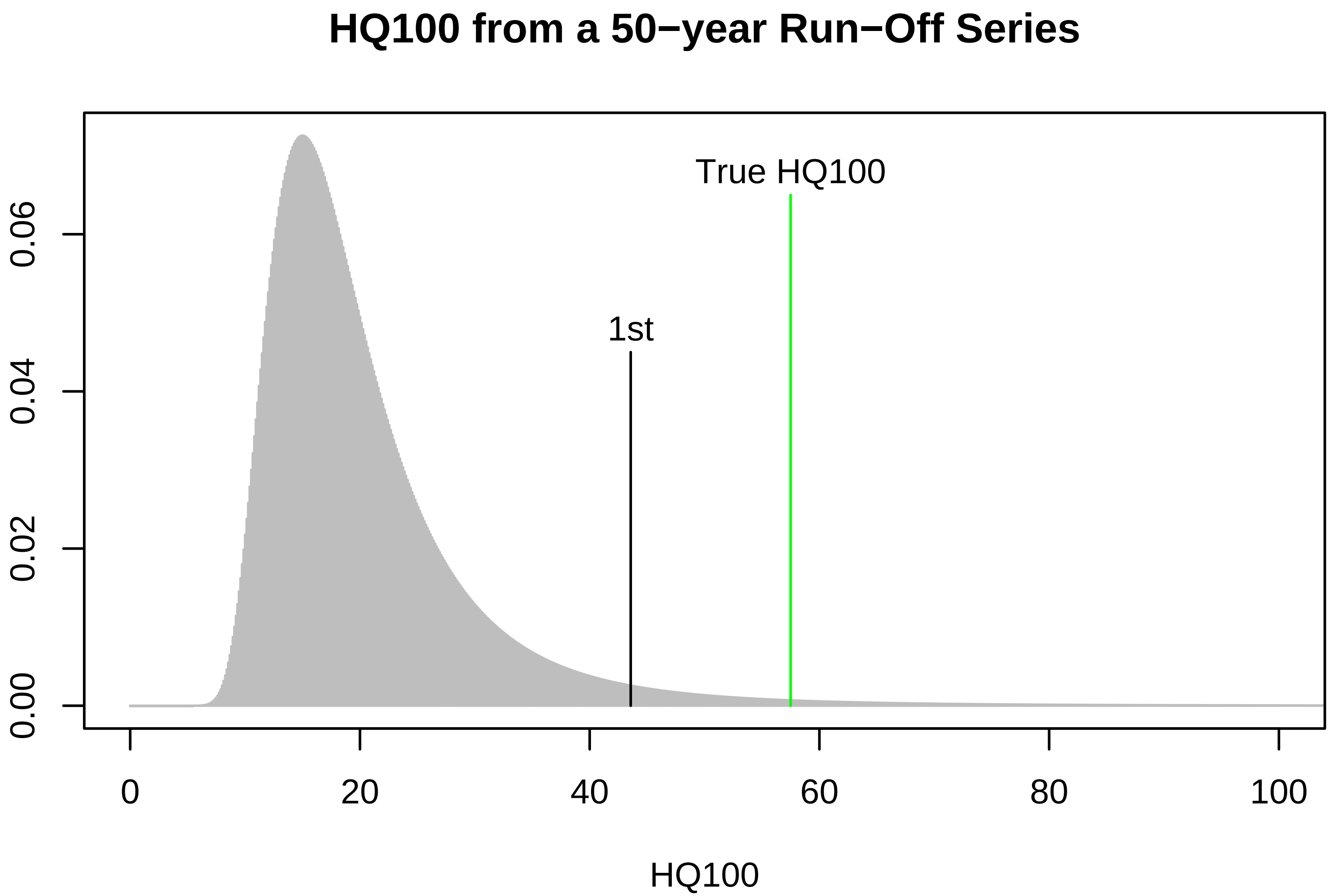

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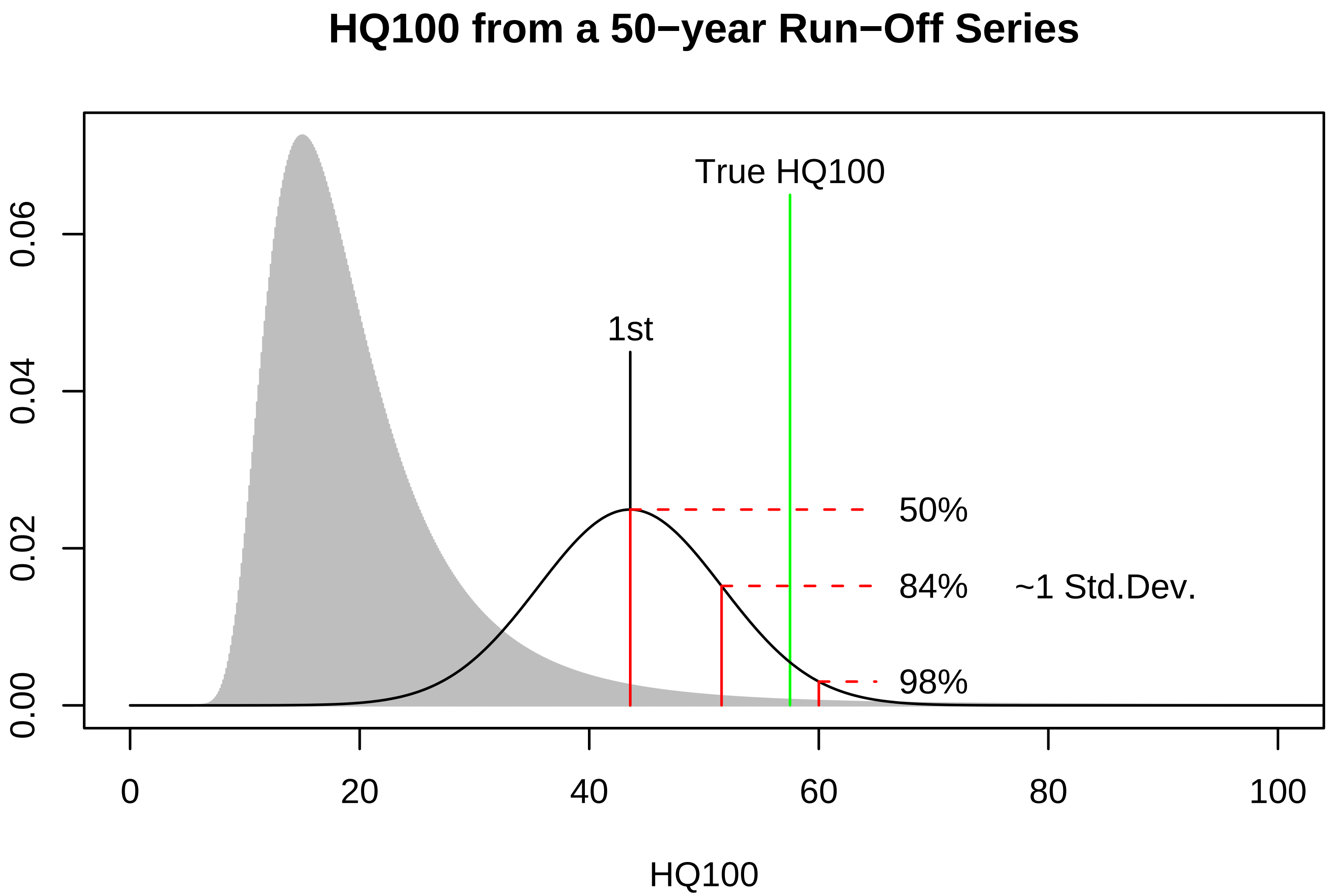

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- Why do we need confidence intervals?
  - Imagine a run-off series, incredibly long and perfectly stationary
  - 2 Persons know only **50 years** (different) and estimate a HQ100
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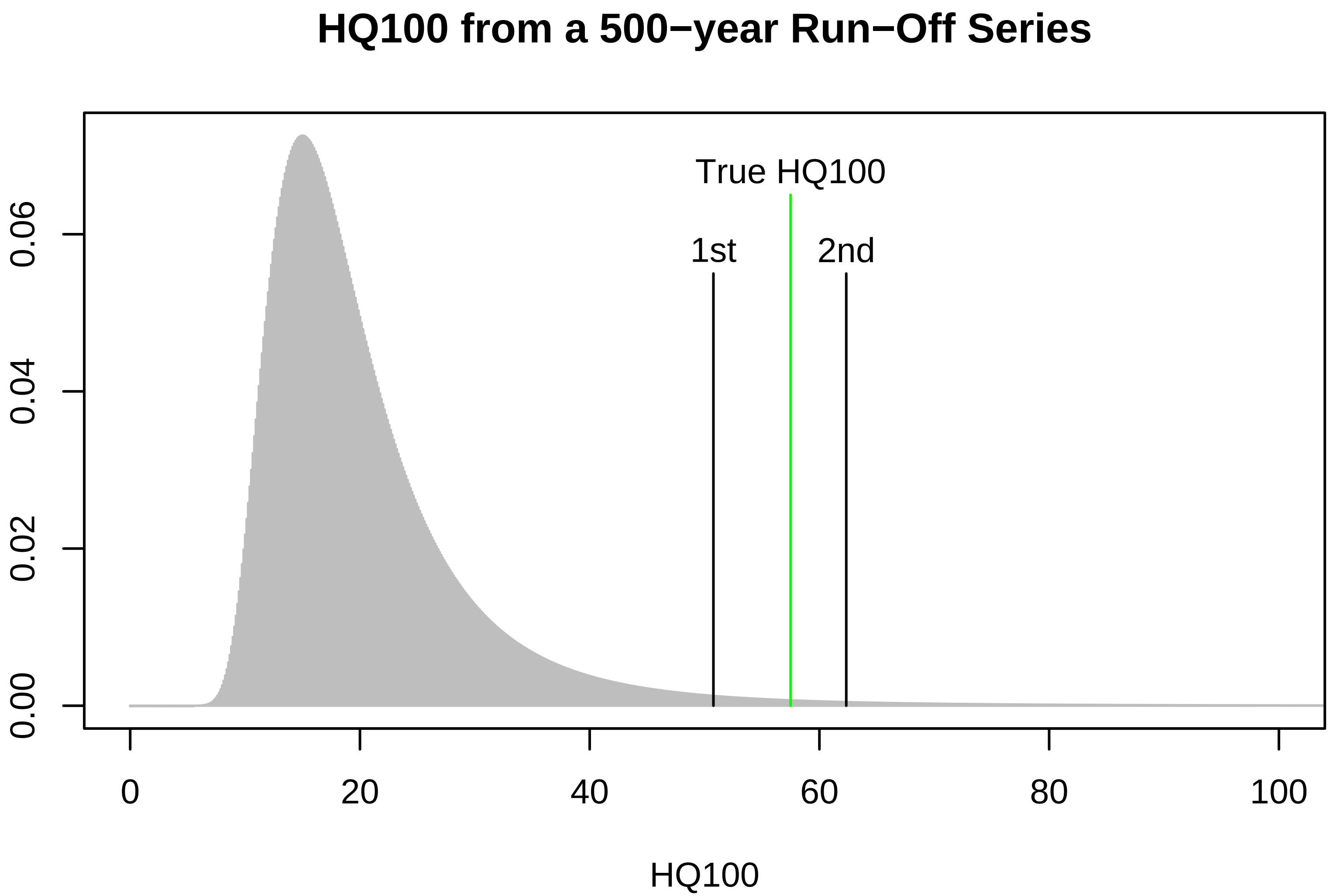

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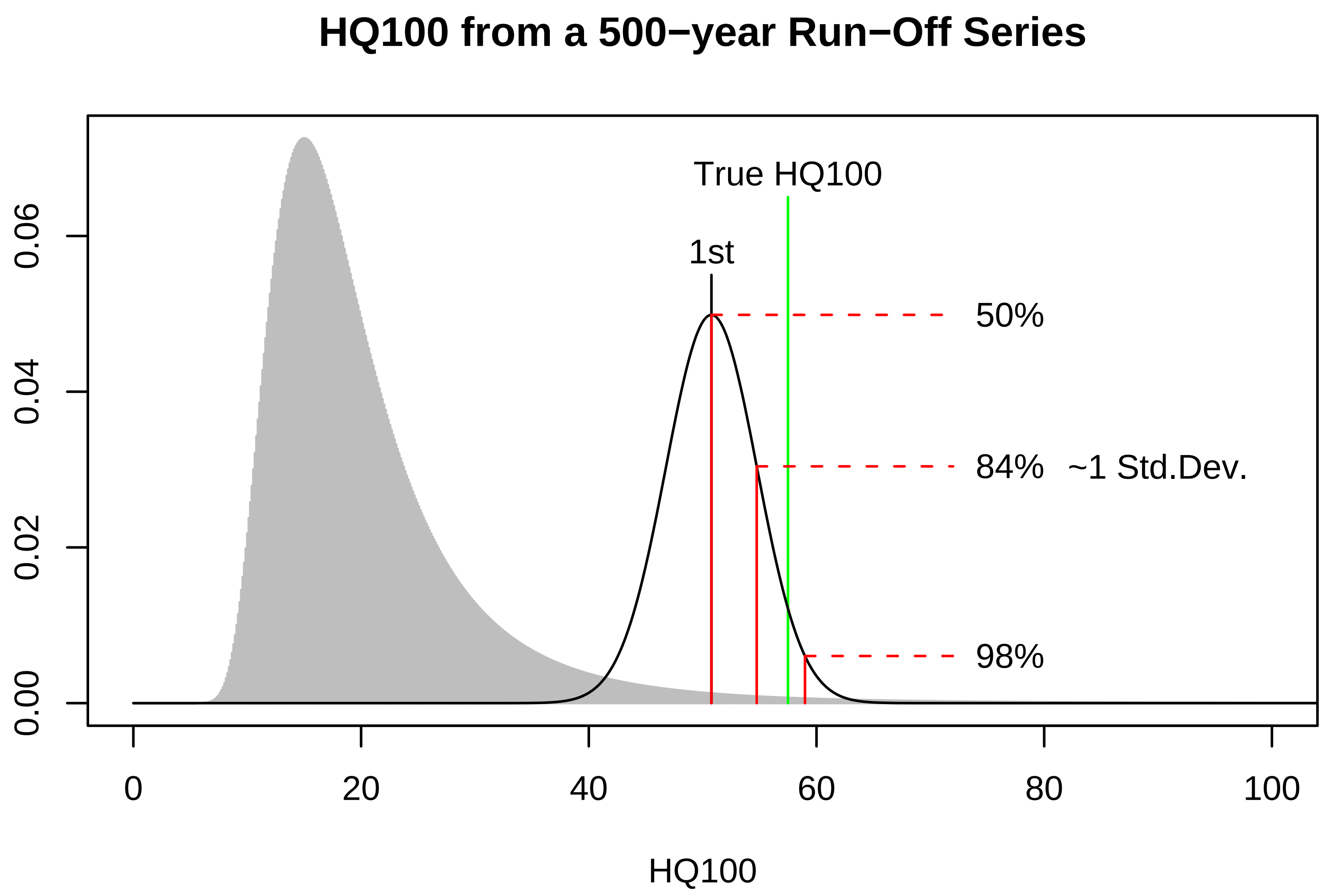

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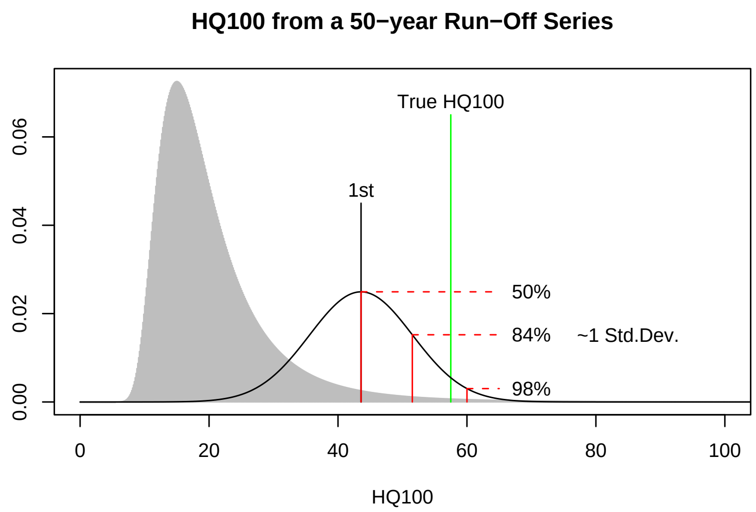

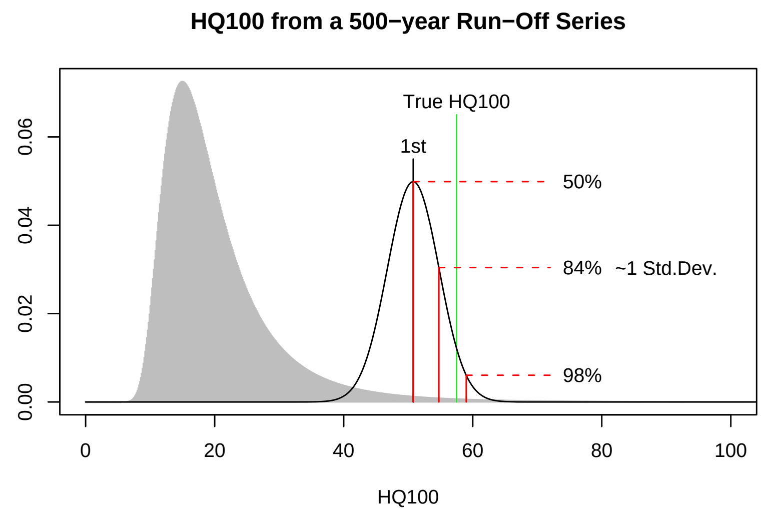

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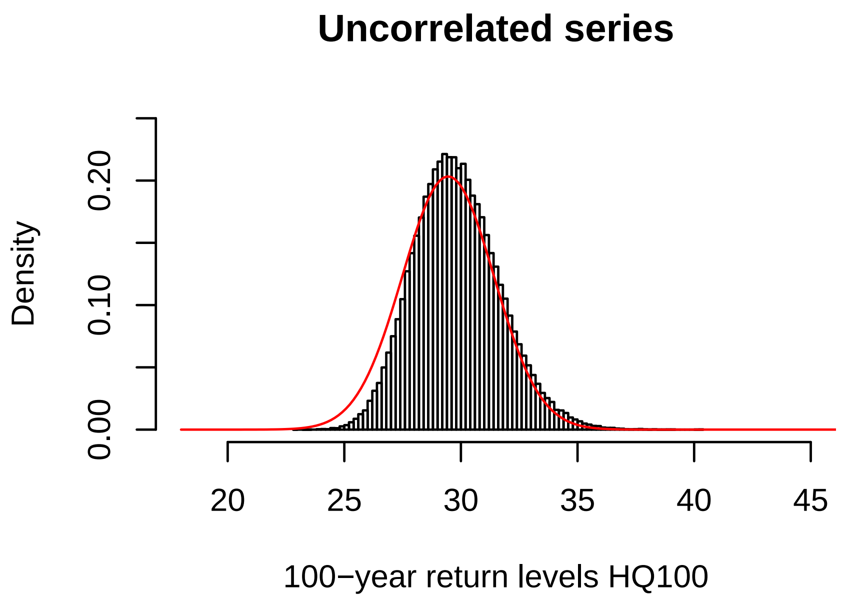

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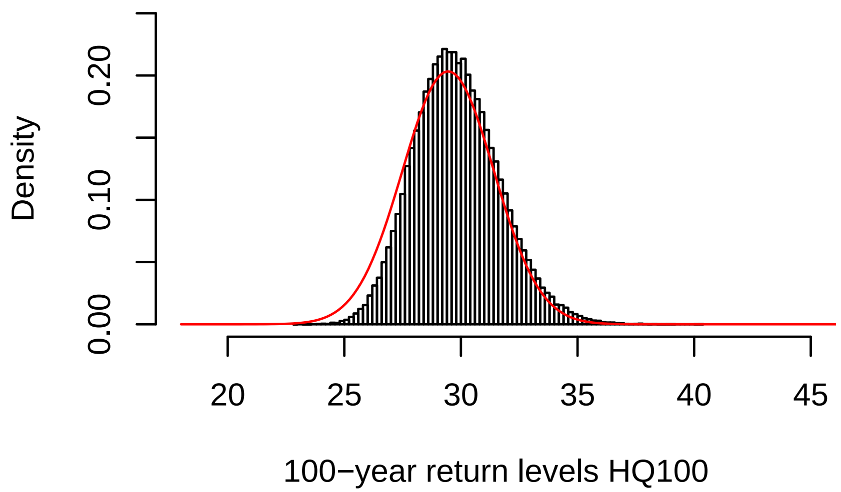
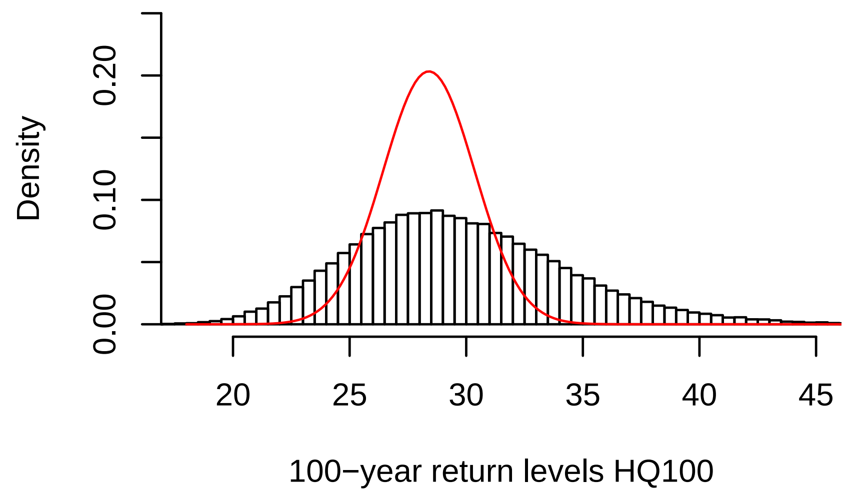
Uncorrelated series



Correlated series



Structure

1. Extreme Value Modelling,
   HQ_{T} Estimation
2. The Problem:
   Confidence Intervals for Correlated Data
3. The Bootstrap Strategy
4. A Test of the Strategy
5. Two Examples
6. Summary/Outlook

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GEV Modelling, Return-Level Estimation

1. Fisher-Tippett (Three-Types Theorem) (roughly):

> For increasing block size $N \to \infty$ the maxima $M_N = \max\{X_1, \ldots, X_N\}$ are distributed according to the GEV.

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$$G(z) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}$$

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3. TTT motivates a Model (GEV) to describe the maxima

Estimating the Parameters

- Graphical methods (probability plots)

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- Graphical methods (probability plots)
- Moment-based techniques (e.g. L-Moments)
- Order statistics
- Likelihood-based
  - All-round utility and adaptable to complex models (e.g. instationarities, covariates)
  - Estimator asymptotically normal distributed $\rightarrow$ confidence intervals
  - MLE for HQ_T

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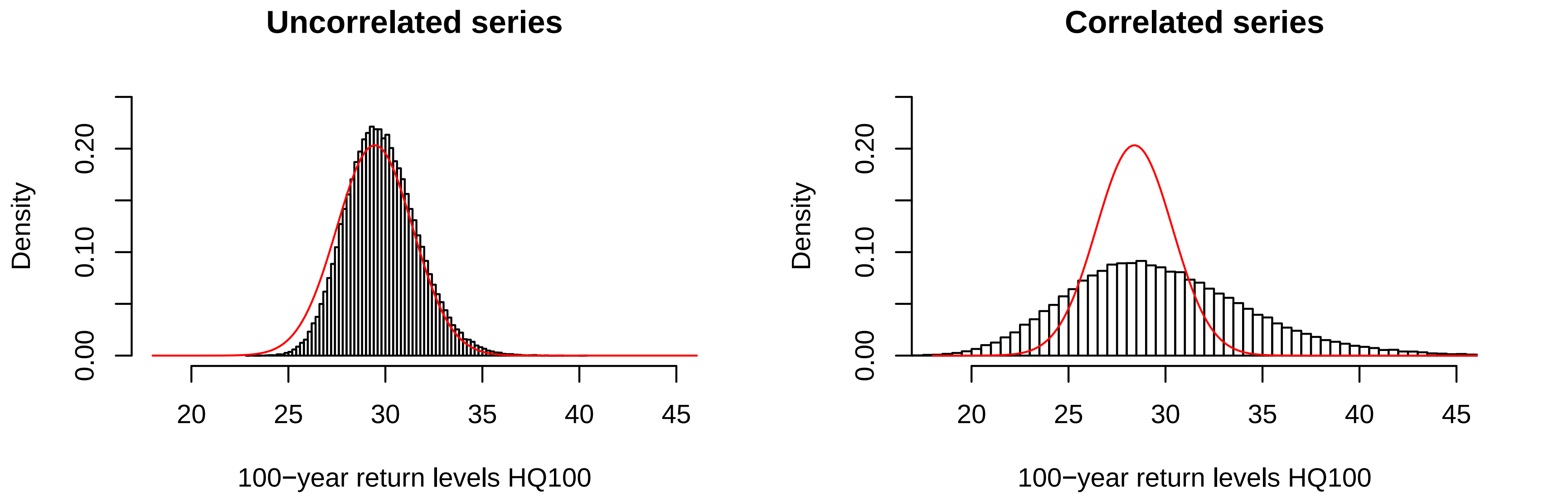
The Problem

- The asymptotic ($N \to \infty$) confidence intervals are not a good approximation for correlated data



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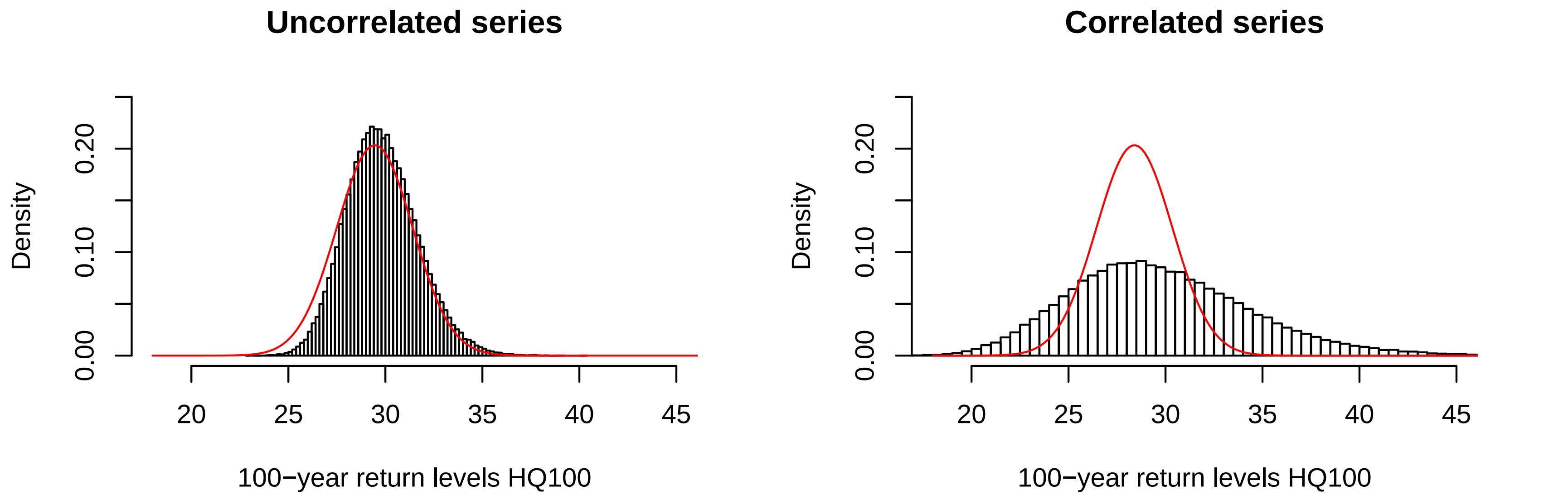

- Idea:
  - Build a model for the maxima series, Distribution and ACF
  - Estimate variability with Monte-Carlo ensemble

The Bootstrap Strategy

- Parametric Bootstrap:

  $$\text{Data} \xrightarrow{\text{fit}} \text{param. Model} \xrightarrow{\text{simulation}} \text{Ensemble}$$

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     iterative Fourier based algorithm, used in nonlinearity testing

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- Are maxima correlated?
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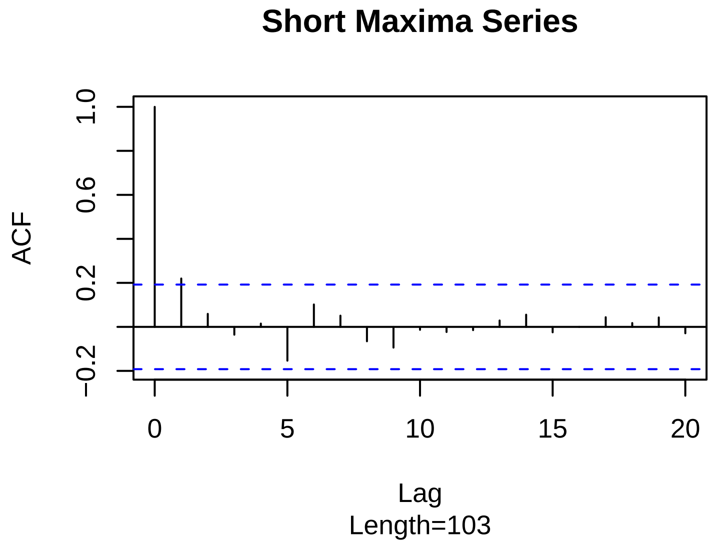

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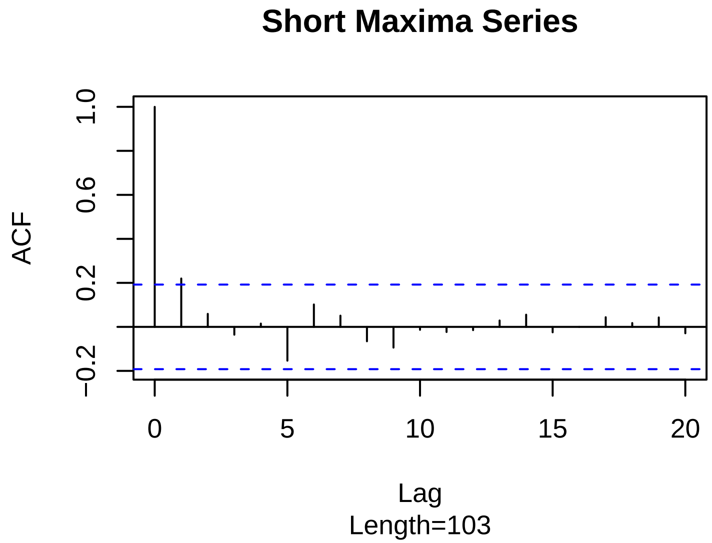

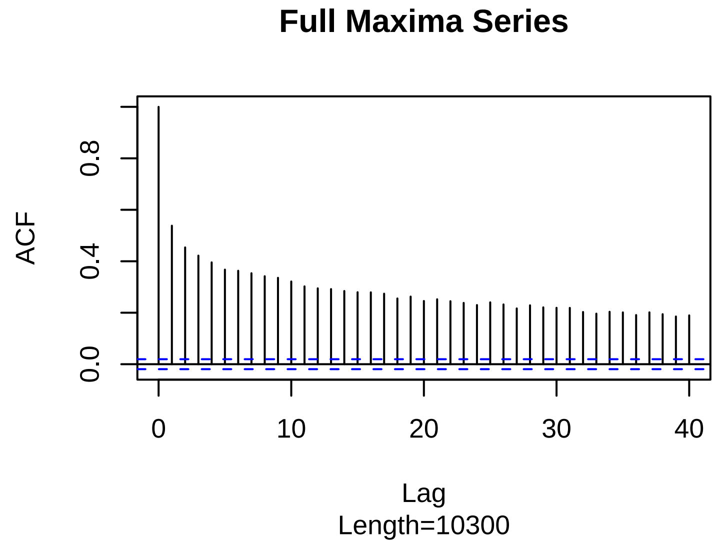

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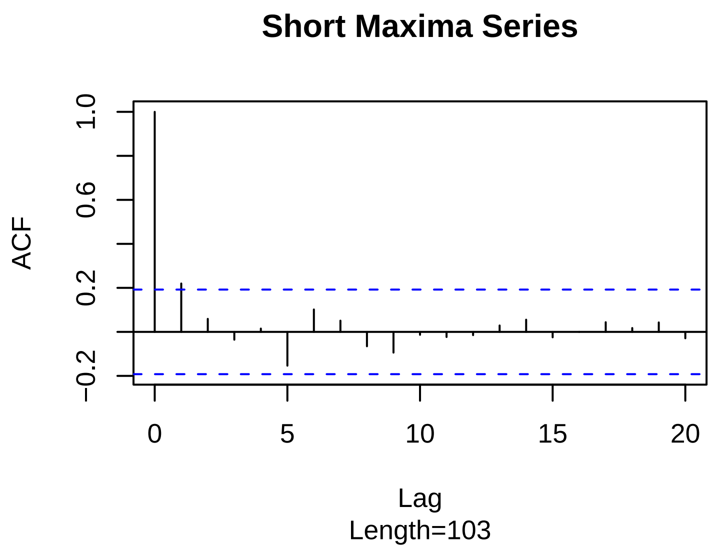

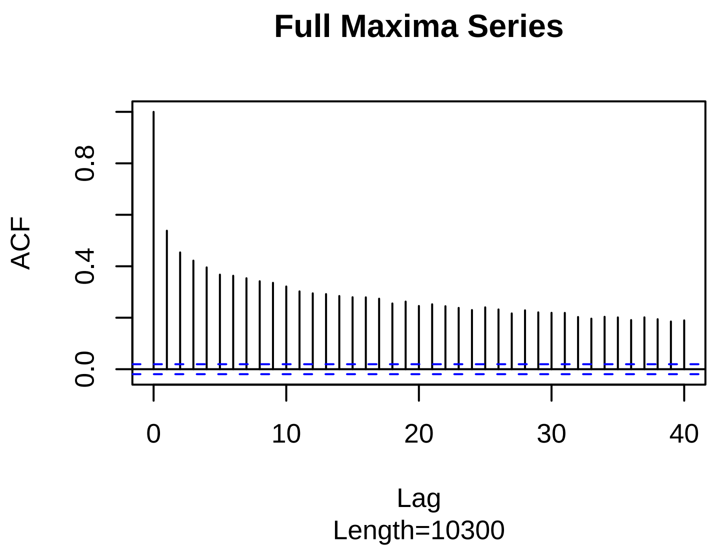

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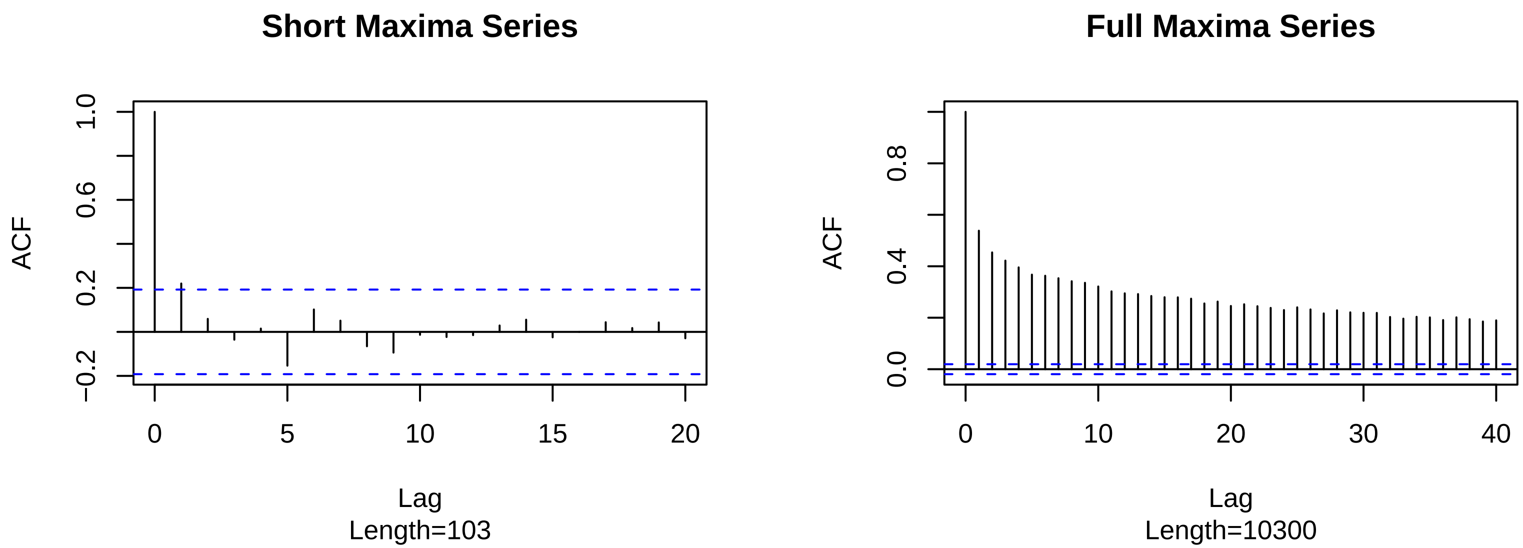

- Need longer maxima series to model ACF
- Trick:
  - Model daily series with FARIMA
  - Generate a very long series and extract maxima
  - Model this long maxima series with FARIMA

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6. Estimate parameters (HQ_{100}) for all ensemble members
   $\rightarrow$ ensemble for HQ_{100}

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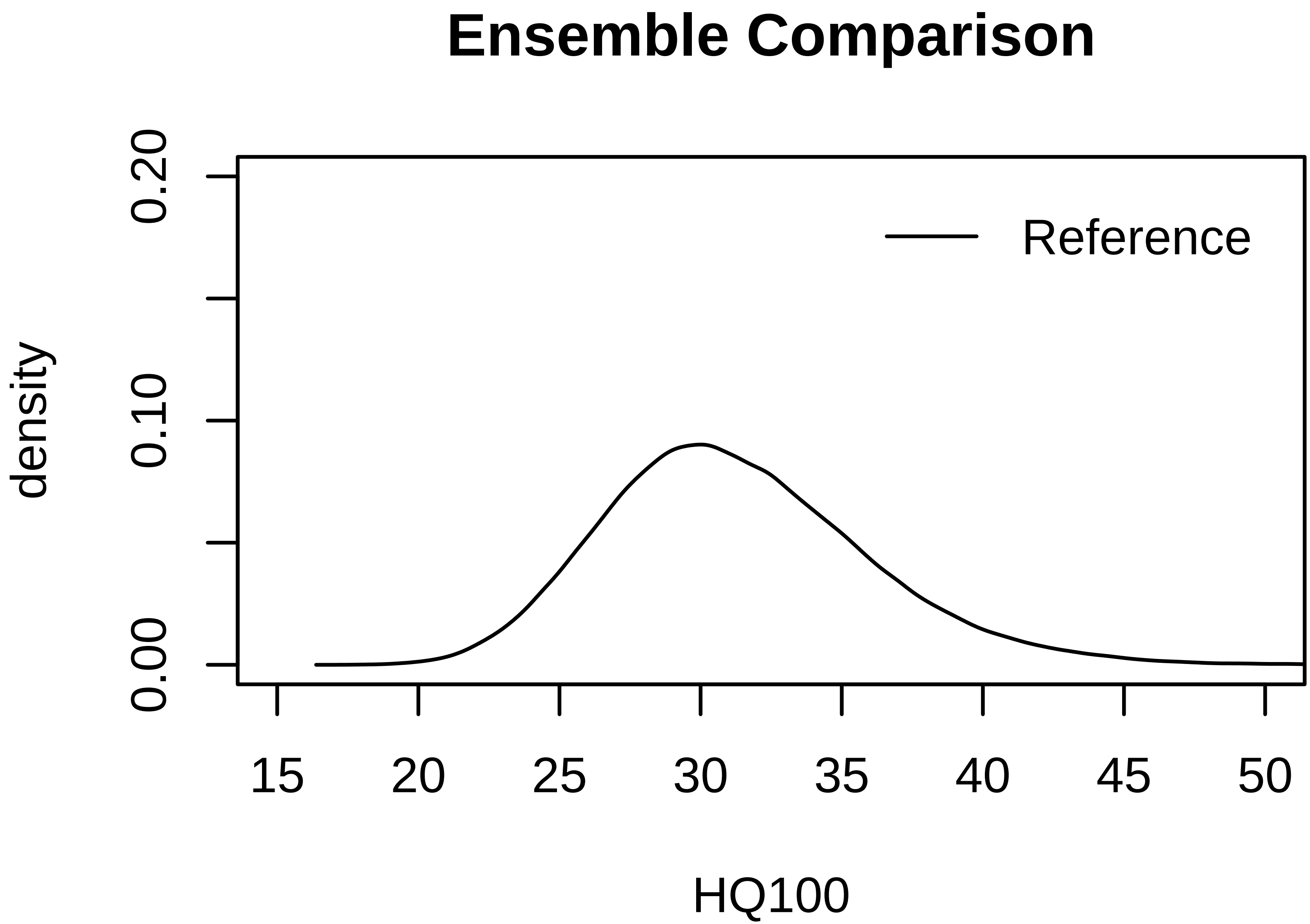
  - Generate an ensemble from it (100.000 runs)
  - Extract maxima and estimate HQ_{100}
    $\rightarrow$ reference ensemble
    represents the estimator's variability

Result



Ensemble Comparison
Reference
density
0.00
0.10
0.20
15
20
25
30
35
40
45
50
HQ100

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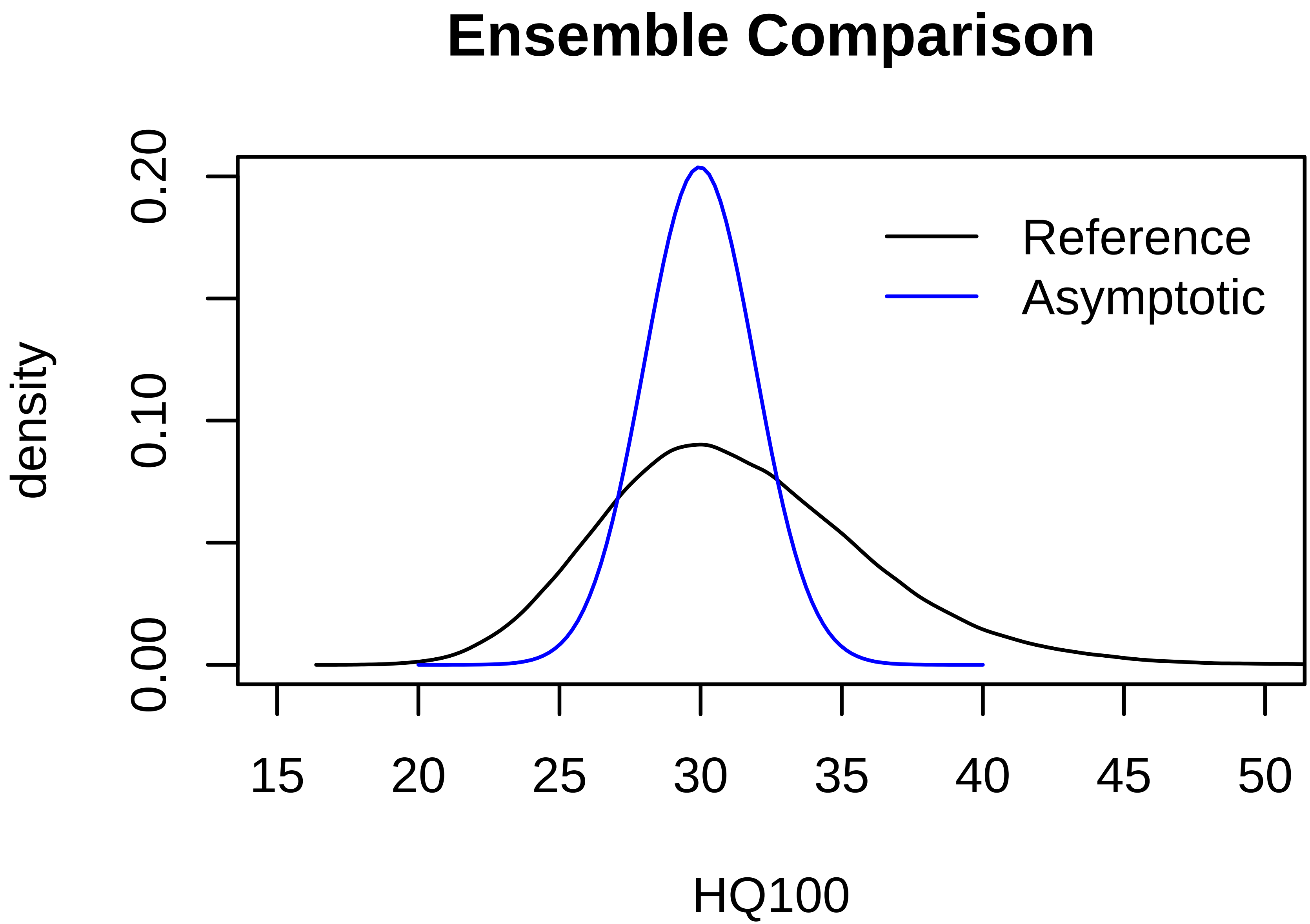
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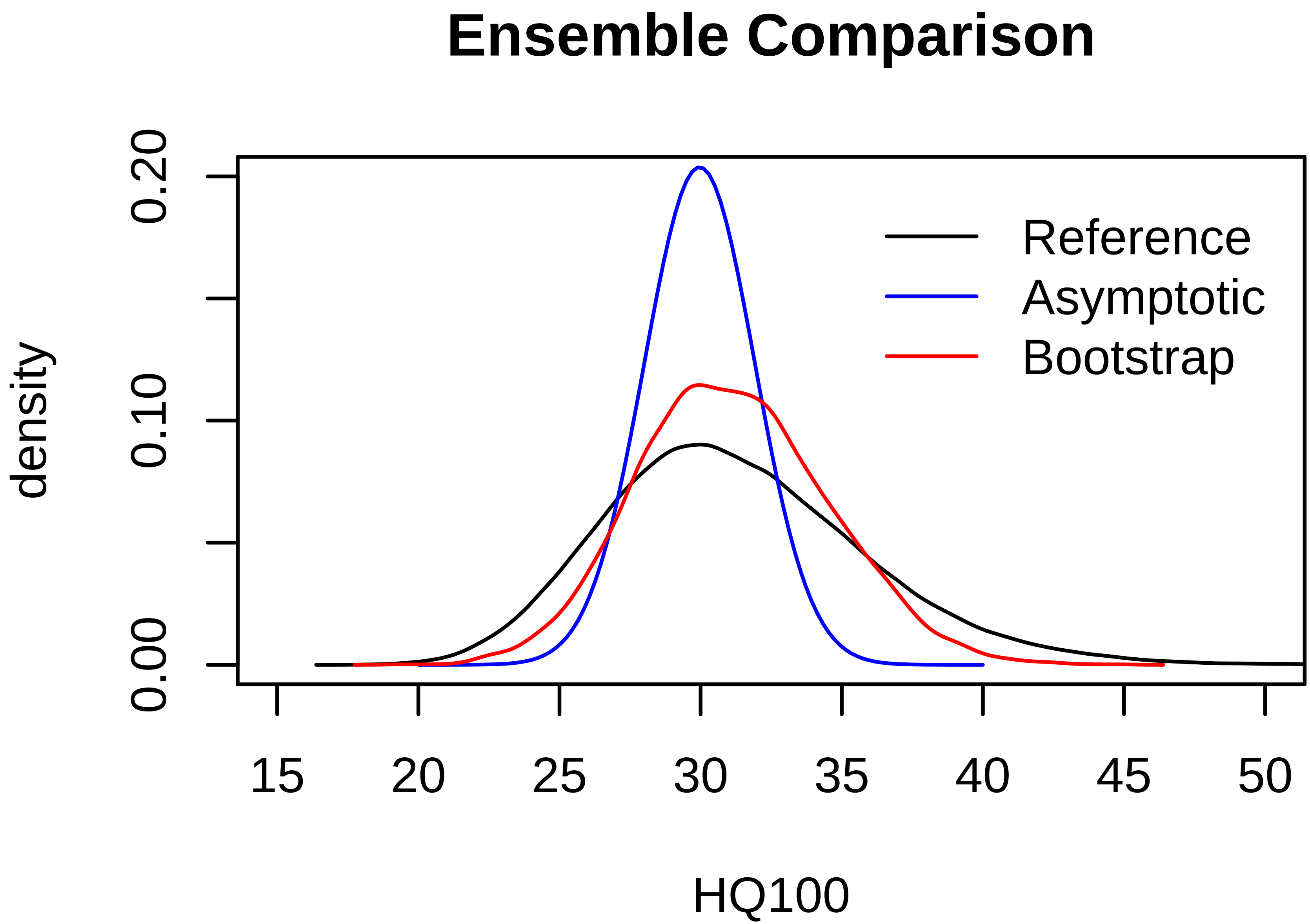
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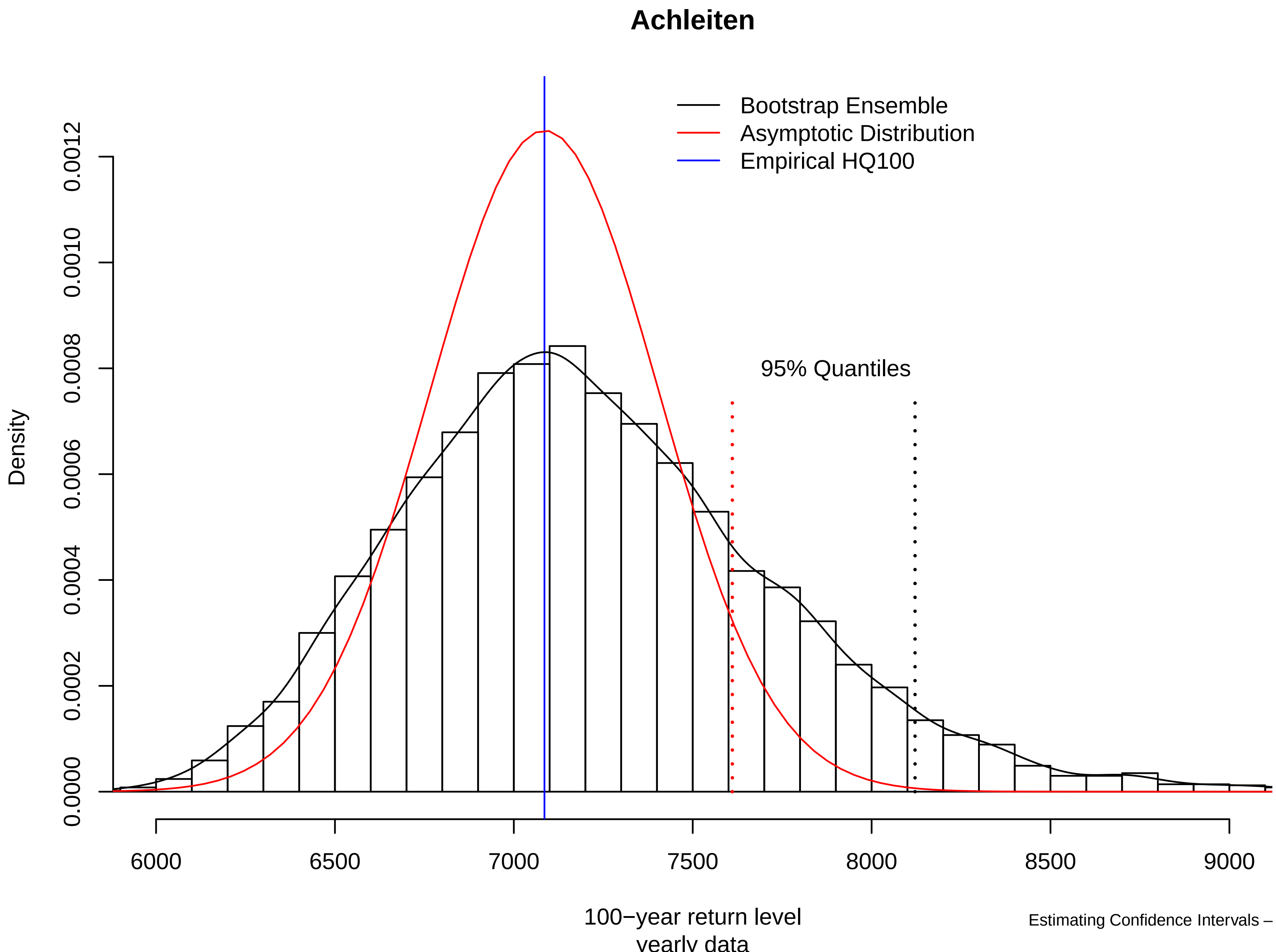
Achleiten

- The Data
  - Gauge Achleiten, Danube River, southwest Germany (Passau)
  - Catchment size: 76.653km^2
  - $\bar{Q} \approx 1430$m^3/s
  - 103 years daily data
- The Models
  - FARIMA[2,d,1], d=0.43 (H=0.93)
  - GEV, HQ$_{100} = 7086$m^3/s

Achleiten


Achleiten
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Asymptotic Distribution
Empirical HQ100
95% Quantiles
Density
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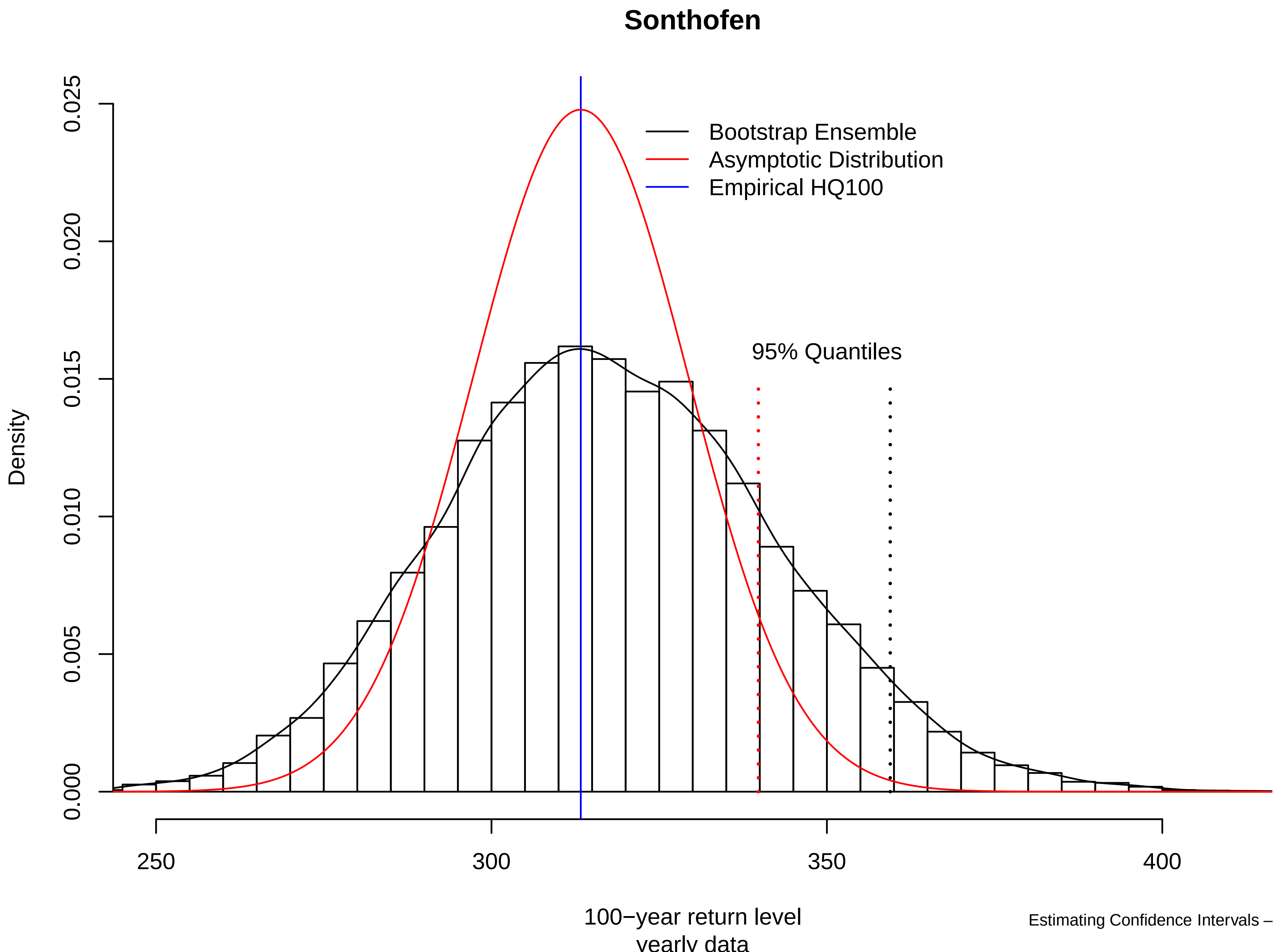
0.0008
0.0010
0.0012
6000
6500
7000
7500
8000
8500
9000
100−year return level
yearly data

Sonthofen

- The Data
  - Gauge Sonthofen, Iller, south Germany (Allgäu)
  - Catchment size: 388km^2
  - $\bar{Q} \approx 21$m^3/s
  - 102 years daily data
- The Models
  - FARIMA[2,d,2], d=0.24 (H=0.74)
  - GEV, HQ$_{100} = 313$m^3/s

Sonthofen



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- Better than asymptotic confidence intervals
- Relevant for run-off series

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