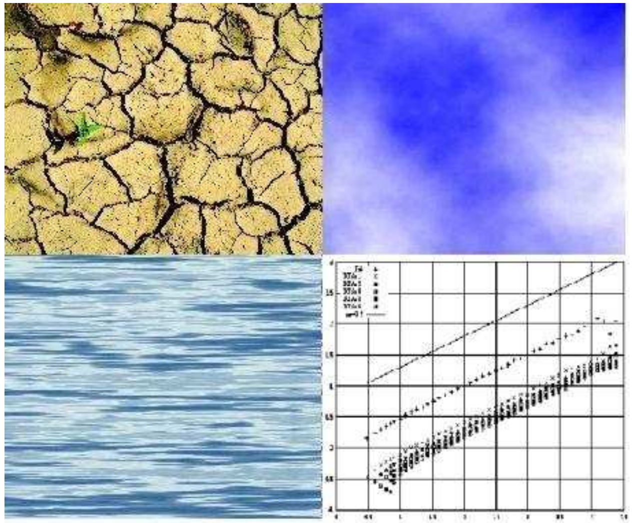


R&D Research Project: Scaling analysis of hydrometeorological time series data

S. Endres (BayCEER, Univeristy of Bayreuth)
H. Lange (Skogforsk, Ås, Norway)

Extreme Value Statistics of non-stationary and correlated Runoff Time Series Data

P I K
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN
Bayerisches Landesamt für Umwelt
SKOG FORSK
UNIVERSITÄT BAYREUTH
Bayreuther Zentrum für Ökologie und Umweltforschung
BayCEER
Bar-Ilan University

Contents

General Extreme Value Distribution GEV

General form:

$$G(x) = \exp\left(-\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)_+^{-1/\xi} \right) \equiv \mathrm{GEV}(\mu, \sigma, \xi)$$

μ : Location parameter

σ : Scale parameter

ξ : Shape parameter

$\xi = 0$ Exponential tail (Gumbel)

$\xi > 0$ Heavy tail (Fréchet)

$\xi < 0$ Finite tail (negative Weibull)

General Extreme Value Distribution GEV

General form:

$$G(x) = \exp\left(-\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)_+^{-1/\xi} \right) \equiv \mathrm{GEV}(\mu, \sigma, \xi)$$

Special cases: Gumbel, Fréchet, Weibull Distribution

$$\mathrm{GEV}(0,1,0) = \exp(-\exp(-x))$$

Gumbel distribution

$$\mathrm{GEV}(1,1/\alpha,1/\alpha) = \exp\left(-x^{-\alpha}\right)$$

$$(x > 0, \alpha > 0)$$

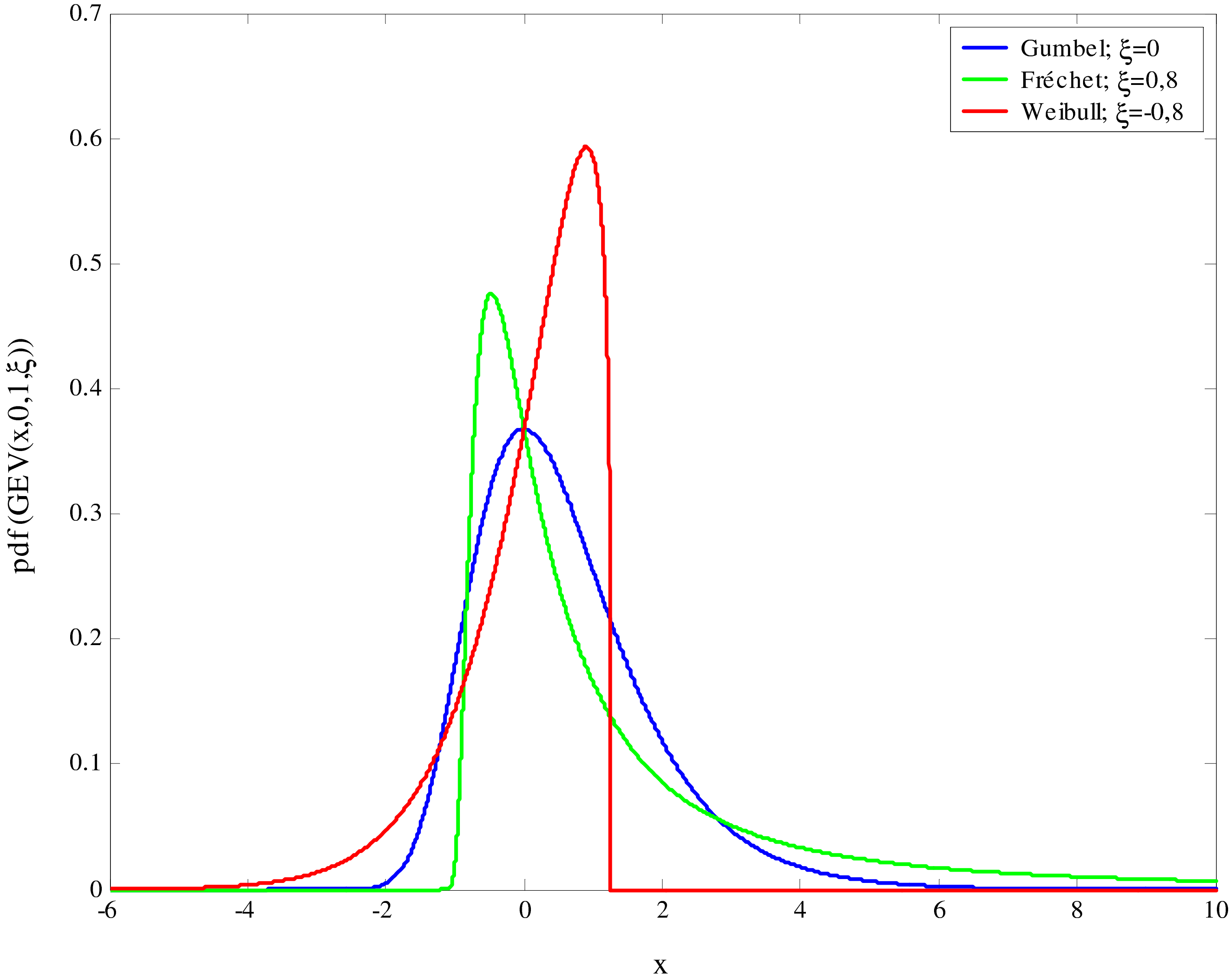
Fréchet distribution

$$\mathrm{GEV}(-1,1/\alpha,-1/\alpha) = \exp\left(-(-x)^{\alpha}\right)$$

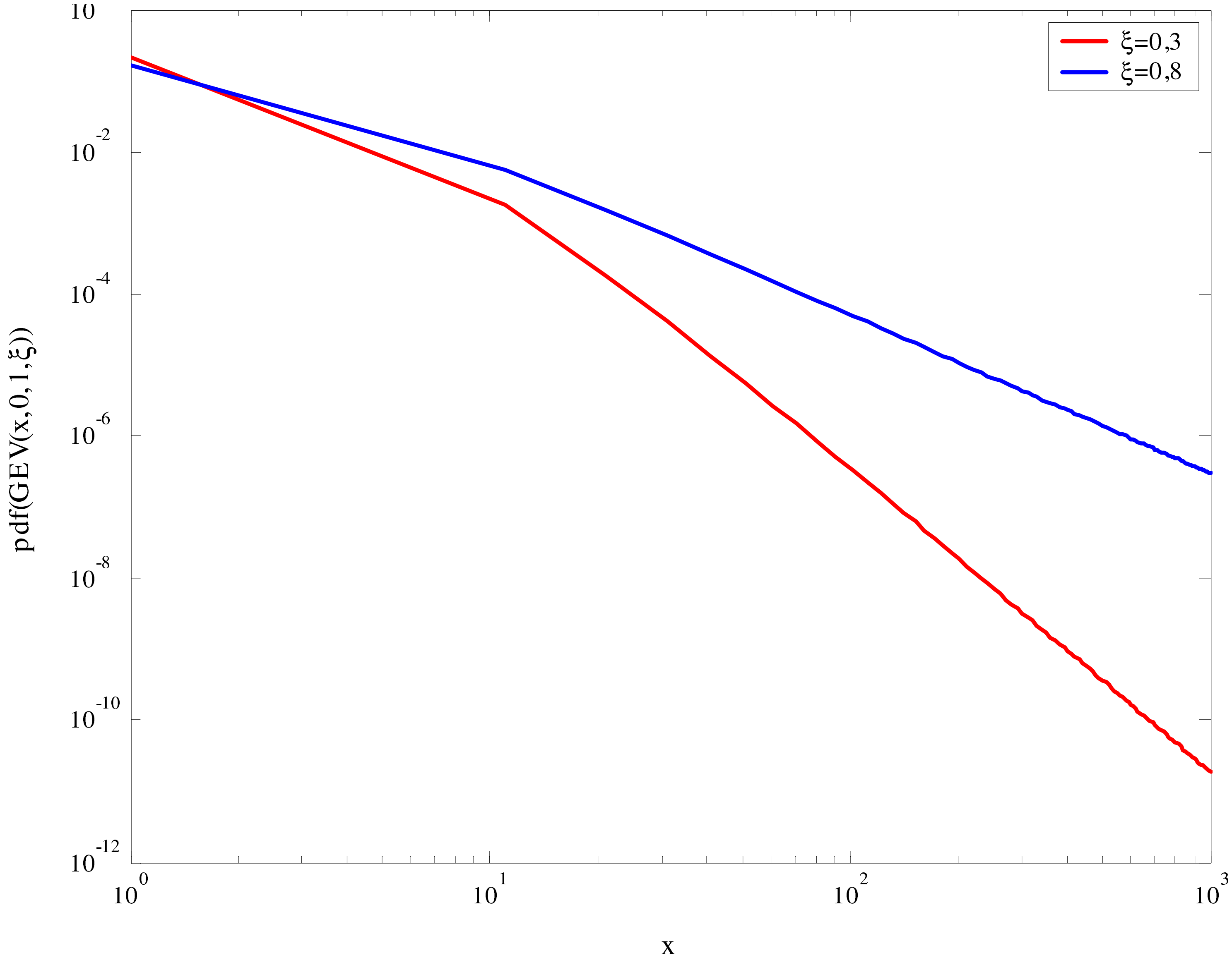
$$(x < 0, \alpha > 0)$$

(negative)
Weibull distribution

Three Types of the GEV



Impact of the shape parameter on tail behaviour



Fitting Method

- Software:

  ‘**EVIM**: A Software Package for Extreme Value Analysis in MATLAB’

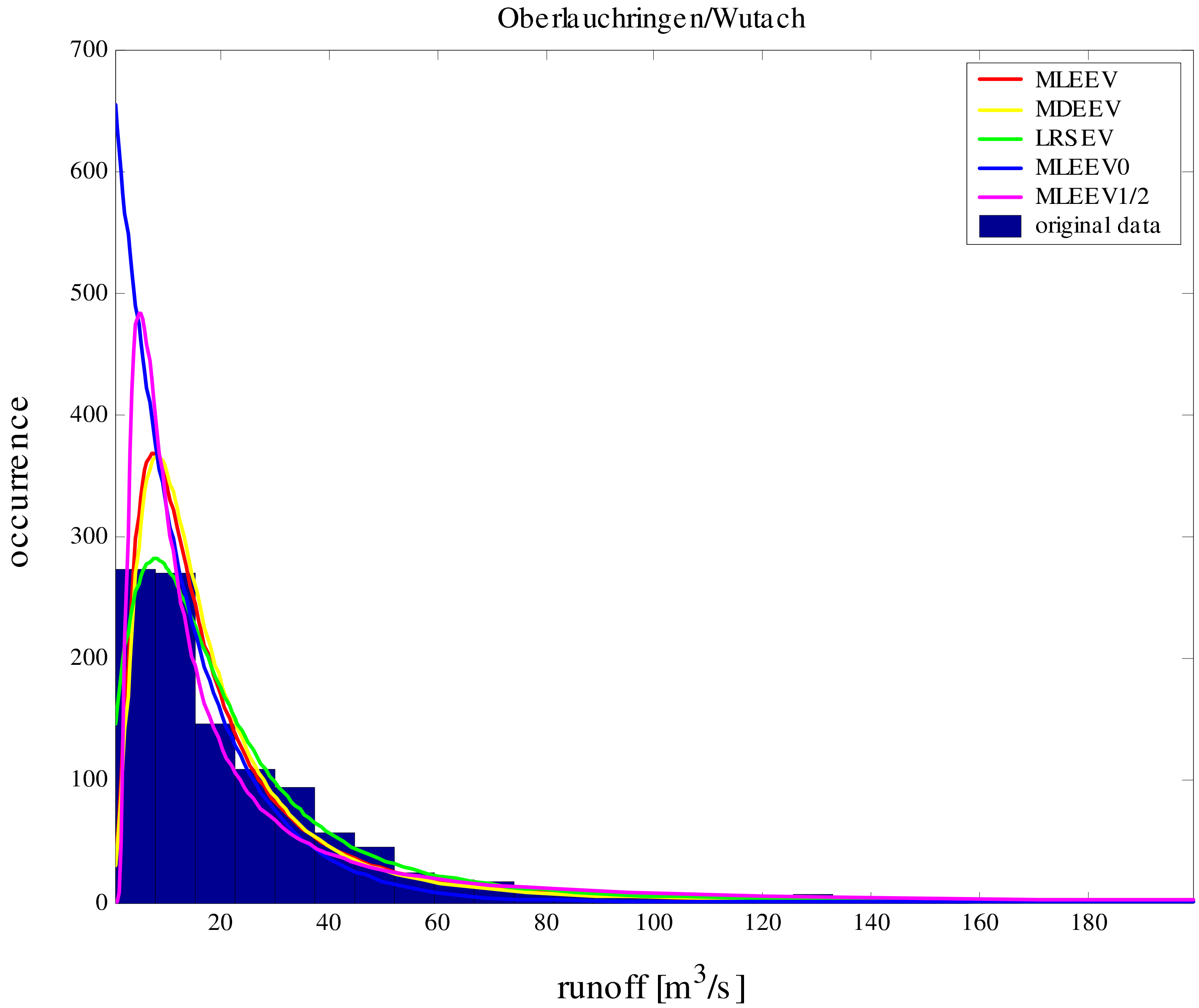
  (April 2001; Ramazan Gençay, Faruk Selçuk, Abdurrahman Ulugülyağci)

→Maximum Likelihood Estimator

→Model selection with Akaike Information Criterion (AIC)

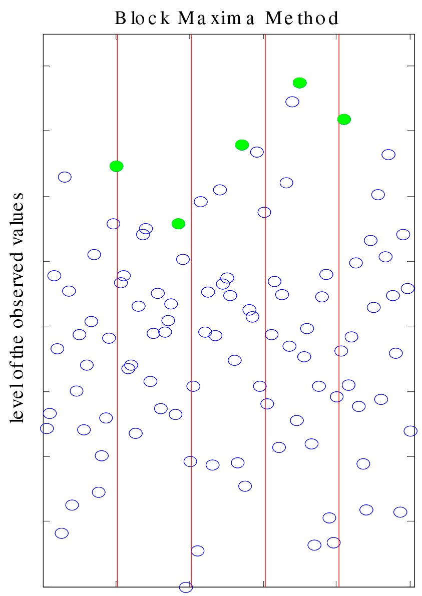
- Download: http://www.bilkent.edu.tr/~faruk

Impact of the Fitting Method



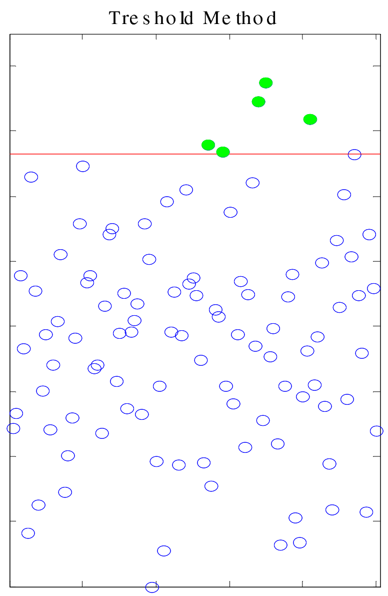
AMS/GEV vs. POT/GP

AMS= Annual Maxima Series



→ Good for observed data

POT= Peak Over Treshold



→ Good for regional interpolated data
(e. g. meteorological data)

Data Set

- 160 runoff time series from Bayern and Baden-Wuerttemberg (basically Danube catchment basin)
- Daily means
- 69 time series without gaps, 120 with gaps < 1 a

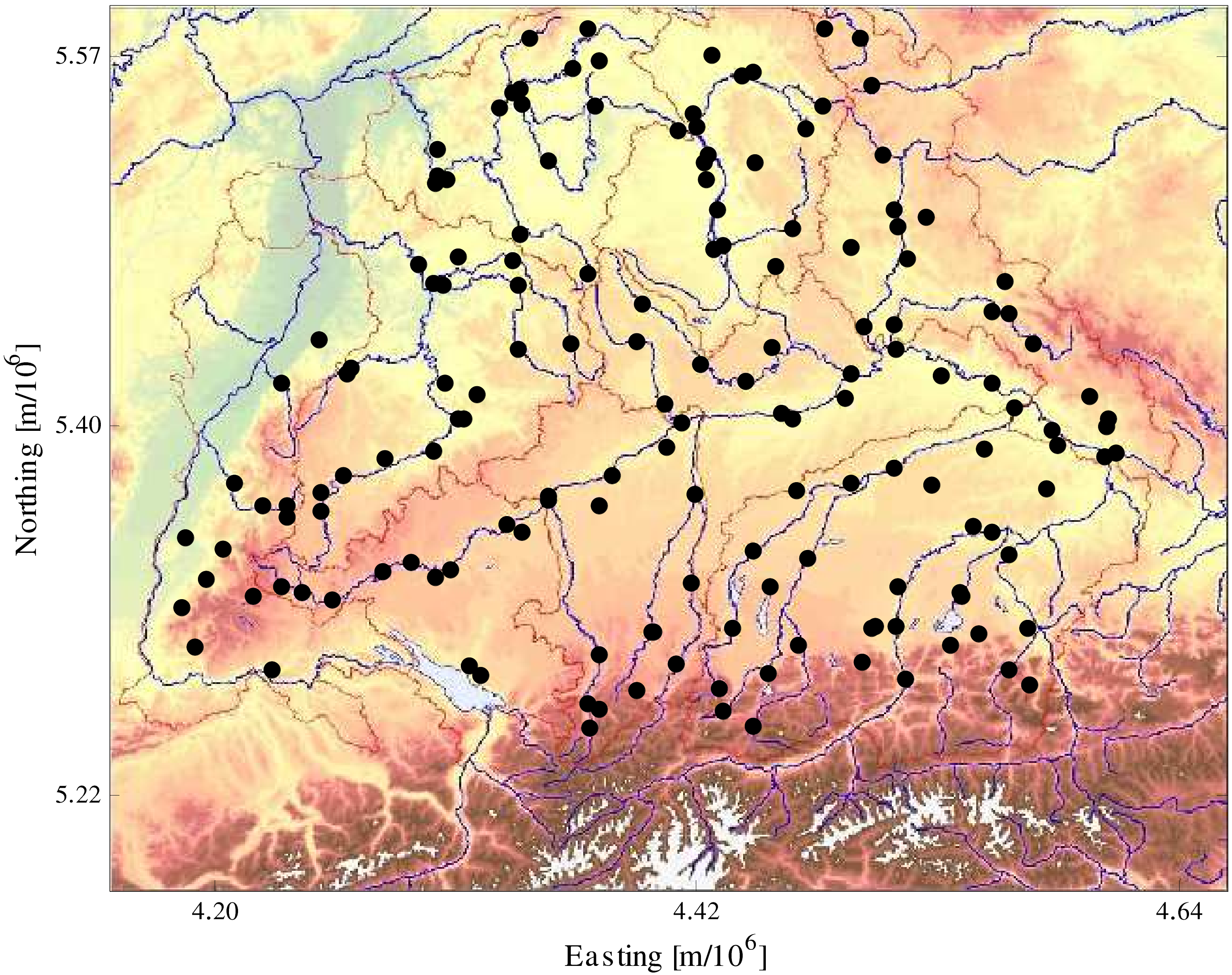
	minimum	mean	maximum
length [a]	10	71	177
catchment area [km^2]	47	3 300	77 000

Sources:

Bayerisches Landesamt für Wasserwirtschaft

GRDC

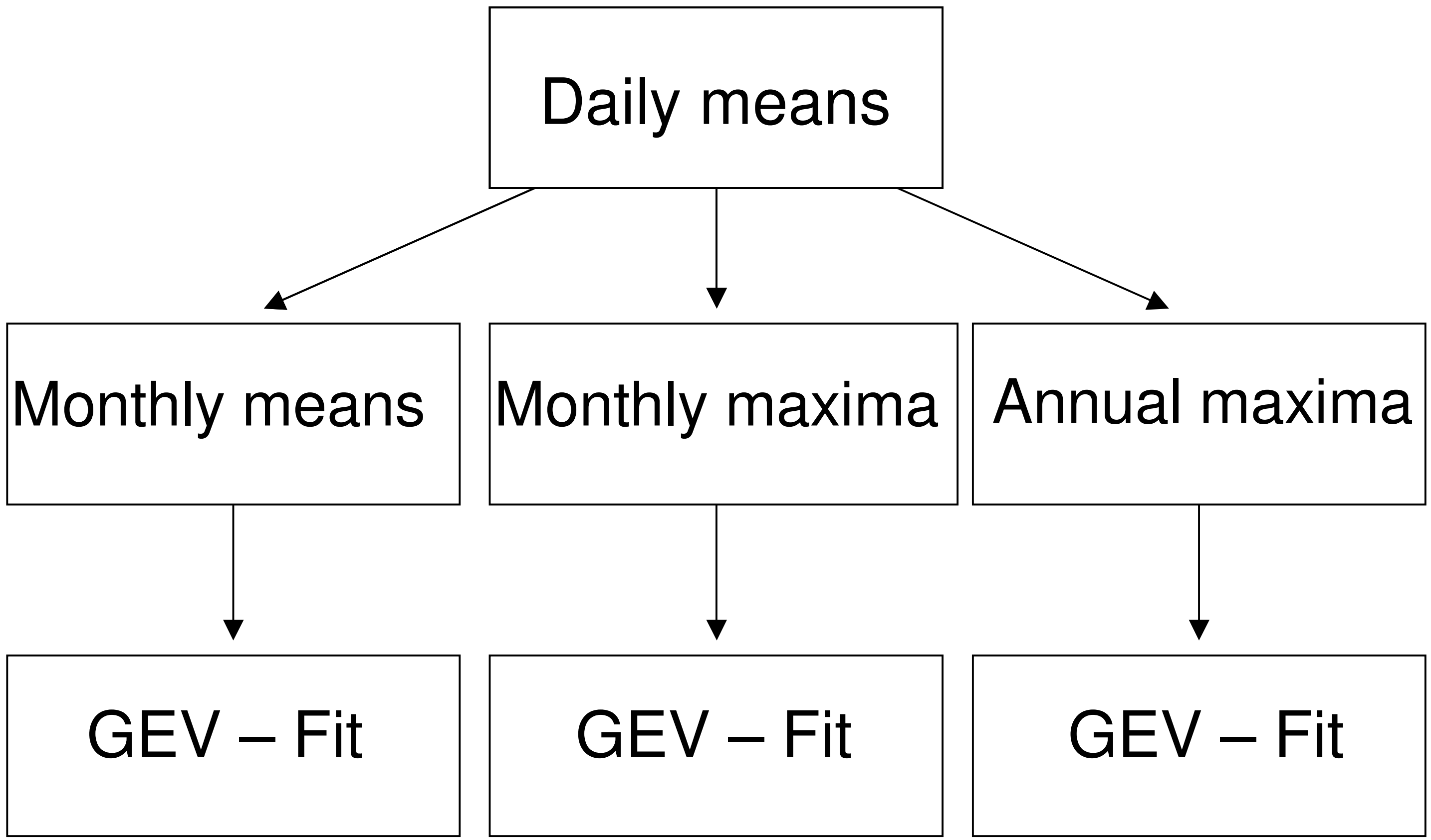
Gauges Overview



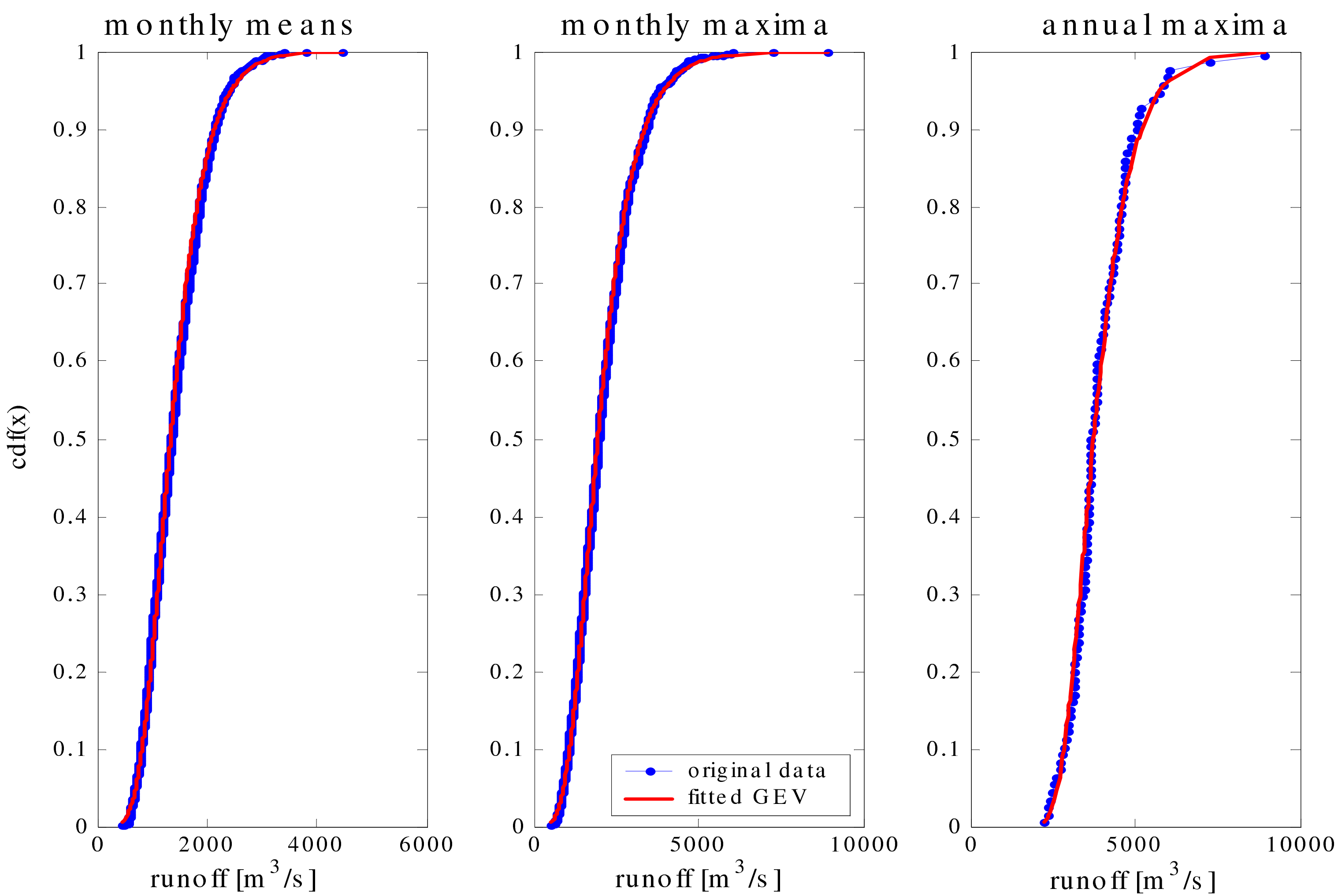
Procedure

Original data:

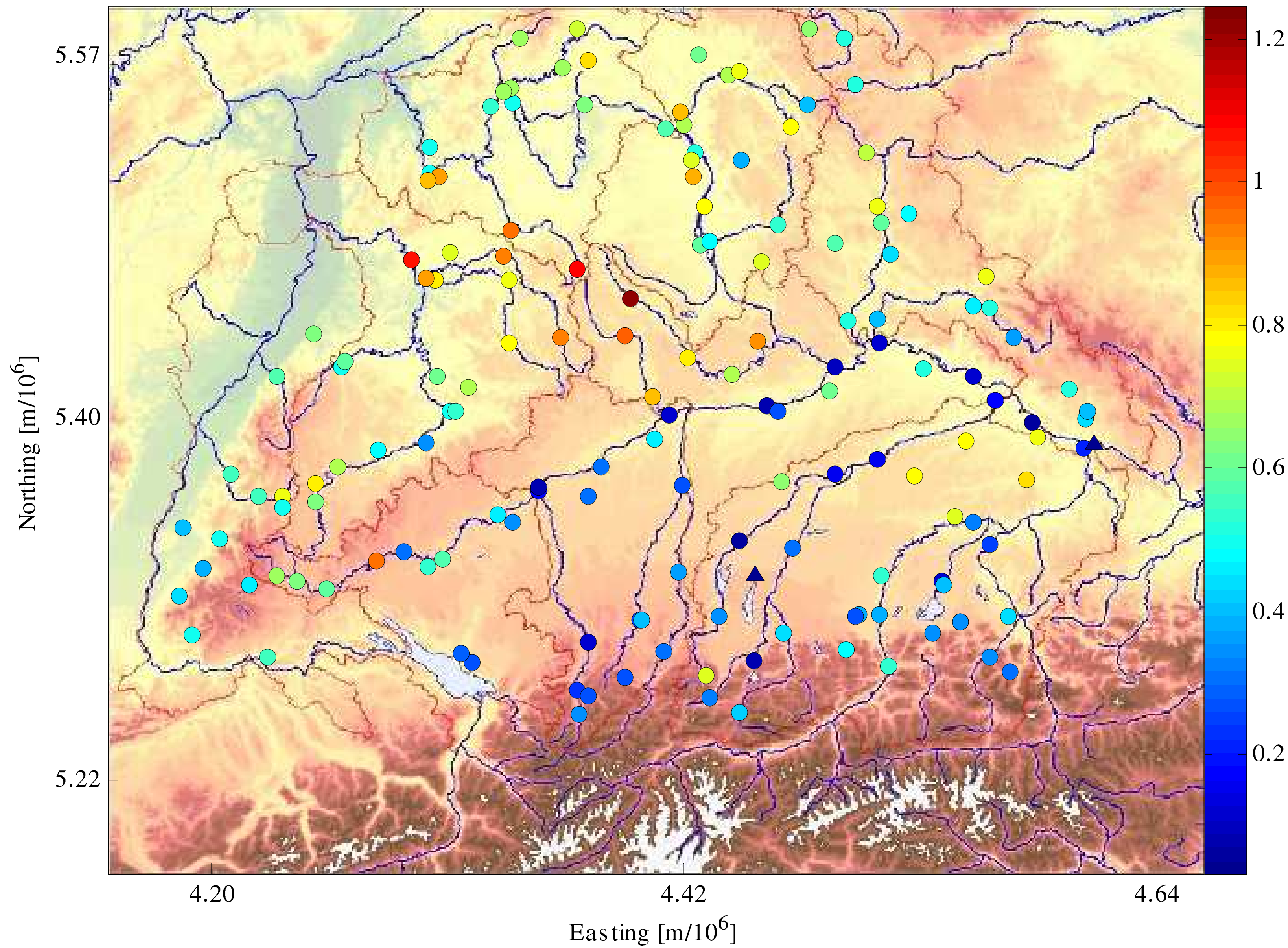
Different decimation and time resolution:



Example: Achleiten/Danube

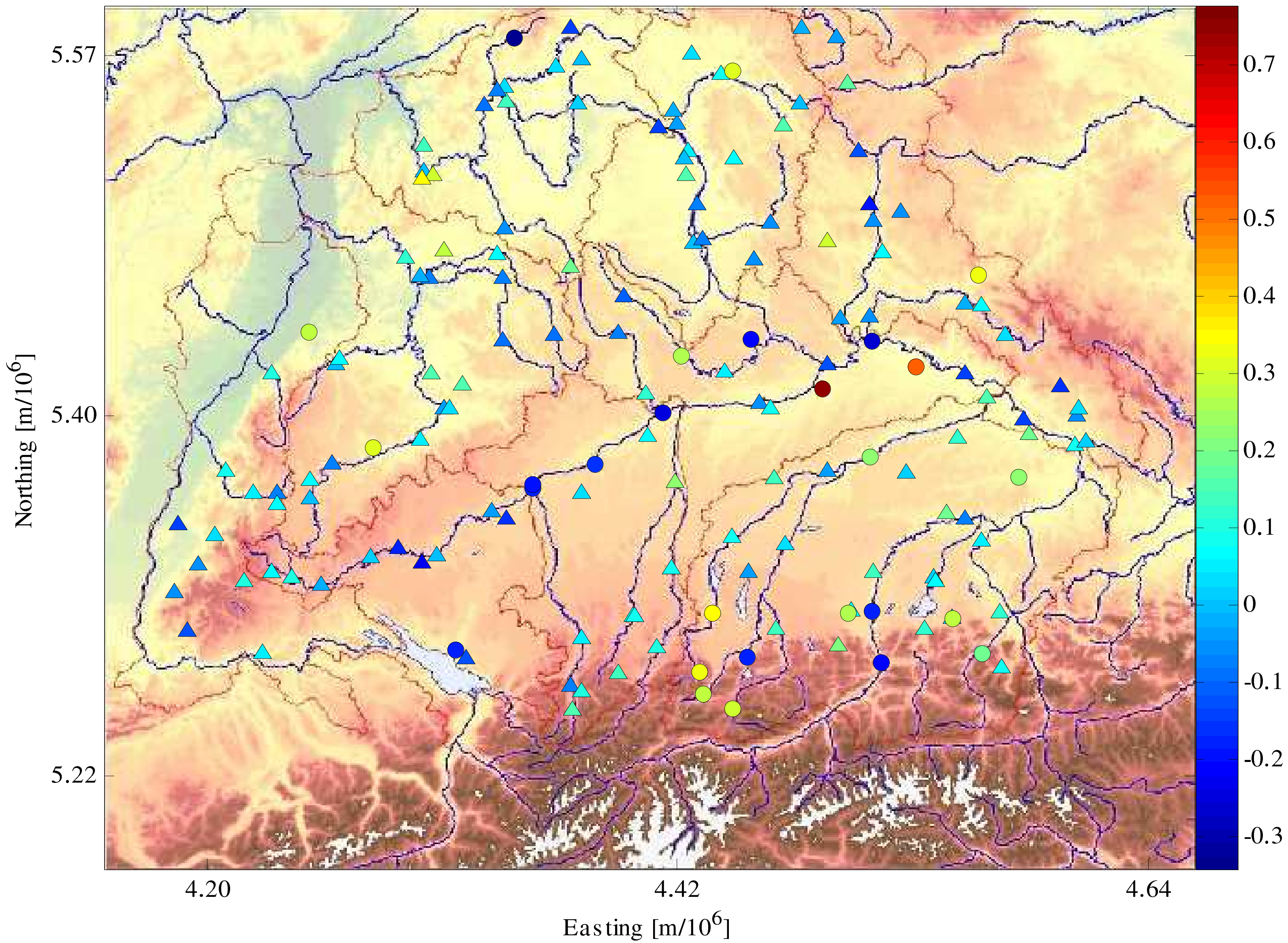


Spatial distribution of ξ: Monthly maxima



△ : Compatible with zero

Spatial distribution of ξ: Annual maxima



△ : Compatible with zero

Are Runoff Data Gumbel Distributed?

	monthly means	monthly maxima	annual maxima
$\xi_e>0$	145	158	16
$\xi_e=0$	15	2	134
$\xi_e<0$	-	-	10

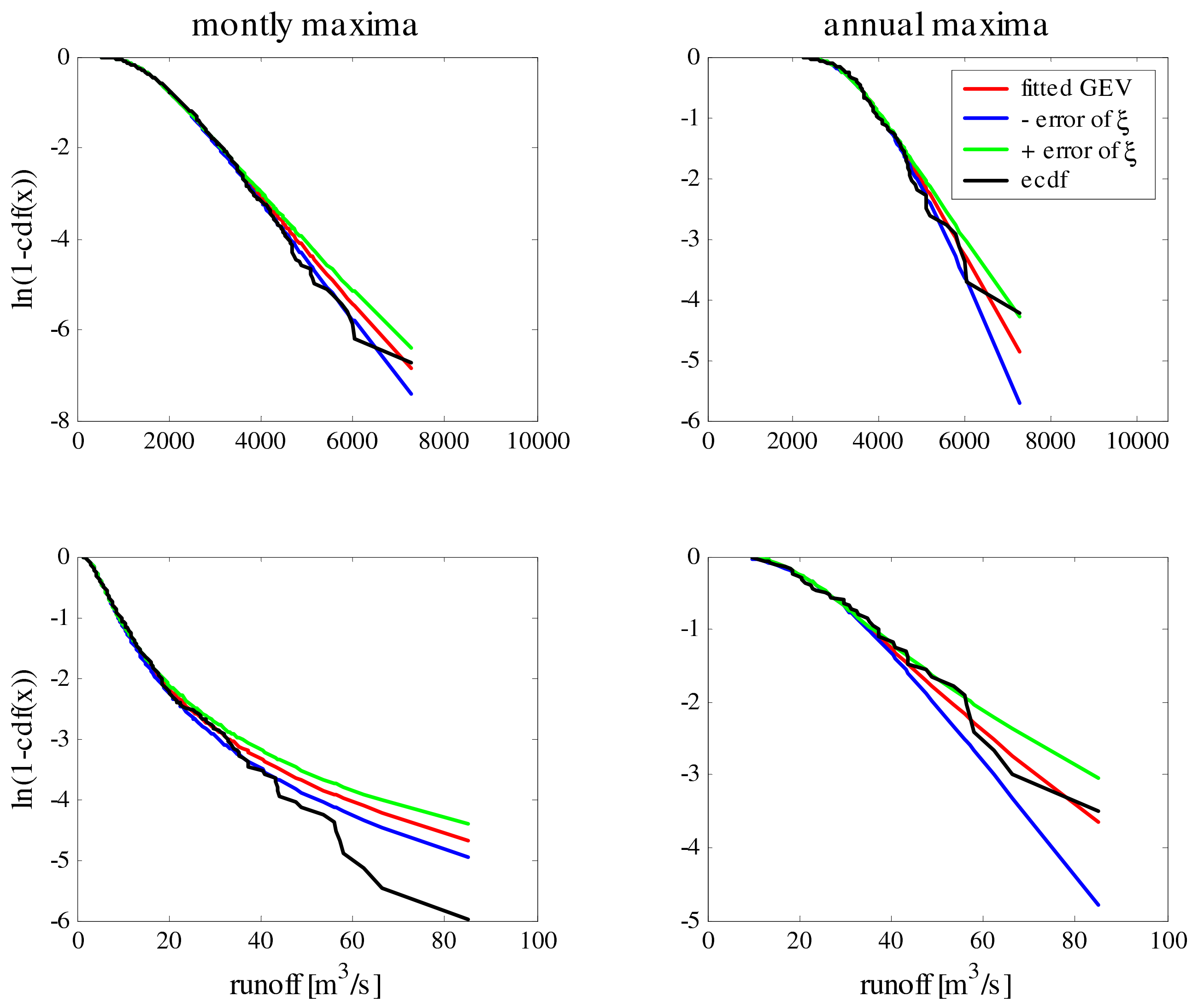
→ annual maxima mostly Gumbel distributed

→ monthly data mostly Fréchet distributed

Quality of the GEV - Fit

Achleiten/Danube:

Anger/Attel:



Dealing with Instationarties

Seasonal effects

→ series for separate month

other effects

→ split the data into halves

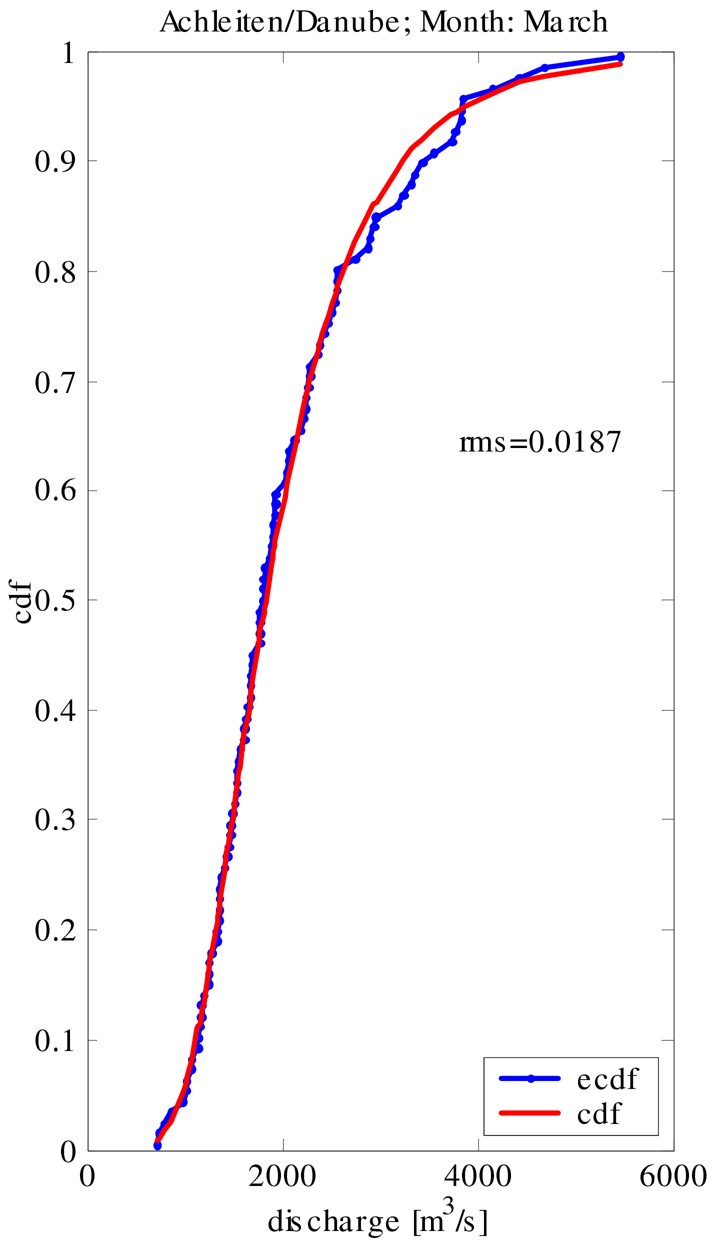
Dealing with Instationarties

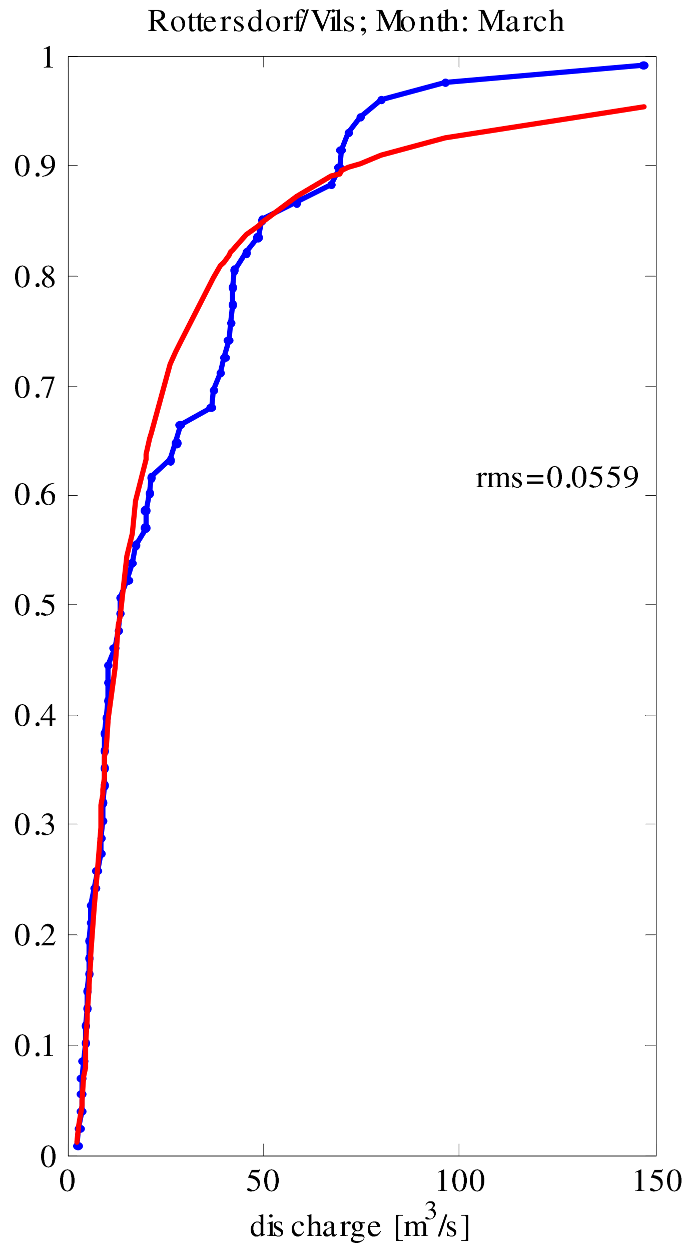
Seasonal effects

→ series for separate month

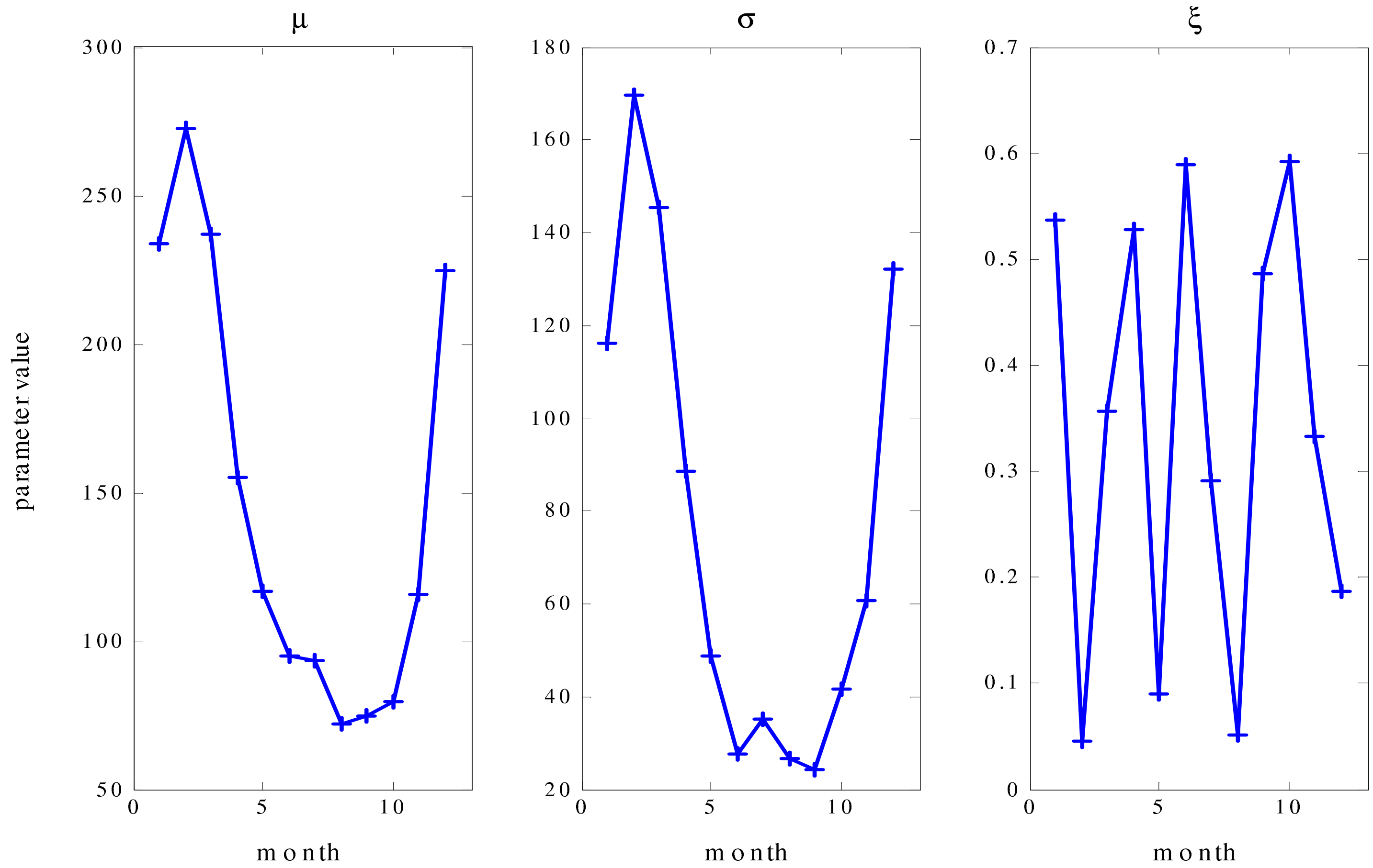
other effects

→ split the data into halves


Achleiten/Danube; Month: March
cdf
discharge [m3/s]
rms=0.0187
ecdf
cdf
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2000
4000
6000

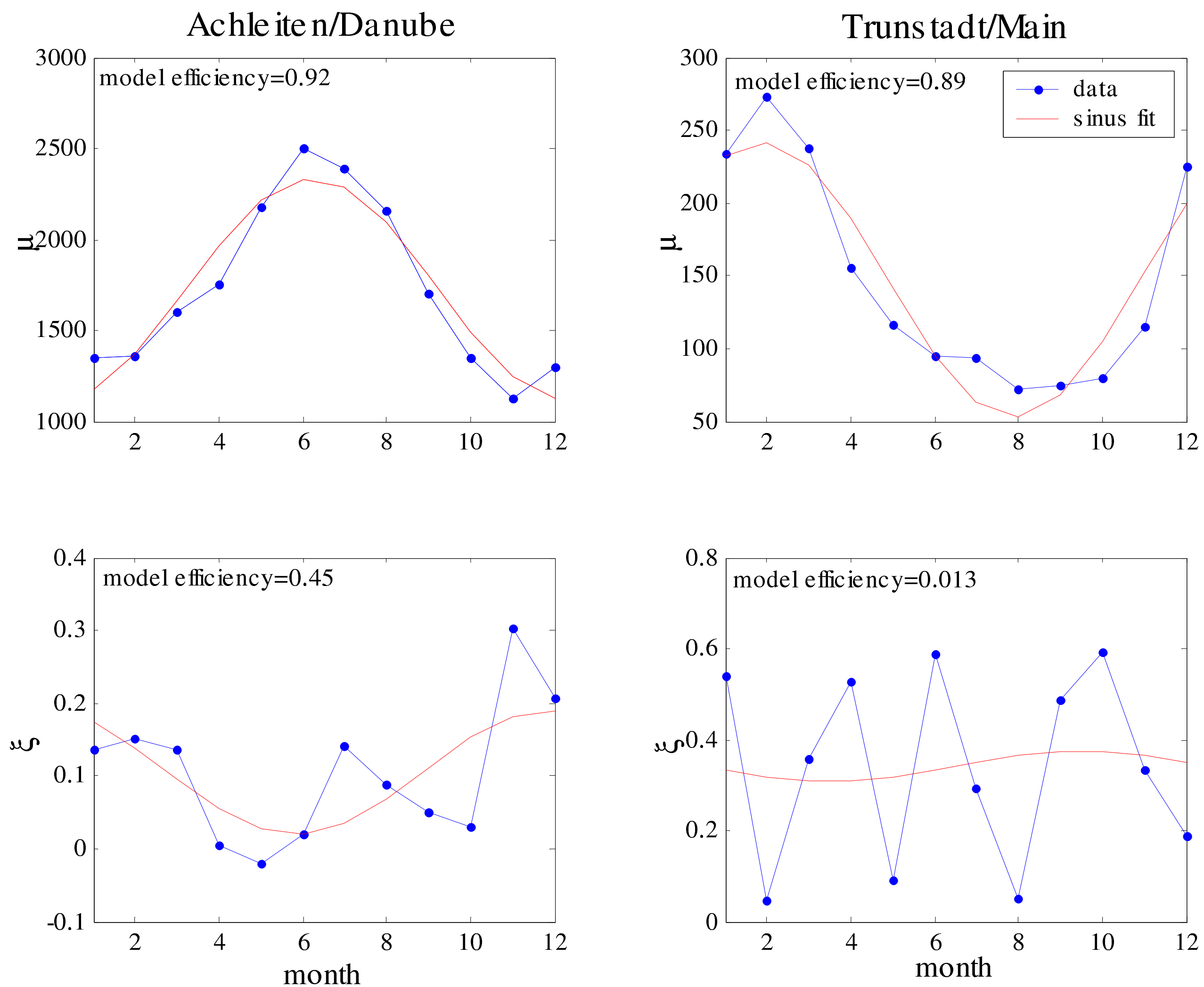

Rottersdorf/Vils; Month: March
discharge [m3/s]
rms=0.0559
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50
100
150

Example: Trunstadt/Main



→ Seasonal variations exist

Sinus Fit

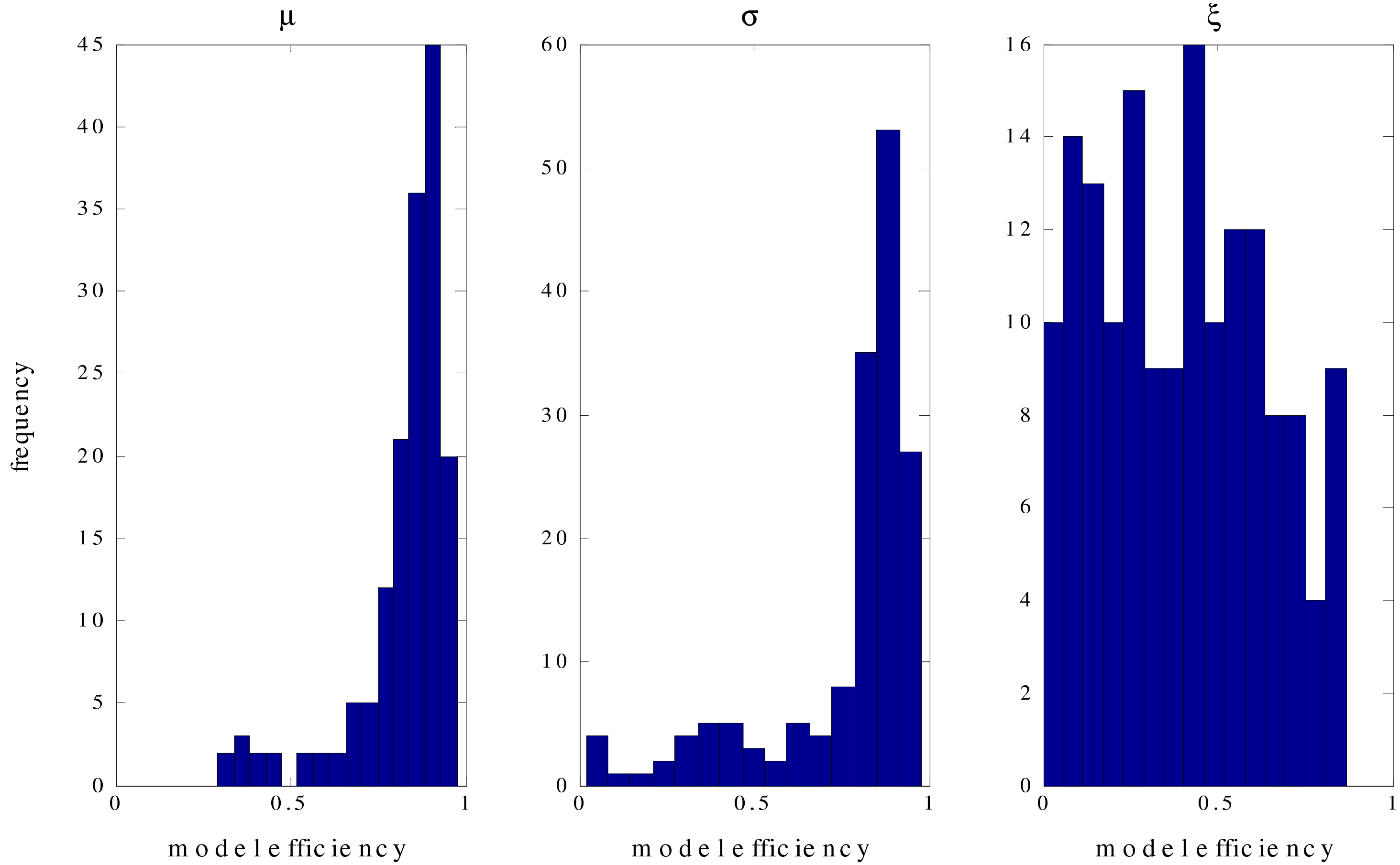


Model efficiency:

$$R = 1 - \frac{\sum (x^{org} - x^{\text{mod}})^2}{\sum \left(x^{org} - \bar{x}\right)^2}$$

→ Seasonal variations are for μ and σ (but not for ξ) periodically in the majority of cases

Histogramms of the Modell Efficiency for the Sinus Fit



→ Seasonal variations are for μ and σ (but not for ξ) periodically in the majority of cases

Dealing with Instationarties

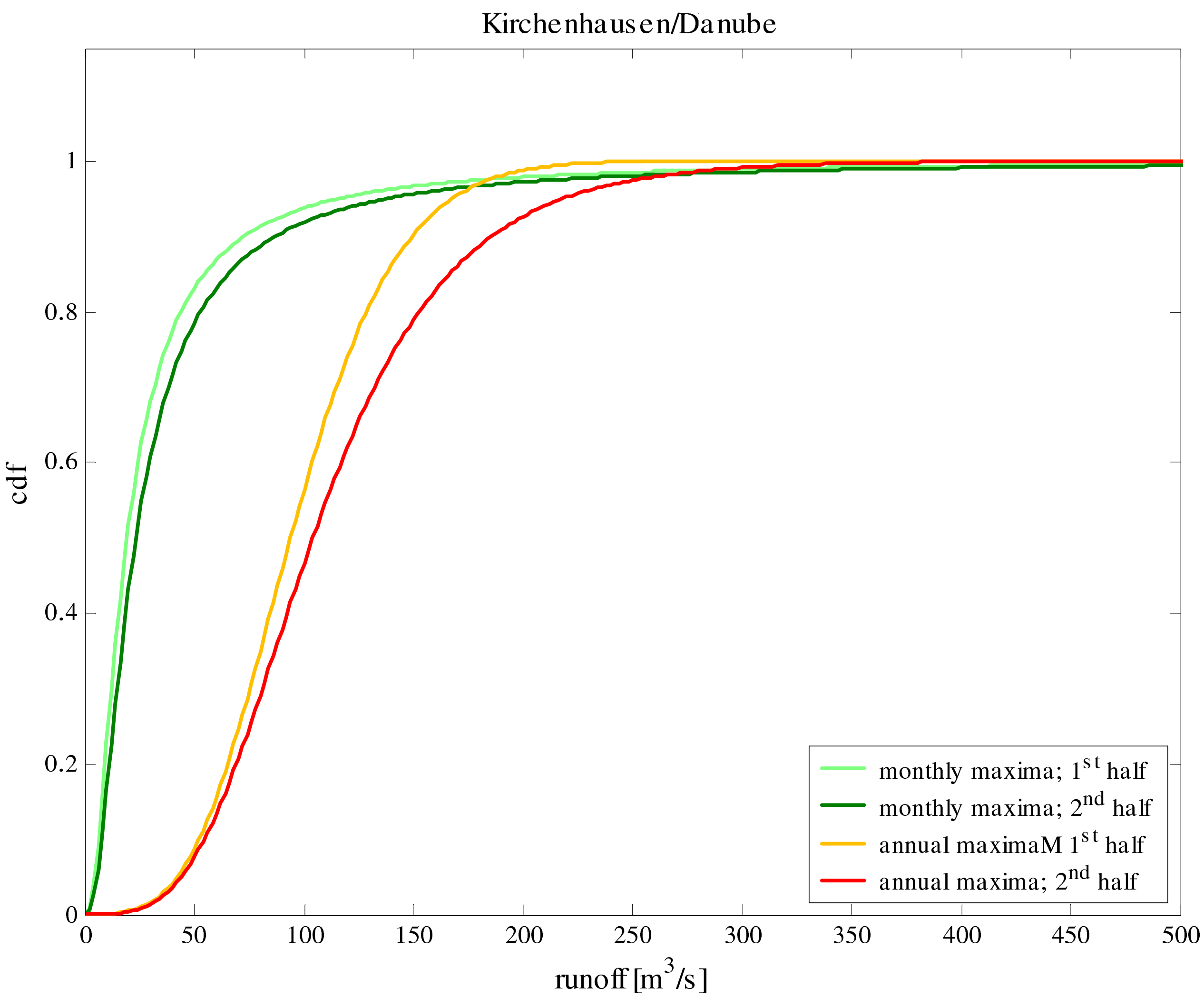
Seasonal effects

→ series for separate month

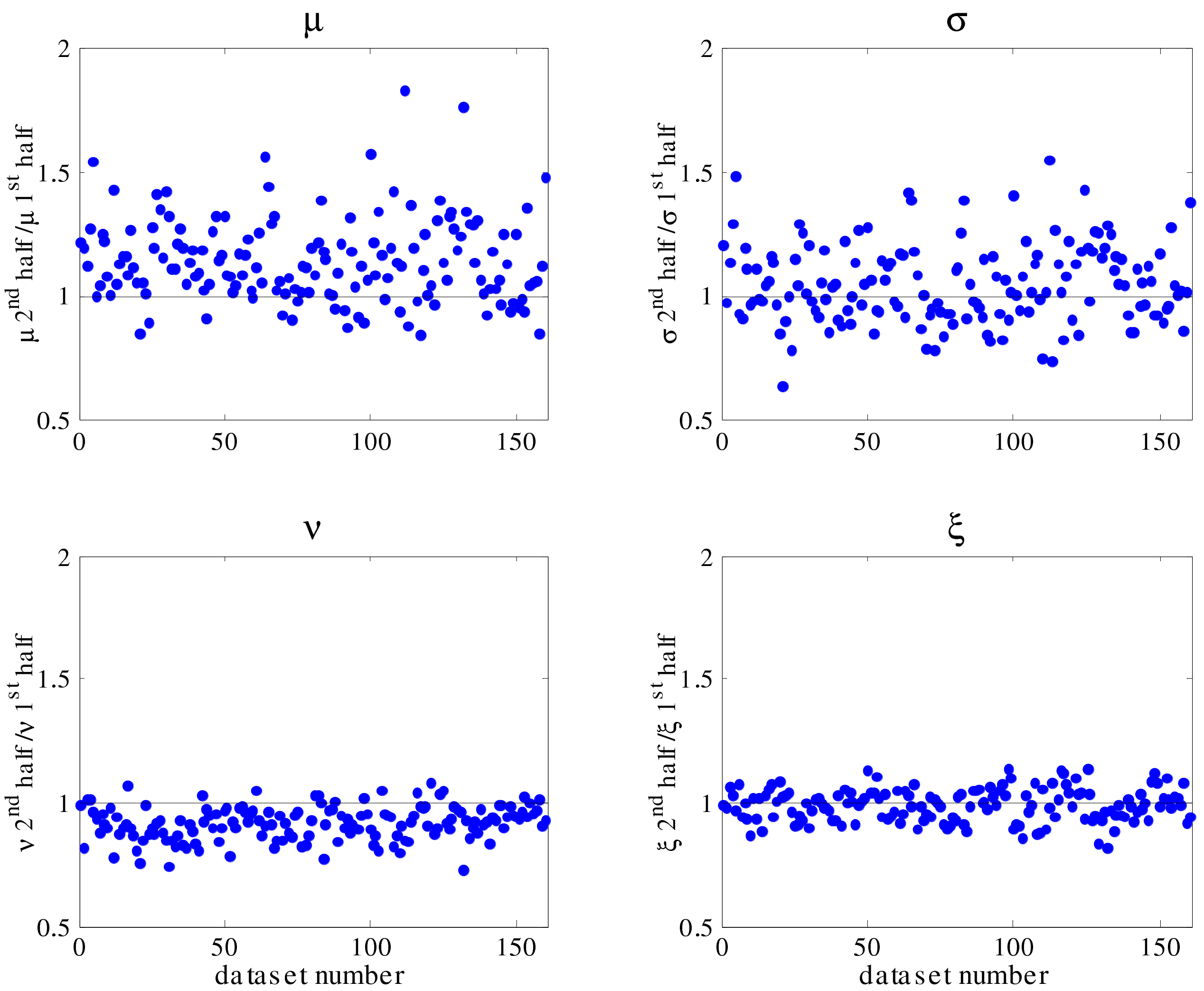
other effects

→ split the data into halves

Changes in Time of the cdf

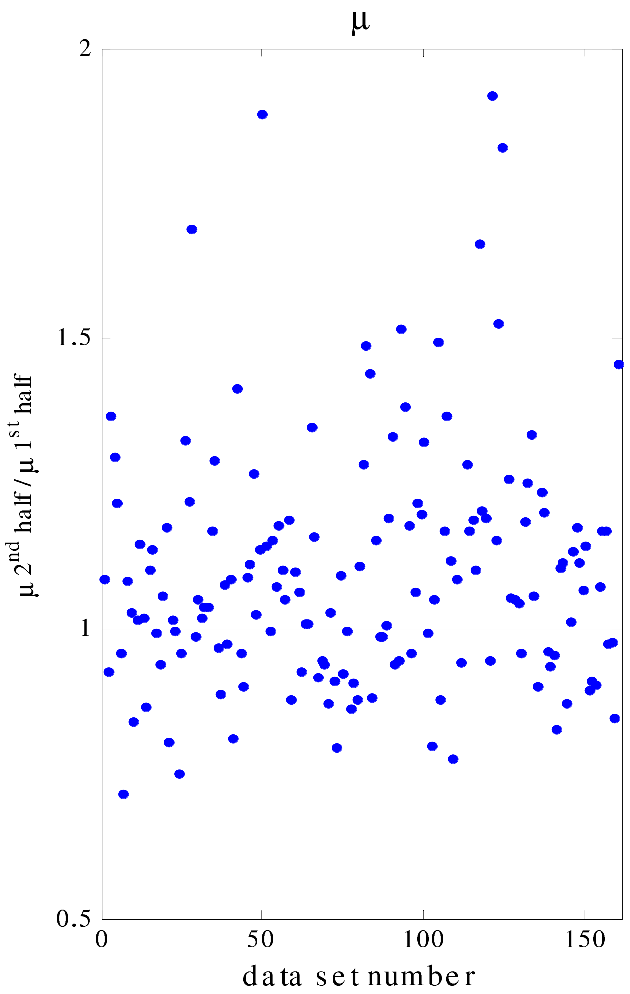


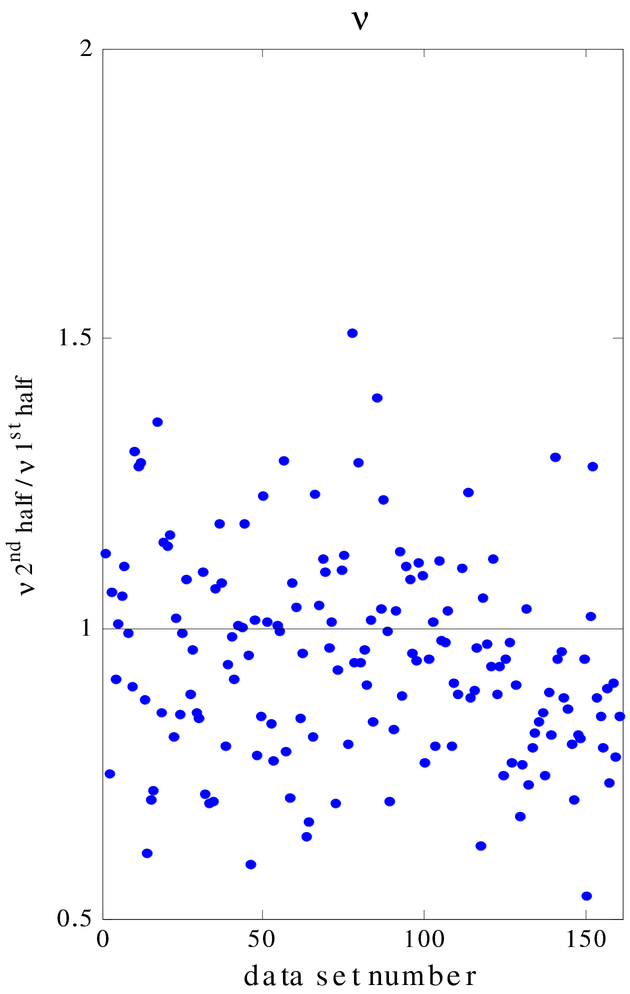
Monthly Maxima



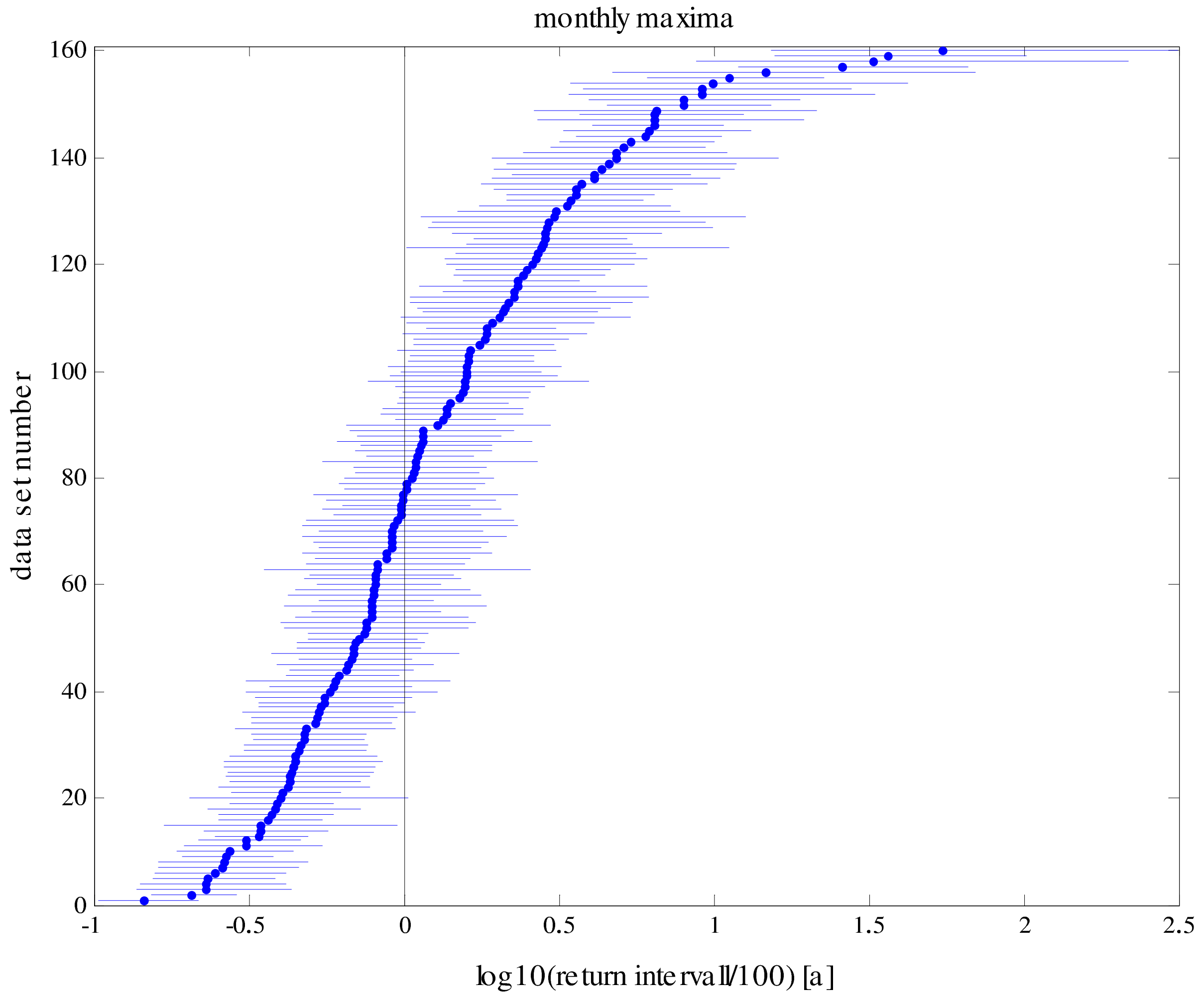
$$\nu = \frac{\sigma}{\mu}$$

Annual Maxima





HQ100



Exceedance probability:

$$E(x) = \Pr(X > x) = 1 - cdf(x)$$

Return interval:

$$T(x) = \frac{1}{E(x)}$$

Return value:

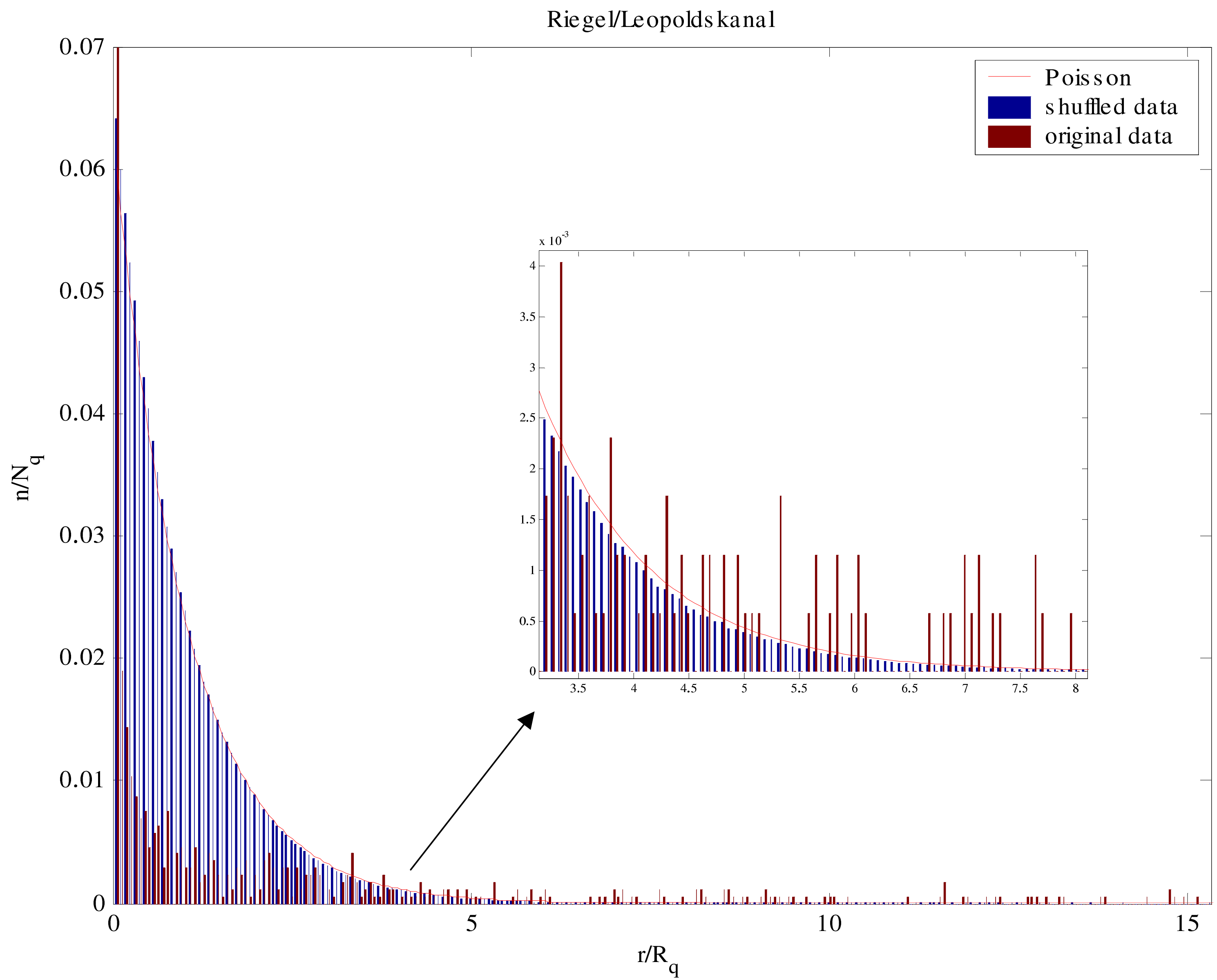
$$x_T = cdf^{-1}(1 - 1/T)$$

HQ100: Level of a runoff event, which occurs statistically every 100 years once

Conclusions

- Monthly maxima mostly <u>not</u> Gumbel distributed
- Decimation big impact on the results and quality of the fit
- GEV parameter fits for μ and σ are strongly seasonal dependent
- No pattern for ξ estimate
- Dominating upward trend for μ
- No change in xi
- Return intervals vary strongly between halves
  - ~ 50% show larger return intervals
  - Asymmetry between shorter and longer return intervals

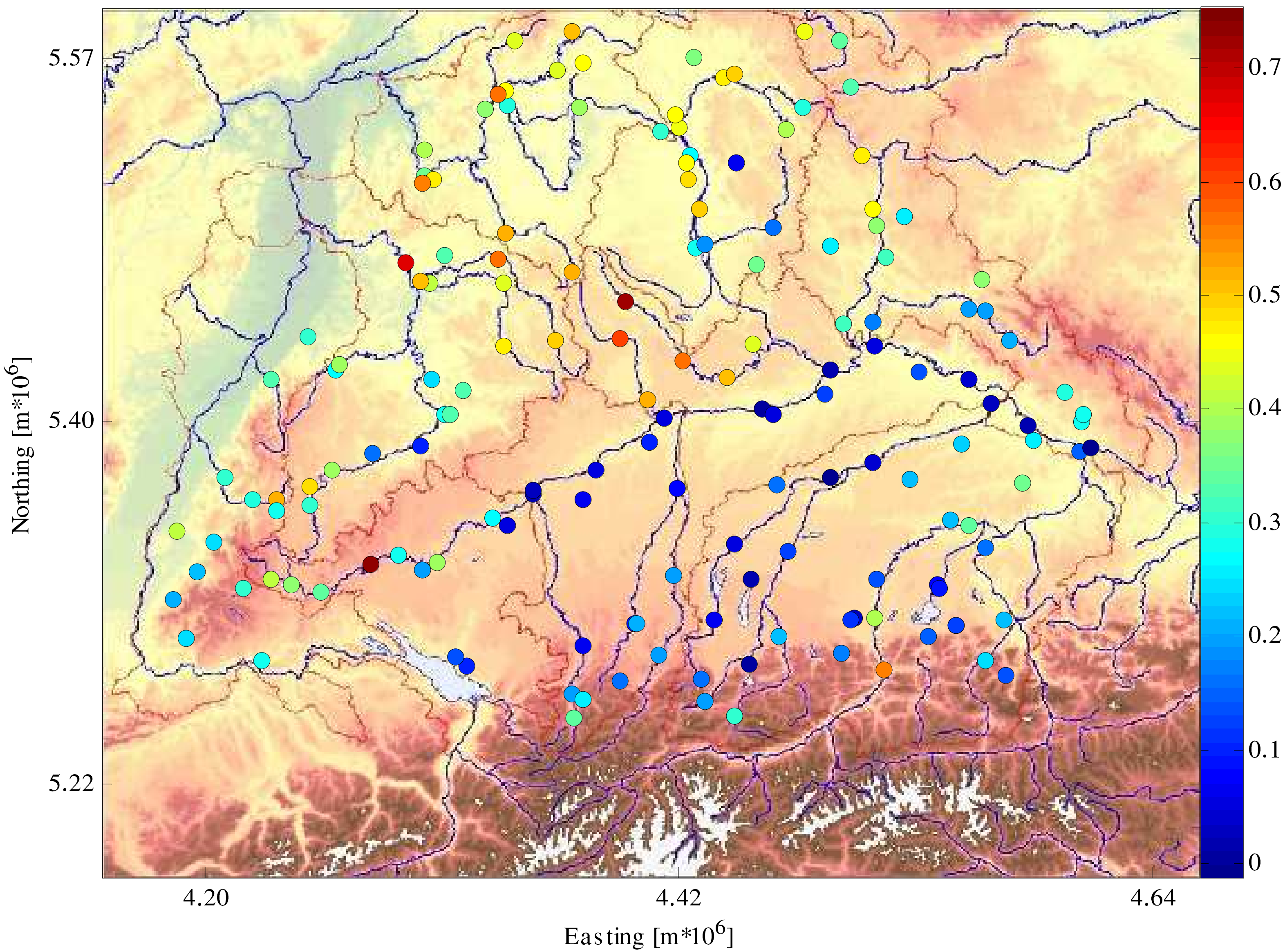
Return Intervalls

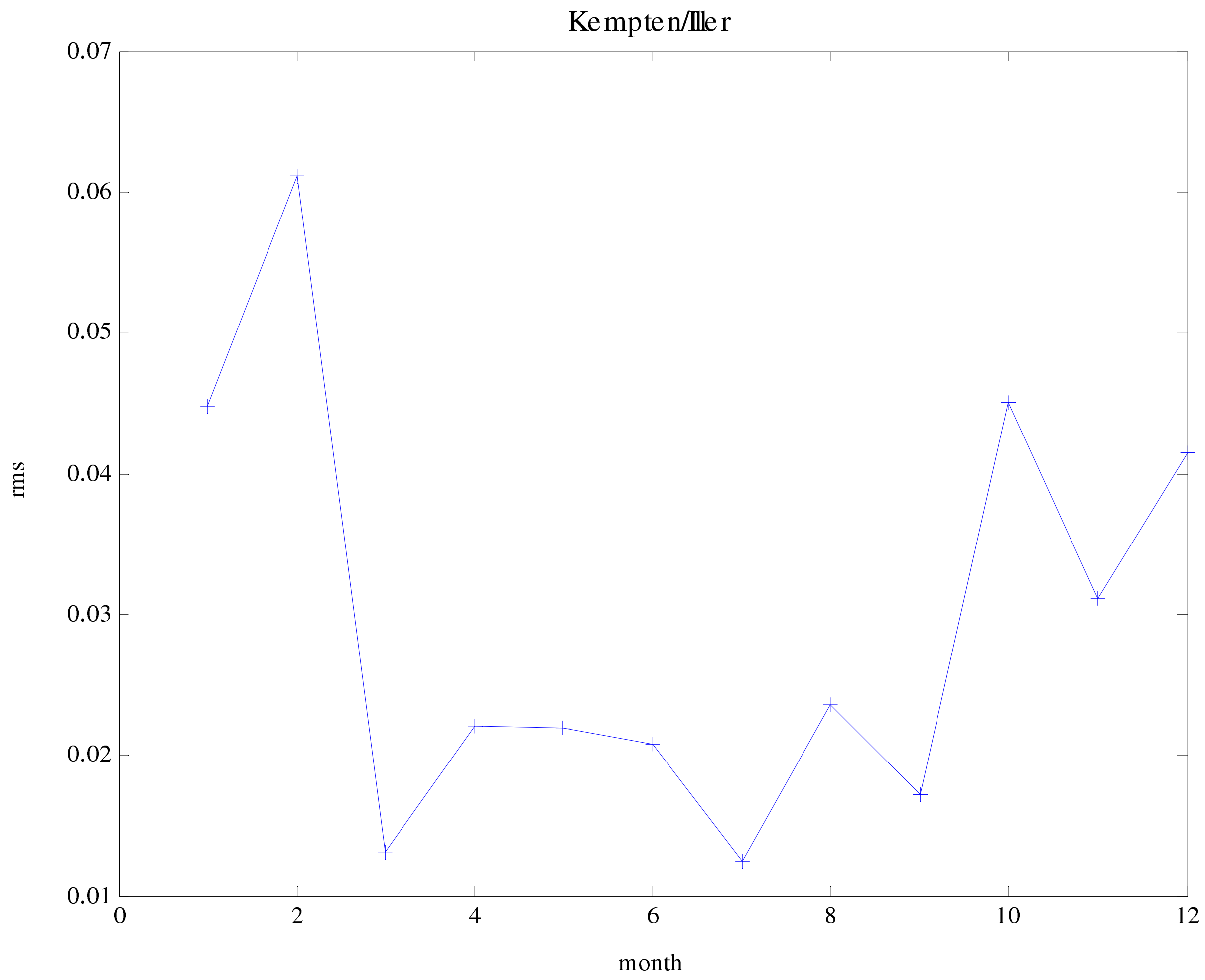


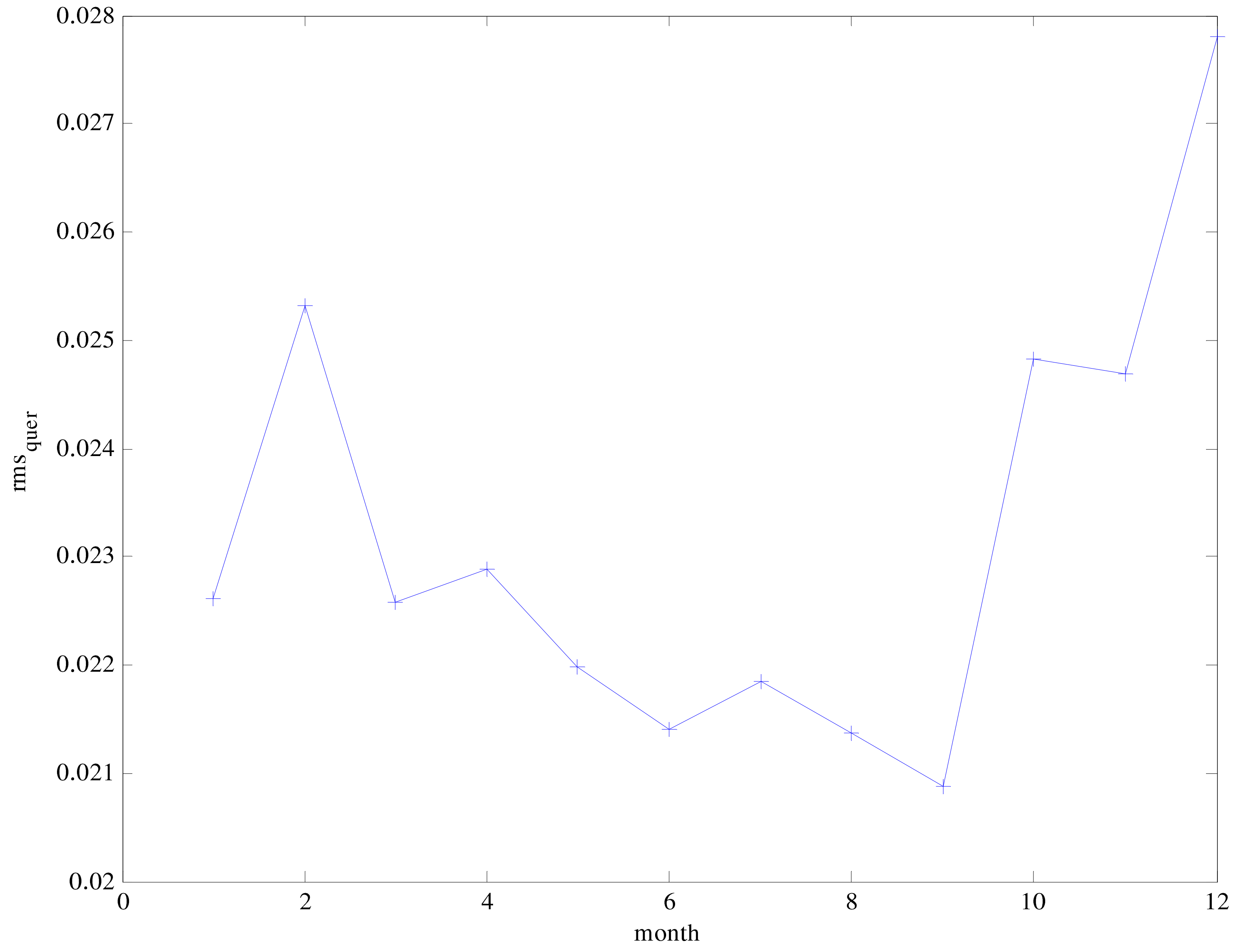
Overview of the Fitted GEV - Parameters

	monthly means			**monthly maxima**			**annual maxima**		
	μ_e	σ_e	ξ_e	μ_e	σ_e	ξ_e	μ_e	σ_e	ξ_e
maximum	1 200	430	0,75	1 700	720	1,3	3 500	750	0,78
mean	42	18	0,28	65	36	0,53	190	66	0,046
minimum	0,34	0,21	-0,012	0,62	0,60	0,033	3.9	2,0	-0,34

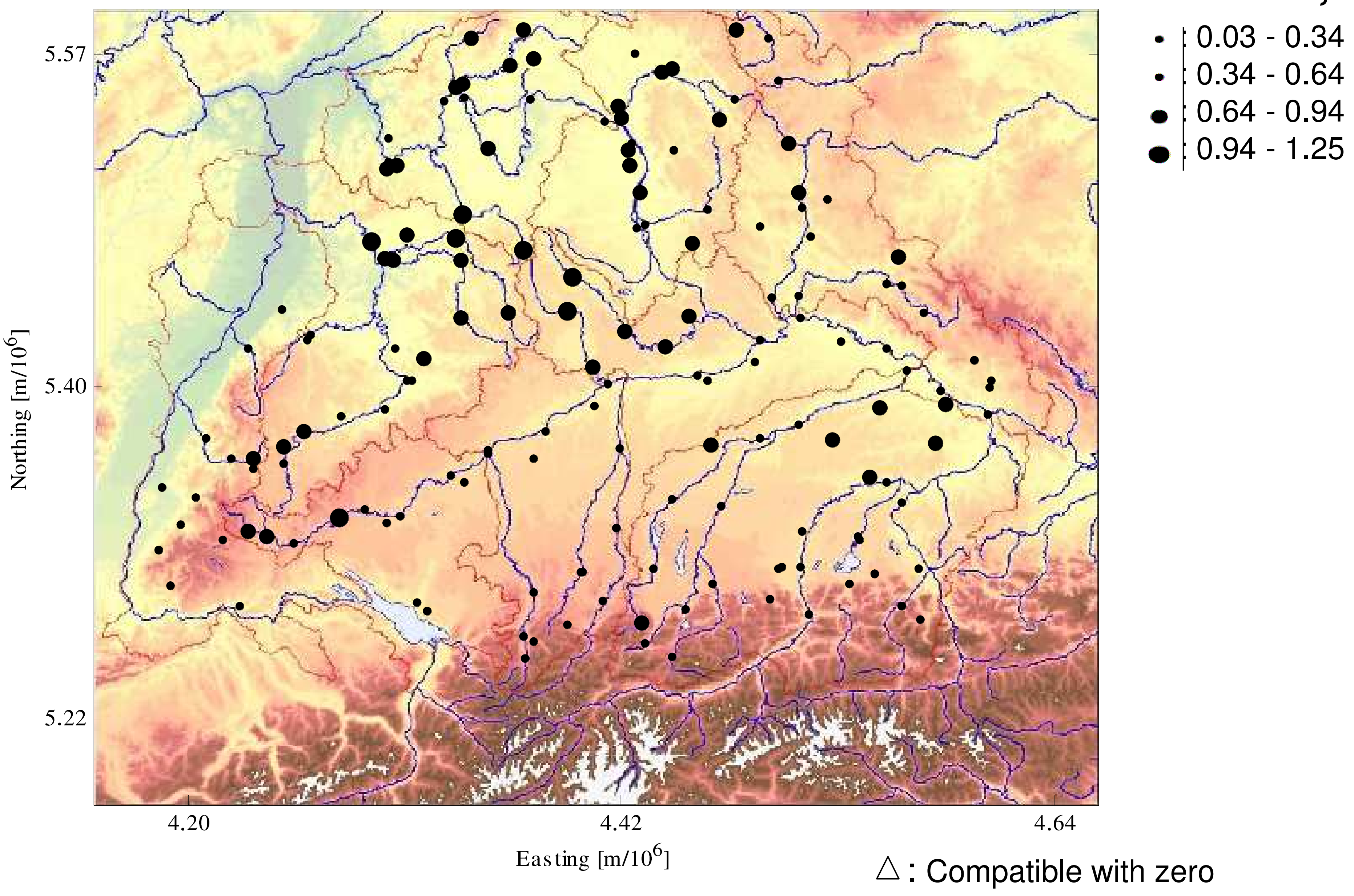
Spatial distribution of ξ: Montly means



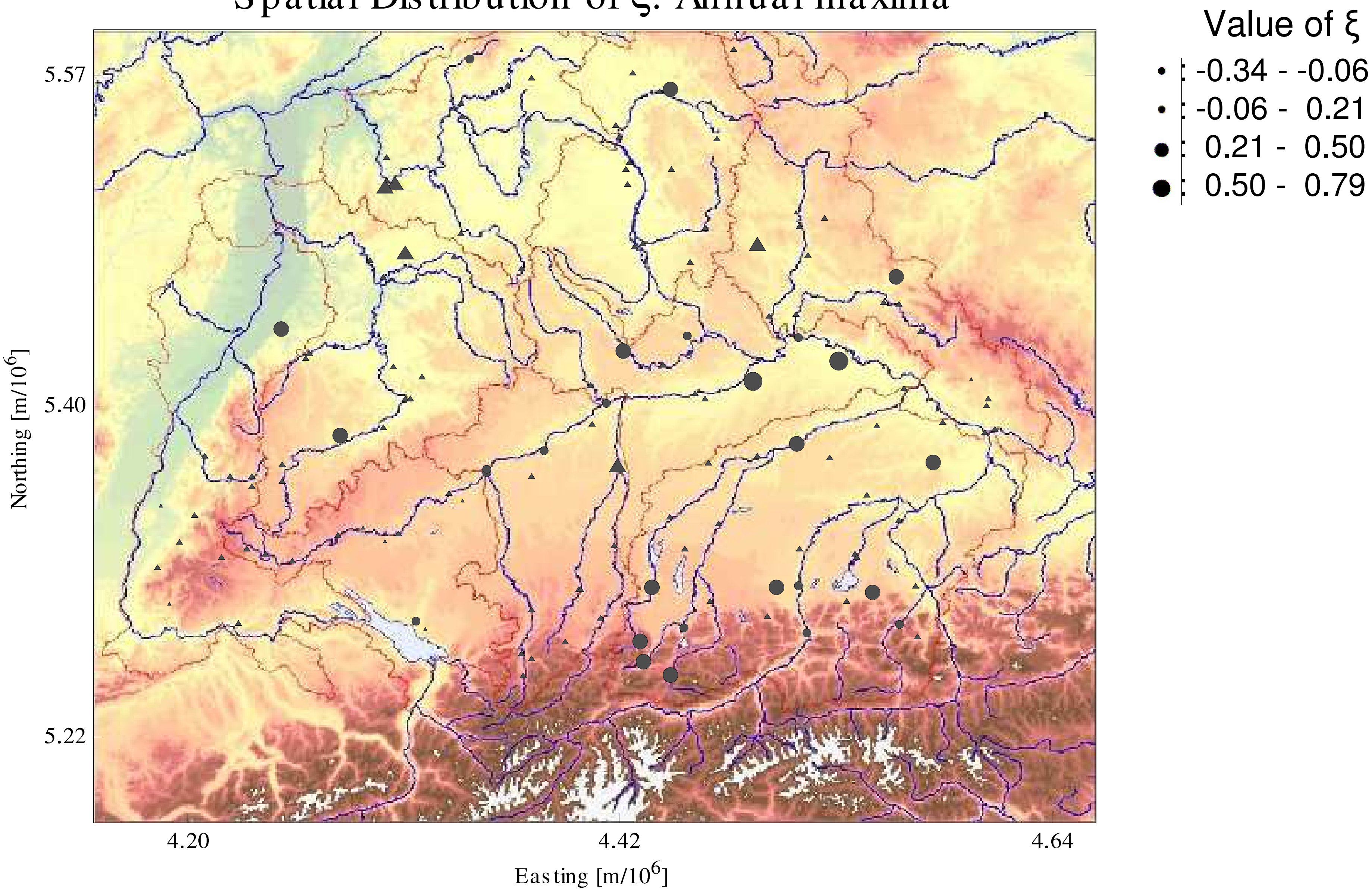

Kempten/Iller
rms
month
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2
4
6
8
10
12


0.028
0.027
0.026
0.025
0.024
0.023
0.022
0.021
0.02
rms_quer
0
2
4
6
8
10
12
month

Spatial Distribution of ξ: Monthly maxima



Spatial Distribution of ξ: Annual maxima



△ : Compatible with zero

Separate Months