Understanding agent-based models of barter economies through specifications

- Applied mathematics, physics: implicit specifications
- Computational economics: narrative, diagrams
- Specification framework for barter economy models
- Trade-driven stocks dynamics
- Prices evolution

Applied mathematics, physics: implicit specifications

$$\begin{aligned} \rho_t + \nabla \cdot (\rho \mathbf{v}) &= 0 \\ (\rho \mathbf{v})_t + \nabla \cdot (\rho \mathbf{v} \circ \mathbf{v}) + \nabla p &= 0 \\ (\rho e)_t + \nabla \cdot ((\rho e + p)\mathbf{v}) &= 0 \end{aligned}$$

- Implicit *problem* specification:
  - $\rho \in \Omega \times \mathbb{R} \to \mathbb{R}_{>0}$, $\rho \mathbf{v} \in \ldots$
  - $\Omega \subset \mathbb{R}^n$
  - Given initial ... on Ω and boundary conditions ..., find ...
- Theories about problem and computer-based approximations of its solutions:
  - Existence, uniqueness, stability ...
  - Analytic solutions for particular problems ...
  - Notions of LTE, accuracy, consistence, stability, convergence of approximations ...

Computational economics: narrative, diagrams

- from *EURACE: A Massively Parallel Agent-Based Model of the European Economy; C. Deissenberg, S. van der Hoog, H. Dawid; Applied Mathematics and Computation, 204, 2, 541–552*, page 545:

  ...

  The market for consumption goods is a decentralized market, with local interaction between the firms and consumers. We assume that the firms send their merchandise to a given set of local shopping malls. All buying and selling occurs at these malls. Firms chose the outlet malls on the basis of expected local demand and profit opportunities. They also take into account the costs involved in servicing a particular mall, such as the transportation costs, the leases for the stores in the mall, and the inventory management costs.

  ...

Computational economics: narrative, diagrams

- from *EURACE: A Massively Parallel Agent-Based Model of the European Economy"; C. Deissenberg, S. van der Hoog, H. Dawid; Applied Mathematics and Computation, 204, 2, 541–552*, page 546:

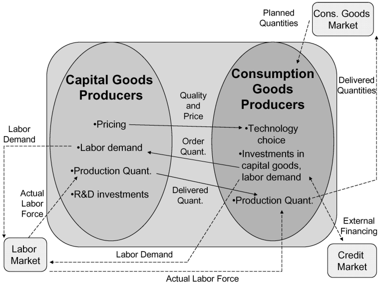

Fig. 1. Interactions between the investment goods producers and the consumption goods producers on the market for capital goods. Arrows show the messages between agents.

Specification framework for barter economy models

- Framework = notation + notions
  - Notation is math + functional and domain independent
  - Notions are domain specific
- Economic equilibrium
- Dynamic barter economies
  - Basic assumptions
  - System's state
  - Transition function
  - Demand function

Economic equilibrium (BE statics, reallocation problem)

- n_G goods, n_A agents
- Given
  - initial stocks $x_0 \in \mathbb{R}_{\geq 0}^{n_G \times n_A}$,
  - utility profile $u \in [\,0, n_A\,) \to \mathbb{R}_{\geq 0}^{n_G} \to \mathbb{R}$,
- stocks $x \in \mathbb{R}_{\geq 0}^{n_G \times n_A}$ are in equilibrium with x_0, u if there are prices $p \in \mathbb{R}_{>0}^{n_G}$ such that

$$\forall\ j \in [\,0, n_G\,) \quad \sum_{i=0}^{i<n_A} x\,i\,j = \sum_{i=0}^{i<n_A} x_0\,i\,j$$

$$\forall\ i \in [\,0, n_A\,) \quad x\,i \in \underset{y \cdot p\ \leq\ (x_0\,i) \cdot p}{\operatorname{argmax}} \quad u\,i\,y$$

Economic equilibrium (statics of BEs, reallocation problem)

- GE theory provides sufficient conditions for equilibrium:
  - Existence under fairly general conditions (Debreu '59)
  - Uniqueness requires restricting conditions (Sonnenschein, Mantel, Debreu '73-'74)
- Ideal utility: every p yields equilibrium stocks (Scarf '60):

$$u\, i\, y = \min_{j \in [0, n_G)} (y\, j) \;/\; (w\, j)$$

$$w = \lambda \left(\sum_{i=0}^{i<n_A} x_0\, i \right) \;\wedge\; \lambda \in \mathbb{R}_{>0}$$

$$x\, i = \frac{(x_0\, i) \cdot p}{w \cdot p} w$$

Dynamic barter economies: basic assumptions

- Discrete time $t \in \mathbb{N}$.
- The system's state is given at $t = 0$. For $t > 0$, it is computed by iterating a *transition function*.
- Transition functions take as arguments the present system's state, a random variable but neither time nor the next state.
- Transition functions are defined in terms of sequences of *elementary* transitions.
- Elementary transitions represent idealized economic processes: goods *trading*, *production*, *consumption*, involving a (small) number of agents independent of n_A (social networks, imperfect knowledge).
- GE anchoring: dynamic models of exchange should converge, for constant prices and ideal utility, towards equilibrium stocks.

Dynamic barter economies: system's state

- Stocks $x \in \mathbb{N} \to \mathbb{R}_{\geq 0}^{n_G \times n_A}$,
- prices $p \in \mathbb{R}_{>0}^{n_G \times n_A}$,
- offer (sector) good tag $g \in [\,0, n_G\,)^{n_A}$,
- utility profile $u \in [\,0, n_A\,) \to \mathbb{R}_{\geq 0}^{n_G} \to \mathbb{R}$,
- trades upper bound (node degree) $c \in \mathbb{N}^{n_G \times n_G}$.

Dynamic barter economies: transition function

$$x\,(t+1) \;=\; x\,t \;+\; f\,c\,u\,g\,p\,(x\,t)\,\omega$$

- f conservative $\;\equiv\; \sum_{i=0}^{i<n_A} (f\,c\,u\,g\,p\,y\,\omega)\;i \;=\; 0$
- f positivity preserving $\;\equiv\; y \geq 0 \;\Rightarrow\; y \;+\; f\,c\,u\,g\,p\,y\,\omega \geq 0$
- f value preserving $\;\equiv$
  $y' = y \;+\; f\,c\,u\,g\,p\,y\,\omega \;\Rightarrow\; (y'\,k)\cdot(p\,k) \;=\; (y\,k)\cdot(p\,k)$
- f monotonic $\;\equiv$
  $y' = y \;+\; f\,c\,u\,g\,p\,y\,\omega \;\Rightarrow\; y'i(g\,i) \;\leq\; yi(g\,i)$
  $y' = y \;+\; f\,c\,u\,g\,p\,y\,\omega \;\Rightarrow\; j \neq g\,i \;\Rightarrow\; y'ij \;\geq\; yij$

Dynamic barter economies: transition function

Transition functions are defined in terms of sequences of elementary transitions:

$$\begin{aligned} x\,(t+1) &= \; x\,t \;+\; f\,c\,u\,g\,p\,(x\,t)\,\omega \\ &= \; \mathrm{fold}\,(h\,u\,g\,p)\,(x\,t)\,(ts\,c\,\omega) \end{aligned}$$

- random trade schedule

  $ts\,c\,\omega \;\in\; List\;[\,0, n_A\,) \times [\,0, n_A\,)$
- elementary trade

  $h\,u\,g\,p \;\in\; \mathbb{R}_{\geq 0}^{n_G \times n_A} \to [\,0, n_A\,) \times [\,0, n_A\,) \to \mathbb{R}_{\geq 0}^{n_G \times n_A}$

Dynamic barter economies: transition function

We can *understand* the mechanics of an elementary trade by reading its *specification*: $\delta y \; = \; h\, u\, g\, p\, y\, (i, i') \; - \; y \; \Rightarrow$

- a trade between i and i' does not affect other agents:

  $\forall k \in [\, 0, n_A \,) \;\; k \neq i \;\wedge\; k \neq i' \;\Rightarrow\; \delta y\, k = 0$

- trades are exchanges of "offer" goods:

  $$\forall j \in [\, 0, n_G \,) \;\; j \neq g\, i \;\wedge\; j \neq g\, i' \;\Rightarrow\; \delta y\, i\, j = \delta y\, i'\, j = 0$$
  $$\delta y\, i\, (g\, i) \; = \; -\delta y\, i'\, (g\, i) \; \leq \; 0$$
  $$\delta y\, i\, (g\, i') \; = \; -\delta y\, i'\, (g\, i') \; \geq \; 0$$

- trades are win-win exchanges:

  $$(-\delta y\, i\, (g\, i)) \cdot (p\, i\, (g\, i)) \; \leq \; (\delta y\, i\, (g\, i')) \cdot (p\, i\, (g\, i'))$$
  $$(-\delta y\, i'\, (g\, i')) \cdot (p\, i'\, (g\, i')) \; \leq \; (\delta y\, i'\, (g\, i)) \cdot (p\, i'\, (g\, i))$$

- trades are budget constrained (positivity preserving):

  $$-\delta y\, i\, (g\, i) \; \leq \; y\, i\, (g\, i)$$
  $$-\delta y\, i'\, (g\, i') \; \leq \; y\, i'\, (g\, i')$$

Dynamic barter economies: demand function

Dynamic models of exchange should converge, for constant prices and ideal utility, towards equilibrium stocks:

- $d\ u\ g\ p\ y\ i\ j \ \geq\ 0$
- $y\ i \ \in \operatorname*{argmax}_{z\cdot(p\ i)\leq(y\ i)\cdot(p\ i)} u\ i\ z \ \Rightarrow\ d\ u\ g\ p\ y\ i\ =\ 0$
- natural demand (unique optimal stocks):

$$d\ u\ g\ p\ y\ i\ j = \begin{cases} 0 & \text{if } j = g\ i \\ \max 0 \left(\left(\operatorname*{argmax}_{z\cdot(p\ i)\leq(y\ i)\cdot(p\ i)} u\ i\ z \right) j - y\ i\ j \right) & \text{otherwise} \end{cases}$$

Trade-driven stocks dynamics: results

$$x\,(t+1) \;=\; \text{fold}\,(h\,u\,g\,p)\,(x\,t)\,(ts\,c\,\omega)$$

- constant prices, h demand-based cooperative trade

  $\Rightarrow$

  h stocks-value preserving, monotone

- h stocks-value preserving, monotone and natural demand

  $\Rightarrow$

  total demand non-increasing

- zero total demand $\Rightarrow$ stationary point
- zero total demand, rationing $\Rightarrow$ equilibrium
- no rationing $\rightarrow$ no convergence (towards zero total demand)

Trade-driven stocks dynamics: without rationing

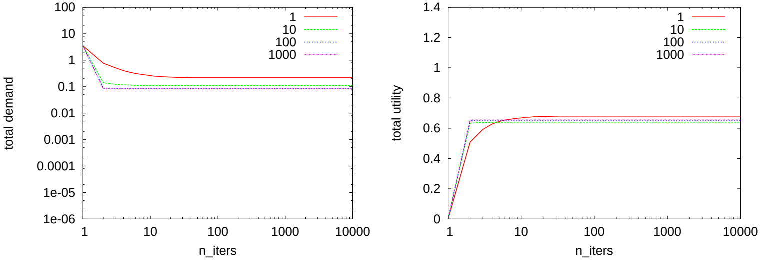

Figure: Elementary trade based transition function: total demand (left) and total utility (right) versus number of iterations for sector-to-sector trades upper bounds of 1, 10, 100 and 1000

Trade-driven stocks dynamics: with rationing

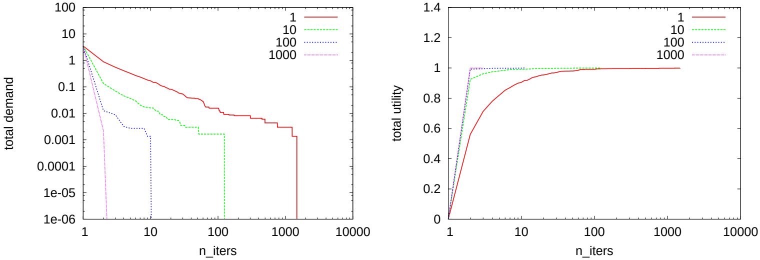

Figure: Elementary trade based transition function: total demand (left) and total utility (right) versus number of iterations for sector-to-sector trades upper bounds of 1, 10, 100 and 1000

Prices evolution: replicator dynamics, nested iteration

$$p\,0 = p_0,$$
$$p\,(\tau+1) = \text{fold}\,(cm\,u\,mp\,mf\;(x\,nt))\,(p\,\tau)\,(cms\,cf\;\omega)$$
$$x\,0 = x_0$$
$$x\,(t+1) \;=\; \text{fold}\,(h\,u\,g\,(p\,\tau))\,(x\,t)\,(ts\,c\,\omega)$$

- copy-mutate algorithm $cm\,u\,mp\,mf\;(x\,nt)$
- mutation probability $mp \in [0,1]$
- mutation factor $mf \in (0,1]$
- copy-mutate schedule $cms\,cf\;\omega \in List\;[\,0, n_A\,) \times [\,0, n_A\,)$
- copy fraction $cf \in [0,1]$

Conclusions

- In agent-based computational economics, specifications can partially compensate for the lack of model equations and of model theory. In particular, they can be useful to:
- understand, explain and share models,
- assist model implementation, refactoring, extension,
- design and setup "crucial" numerical experiments,
- interpret model output.