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# Towards a Formal Framework of Vulnerability to Climate Change.

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## F A V A I A

Formal Approaches to Vulnerability Assessment that Informs Adaptation

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## Abstract

There is confusion regarding the notion of “vulnerability” in the climate change scientific community. Recent research has identified a need for formalisation, which would support accurate communication and the elimination of misunderstandings that result from ambiguous interpretations. Moreover, a formal framework of vulnerability is a prerequisite for computational approaches to its assessment. This paper presents an attempt at developing such a formal framework. We see vulnerability as a relative concept, in the sense that accurate statements about vulnerability are possible only if one clearly specifies *(i)* the entity that is vulnerable, *(ii)* the stimulus to which it is vulnerable and *(iii)* the preference criteria to evaluate the outcome of the interaction between the entity and the stimulus. We relate the resulting framework to the IPCC conceptualisation of vulnerability and two recent vulnerability studies.

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# 1 Introduction

Over the past two decades a multitude of studies have been conducted aimed at understanding how climate change might affect a range of natural and social systems, and at identifying and evaluating options to respond to these effects. These studies have highlighted differences between systems in what is termed “vulnerability” to climate change between systems, although without necessarily defining this term. As shown by Füssel and Klein [1], the meaning of vulnerability within the context of climate change has evolved over time. In the Third Assessment Report of the Intergovernmental Panel on Climate Change (IPCC), vulnerability to climate change was described as “a function of the character, magnitude and rate of climate variation to which a system is exposed, its sensitivity and its adaptive capacity” [2, p. 995]. Straightforward as it may seem, this conceptualisation of vulnerability has proven difficult to make operational in vulnerability assessment studies. Moreover, it appears to be at odds with those conceptualisations developed and used outside the climate change community.

It is increasingly argued that many climate change vulnerability studies, whilst effective in alerting policymakers to the potential consequences of climate change, have had limited usefulness in providing local guidance on adaptation [*e.g.*, 3] and that the climate change community could benefit from experiences gained in food security and natural hazards studies [4]. As a result, the climate change community is currently engaged in a process of analysing the meaning of vulnerability and redefining it such that assessment results would be more meaningful to those wishing to reduce vulnerability. The clearest preliminary conclusion reached to date is that there is much confusion.

We wish to take up the challenge put forward by O’Brien *et al.* [5], who suggested that “one way of resolving the prevailing confusion is to make the differing interpretations of vulnerability more explicit in future assessments, including the IPCC Fourth Assessment Report” (p. 12). We feel that there is an opportunity and indeed a need to cast the discussion in a formal, mathematical framework. By a “formal framework” we mean a framework that defines vulnerability using mathematical concepts that are independent of any knowledge domain and applicable to any system under consideration. The use of mathematics also avoids some of the limitations of natural language. For example, natural language can obscure ambiguities or circularities in definitions [*e.g.*, 6].

Inspired by conceptual work done by, amongst others, Jones [7], Brooks [8], Luers *et al.* [9], Turner *et al.* [10], Jaeger [11] and Downing *et al.* [12] and using vulnerability to climate change as our point of departure, we see this paper as taking a first step towards instilling some rigour and consistency into vulnerability assessment. Our motivation for developing a formal framework of vulnerability is fourfold:

- A formal framework can help to ensure that the process of examining, interpreting and representing vulnerability is carried out in a systematic fashion, thus limiting the potential for analytical inconsistencies.
- A formal framework can improve the clarity of communication of methods and results of vulnerability assessments, thus avoiding misunderstandings amongst researchers and between researchers and stakeholders (especially if the language used for communication is not the native one of all involved).
- By encouraging assessments to be systematic and communication to be clear, a formal

framework will help users of assessment results to detect and resolve any inaccuracies and omissions.

- A formal framework is a precondition for any computational approaches to assessing vulnerability and will allow modellers to take advantage of relevant methods in applied mathematics, such as systems theory and game theory.

We are well aware of two important caveats involved in developing a formal framework of vulnerability. First, the framework could be perceived as being prescriptive, limiting the freedom and creativity of researchers to generate and pursue their own ideas on vulnerability. Second, it could be seen as being developed for illicit rhetorical purposes, namely to throw sand in the eyes of those unfamiliar with mathematical notation. In spite of these caveats, we hope that the development of a formal framework of vulnerability will turn out to be a worthy undertaking, offering an opportunity for rigorous interdisciplinary research that can have important academic and social impacts. If this opportunity is seized by many, the risks represented by the two caveats will be minimised.

This paper is organised as follows. Section 2 investigates the “grammar” of vulnerability for three cases of increasing complexity; it extracts the building blocks for the actual formalisation of vulnerability and related concepts, which is presented in Section 3. Section 4 relates the framework thus developed to the approach taken to vulnerability assessment by the IPCC and in two recent studies. Section 5 presents conclusions and recommendations for future work.

## 2 Grammatical investigation

Before analysing the current technical usage of the concept of vulnerability within the climate change community, we start with an analysis of the everyday meaning of the word. The reason for this is that we consider it likely that the technical usage represents a refinement of the everyday one and that, similarly, we could start with definitions that capture the more general meaning of vulnerability and refine them in order to represent the technical meaning.

### 2.1 Oxford Dictionary of English

The latest edition of the Oxford Dictionary of English gives the following definition of “vulnerable” [13, p. 1977]:

1. Exposed to the possibility of being attacked or harmed, either physically or emotionally,
2. Bridge (of a partnership): liable to higher penalties, either by convention or through having won one game towards a rubber.

The Oxford Dictionary of English provides the following example sentence with the first definition: “Small fish are vulnerable to predators”.

It follows from the definitions and the example sentence that vulnerability is a relative property: it is vulnerability *to* something. In addition, both the definitions and the example sentence make it clear that vulnerability has a negative connotation and therefore presupposes a notion

of “bad” and “good”, or at least “worse” and “better”. It also follows that vulnerability refers to a potential event (of, for example, being harmed), and not to the realisation of this event.

The example sentence has been formalised as the Lotka-Volterra predator-prey model [14, 15]. We interpret the statement as expressing a relation between a system of small fish exposed to predators. The system of small fish has no endogenous controls: it does not incorporate feedbacks that would represent the lessons learnt by small fish from earlier encounters with predators. It is therefore less complex than systems studied in relation to climate change, where emphasis is placed on the possibilities of adaptation beyond simple reaction, which systems either have or can develop.

## 2.2 Vulnerability in context: a non-climate example

The small fish in the Oxford Dictionary of English have no means of defending themselves against predators. They cannot respond in any effective way once they realise that a predator has chosen them for lunch, with fatal consequences. Many natural and human systems, however, will be able to react to imminent threats or experience non-fatal consequences. Consider a motorcyclist riding his motorcycle on a winding mountain road, with the mountain to his left and a deep cliff to his right. There is no other traffic, but unbeknownst to the motorcyclist an oil spill covers part of the road ahead of him, just behind a left-hand curve.

In natural language we would say that the oil spill represents a hazard and that the motorcyclist is at risk of falling down the cliff and being killed. Expanding on the example sentence of the Oxford Dictionary of English, we would consider that the motorcyclist is vulnerable to the oil spill with respect to falling down the cliff and being killed.

We would normally say that a second motorcyclist who drives more slowly or more carefully is less vulnerable to the oil spill. One challenge of formalising vulnerability is to account for such comparative statements.

The situation may be considerably more complex if we expand the time horizon. What about a third motorcyclist, who has heard about the oil spill on the road and has been able to prepare for it by buying new tires and improving his driving skills? Can his condition be meaningfully compared to that of the first two, who are confronted with an immediate hazard? What about a fourth motorcyclist, who has been informed but has no money to buy new tires?

As the remainder of this paper will show, vulnerability to climate change operates on all these different time scales, and introduces new aspects, such as the ability of the vulnerable entity to act proactively to avoid future hazards (by mitigating climate change or by enhancing adaptive capacity).

## 2.3 Vulnerability to climate change

Climate change will affect many groups and sectors in society, but different groups and sectors will be affected differently, for three important reasons. First, the direct effects of climate change will be different in different locations. Climate models project greater warming at high latitudes than in the tropics, sea-level rise will not be uniform around the globe and precipitation patterns will shift such that some regions will experience more intense rainfall, other regions more prolonged dry periods and again other regions both.

Second, there are differences between regions and between groups and sectors in society, which determine the relative importance of such direct effects of climate change. More intense rainfall in some regions may harm nobody; in other regions it could lead to devastating floods. Increased heat stress can be a minor inconvenience to young people; to the elderly it can be fatal. Extratropical storms can kill tens of thousands of people in South Asia; in the United States they lead to thousands of millions of dollars worth of damage.

Third, there are differences in the extent to which regions, groups and sectors are able to prepare for, respond to or otherwise address the effects of climate change. When faced with the prospect of more frequent droughts, some farmers will be able to invest in irrigation technology; others may not be able to afford such technology, lack the skills to operate it or have insufficient knowledge to make an informed decision. The countries around the North Sea have in place advanced technological and institutional systems that enable them to respond proactively to sea-level rise; small island states in the South Pacific may lack the resources to avoid impacts on their land, their people and their livelihoods.

As stated before, the IPCC Third Assessment Report described vulnerability to climate change as “a function of the character, magnitude and rate of climate variation to which a system is exposed, its sensitivity and its adaptive capacity” [2, p. 995]. This is consistent with the explanation above as to why different groups and sectors will be affected differently by climate change. Differences in exposure to the various direct effects of climate change (*e.g.*, changes in temperature, sea level and precipitation) and different sensitivities to these direct effects lead to different potential impacts on the system of interest. The system’s adaptive capacity (*i.e.*, the ability of a system to adjust to climate change to moderate potential damages, to take advantage of opportunities or to cope with the consequences [2, p. 982]) then determines its vulnerability to these potential impacts. These relationships are made visible in Figure 1.

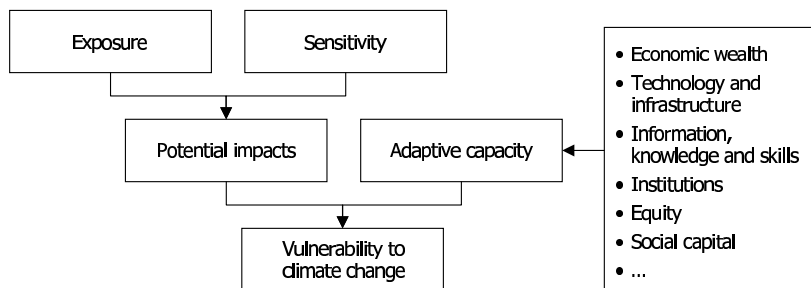


Figure 1: Conceptualisation of vulnerability to climate change in the IPCC Third Assessment Report.

### 3 Formalisation of vulnerability and related concepts

An important result of the grammatical investigation is that the concept of vulnerability is a relative one: it is the vulnerability of an *entity* to a specific *stimulus* and with respect to certain *preference criteria*. We would expect any formalisation of vulnerability to represent these three primitives, which leads us to looking at the way the concepts of “entity”, “stimulus” and “preference criteria” can themselves be formalised. In addition, more complex notions, such as “adaptive capacity”, will require the formalisation of the entity’s ability to act.

| Symbol                | Meaning  |
|-----------------------|--|
| $f(x) = y$            | The value of $f$ for the element $x$ is $y$  |
| $f : X \rightarrow Y$ | $f$ is a function defined on the set $X$ and taking values into the set $Y$                                  |
| $x^k$                 | The value of $x$ at the (discrete) time step $k$   |
| $x \preceq y$         | $x$ is worse than $y$ , or $x$ and $y$ are equally bad   |
| $x \prec y$           | $x$ is worse than $y$  |
| iff                   | if and only if   |
| $\text{id}_X$         | The identity function, <i>i.e.</i> , $\text{id}_X(x) = x$ for all $x \in X$                                  |
| $x \in X$             | $x$ is an element of the set $X$   |
| $X \times Y$          | The set of ordered pairs having as first element an element of $X$ and as second element an element of $Y$   |
| $X + Y$               | The set of ordered pairs $(x, 0)$ and $(y, 1)$ , where $x$ is an element of $X$ and $y$ is an element of $Y$ |
| $\{x \mid \dots\}$    | The set of all $x$ that satisfy ...  |
| $\exists x : \dots$   | There exists at least one $x$ such that ...  |
| $\forall x : \dots$   | For all $x$ such that ...  |

Table 1: Mathematical symbols and their meanings.

This section presents a stepwise formalisation of vulnerability. It uses mathematical notation, with which not every reader may be familiar. For those readers Table 1 explains the mathematical symbols used in this section in the order in which they appear.

### 3.1 System with exogenous input

The mainstream mathematical interpretation of an entity is that of a dynamical system in a given state. This is the interpretation we will adopt here. The stimuli to which such a system can be subjected are then naturally represented by the exogenous input and the preference criteria by a preorder on the set of outputs.

The particular kind of dynamical system we consider is the simplest possible one that can serve our needs: a discrete, deterministic system. That does not mean that our framework will be limited to this simple (and somewhat simplistic) case. Formally it is easy to extend the framework to deal with continuous-time, non-deterministic or stochastic systems.

The system that we consider evolves sequentially in time according to:

$$x^{k+1} = f(x^k, e^k) \quad (1)$$

where

- $f : X \times E \rightarrow X$  is called the *transition function* of the system
- $X$  is the set of states of the system
- $E$  is the set of exogenous inputs to the system
- $k$  is the time step (we consider a discrete system).

The system is considered as generating output depending on its state, according to an *output function*  $g$ :

$$y^k = g(x^k) \quad (2)$$

where

$$g : X \rightarrow Y$$

$Y$  is the set of outputs.

These outputs can be thought of as indicators of the state and are in general considered measurable or observable quantities.

Where no possibility of confusion arises, we denote a system such as the above by the pair  $(f, g)$  of the transition and the output functions.

We denote the exogenous input that represents the stimuli by  $e$ , which is an element of  $E$ , and the preorder on the set  $Y$  of outputs by  $\preceq$ . A preorder is a relation that is reflexive (for all  $x$  we have that  $x \preceq x$ ) and transitive (for all  $x, y, z$ : if  $x \preceq y$  and  $y \preceq z$ , then  $x \preceq z$ ). We will use  $\prec$  to denote the corresponding strict preorder, defined as:  $y \prec y'$  iff  $y \preceq y'$  and *not* ( $y' \preceq y$ ).

We now have a framework that is in principle capable of dealing with the three primitives, but what about vulnerability itself? We are looking for something that can capture the intuitive meaning of “the given system in state  $x^k$  is vulnerable to exogenous input  $e^k$  with respect to the preorder  $\preceq$  if...”. When the system is in state  $x^k$  and is subjected to  $e^k$ , it will undergo a transition to  $x^{k+1}$ , as given by the transition function in (1). We can measure  $y^{k+1}$ , which can be worse, the same, better or not comparable to  $y^k$ . Since we have seen that “vulnerability” is used only with negative connotations, the following definition has a certain inevitability:

**Definition 1.**

A system  $(f, g)$  in state  $x^k$  is vulnerable to an exogenous input  $e^k$  with respect to preorder  $\preceq$  iff  $y^{k+1} \prec y^k$ .

**Example 1.** The example sentence from the Oxford Dictionary of English can be understood as referring to the vulnerability of the system of small fish in a given environment. In a simple Lotka-Volterra model, the state of this system would be given by the number of small fish present. The output of the system would, in this case, be exactly the same as the state (mathematically  $g = \text{id}_X$ ). We might consider preference criteria based on a threshold value  $N$ , as follows:

$$y \prec y' \quad \text{iff} \quad y < N \quad \text{and} \quad y' > N,$$

that is, the number of fish representing the state  $y$  is below threshold  $N$ , whilst the number of fish representing  $y'$  is above  $N$ . A given exogenous input would be represented by the number of predators in the same environment. The system in a given state is vulnerable to that input if the number of small fish were to decrease below the threshold.

We are interested in the vulnerability of an entity because we would like to make the entity less vulnerable or because we want to compare it to the vulnerability of another entity. In other words, we must be able to compare different vulnerabilities, be they of different entities or of the same entity but under different circumstances.

The latter situation is captured by a system in two different states, so we can define:

**Definition 2.**

A system  $(f, g)$  is more vulnerable in state  $x^k$  than in state  $x'^k$  to an exogenous input  $e^k$  with respect to preorder  $\preceq$  iff:

- i) the system in  $x^k$  is vulnerable to  $e^k$  with respect to  $\preceq$
- ii)  $y^{k+1} \preceq y'^{k+1}$ .

The reason we have  $\preceq$  instead of  $\prec$  in the second condition of this definition is that we want to declare a system more vulnerable in state  $x^k$  than in state  $x'^k$  if it is vulnerable (according to Definition 1) in  $x^k$  and not vulnerable in  $x'^k$ , even if the final outcomes,  $y^{k+1}$  and  $y'^{k+1}$  are in fact identical. This expresses the intuitive idea that a rich man is more vulnerable to losing large sums of money than a poor man.

If  $y^{k+1} \prec y'^{k+1}$  we say that the system is *strictly more vulnerable* in state  $x^k$  than in state  $x'^k$ .

One unsatisfactory aspect of this definition is that it requires the exogenous input to be *exactly* the same when applied to each of the two states. We can imagine many situations in which we are interested in the relative vulnerability of two entities in two different states, subjected to very similar or equivalent input. To be able to handle this situation, we need to define precisely what we mean by “equivalent”: in general, this will require the introduction of an equivalence (or tolerance) relation on the set  $E$ . Techniques for doing so are well known in mathematics [e.g., 16].

**Example 2.** Given the system in Example 1, the system of small fish is strictly more vulnerable to a given number of predators in one state than in another state if the number of small fish in the first state decreases below the considered threshold (i.e., it is vulnerable), whilst in the second state the number of small fish stays above the threshold (i.e., it is not vulnerable).

How can we compare the relative vulnerability of two different systems? A possible approach is given in the following:

**Definition 3.**

A system  $(f, g)$  in state  $x^k$  is more vulnerable to an exogenous input  $e^k$  than a system  $(f', g')$  in state  $x'^k$  is to an exogenous input  $e'^k$  with respect to  $\preceq$  iff:

- i) it is vulnerable to  $e^k$  with respect to  $\prec$
- ii)  $y^{k+1} \preceq y'^{k+1}$ .

This means that we need to compare outputs that belong to different sets. We therefore need to choose a preorder on the direct sum of the output sets  $(Y + Y')$ , which is compatible with the preorders on  $Y$  and  $Y'$ . Definition 2 is obtained as a special case for  $Y = Y'$ ,  $f = f'$ ,  $g = g'$  and  $e^k = e'^k$ . We obtain another typical situation by considering the case when  $x^k = x'^k$ ,  $f = f'$  and  $g = g'$ . This is a simplified representation of a scenario-based vulnerability assessment.

As an aside, the relation “more vulnerable than” is itself a preorder.

**Example 3.** We compare two systems of small fish, assuming that we have defined a preorder as in Example 1 for each system (the thresholds may be different). We define a preorder on the set  $Y + Y'$  in the following way:

$$\begin{aligned}
 (y, 0) \preceq (y', 0) & \text{ iff } y \preceq y' \\
 (y, 1) \preceq (y', 1) & \text{ iff } y \preceq y' \\
 (y, 0) \preceq (y', 1) & \text{ iff } y \text{ is below threshold } t \text{ and } y' \text{ is above } t' \\
 (y, 1) \preceq (y', 0) & \text{ iff } y \text{ is below threshold } t' \text{ and } y' \text{ is above } t.
 \end{aligned}$$

Here a pair  $(y, 0)$  should be read as “value  $y$  taken from set  $Y$ ” and  $(y, 1)$  as “value  $y$  taken from set  $Y'$ ”.

### 3.2 System with endogenous input

Definitions 1 to 3 are sufficient to capture some important aspects of the concept of vulnerability, as used in everyday language. As we have seen, they can be used to give an account of the fish-predators example. However, they are too general to represent adequately the use of the concept of vulnerability in the climate change community. In particular, it is not clear how to interpret terms such as “potential impacts” or “adaptive capacity” from the IPCC conceptualisation (*cf.* Section 2.3). To overcome this limitation, we need to extend our system by including an endogenous input along with the exogenous one, which means that we now have:

$$x^{k+1} = f(x^k, e^k, u^k) \quad (3)$$

where  $u^k$  is an element of  $U$ , the set of available endogenous inputs. The transition function  $f$  will, in general, be *partial*: not all actions are possible at a given point in time. We denote the set of actions that are available at time step  $k$  by  $U^k$ .

The endogenous input represents the response of the system to the exogenous input. We will refer to it alternatively as actions or commands of the system. In most applications the set  $U$  will contain a “do nothing” action.

**Example 4.** The first two motorcyclists in Section 2.2 can be modelled as discrete dynamical systems in the following way: the state will contain all (and only) those physical variables necessary for the specification of the transition function. The endogenous input is a representation of the manoeuvres the motorcyclist can make when confronted with the oil spill. The exogenous input is further represented by the immediate conditions of the road. The transition function will then return the state of the motorcyclist after the encounter with the oil spill. As output we can distinguish between the two end-points of being on the road or down the cliff, and we can consider our preorder to be such that  $y \prec y'$  iff  $y$  represents being down the cliff and  $y'$  represents being on the road.

Formally, in the absence of more information (such as a policy  $\phi$  that would tell us which actions to take under given circumstances, so that we had a direct link  $u^k = \phi(x^k, e^k)$ ; see Section 3.4), the introduction of the command has turned our system into a non-deterministic one. Definition 2 still applies to the non-deterministic system, except that now our states are subsets of  $X$ , and the preorder is between subsets of  $Y$ , as shown by the next example.

**Example 5.** Consider two possible states of the motorcyclist in Example 4:  $x^k$  and  $x'^k$ . If we do not specify the action taken by the motorcyclist in the respective states, all we can do is compute the possible sets of outcomes  $Y^{k+1}$  and  $Y'^{k+1}$ .  $Y^{k+1}$ , for instance, is the set obtained by taking each available endogenous input  $u^k$  and computing the associated value  $g(f(x^k, e^k, u^k))$ :

$$Y^{k+1} = \left\{ y^{k+1} \mid \exists u^k \in U : y^{k+1} = g(f(x^k, e^k, u^k)) \right\}. \quad (4)$$

In (other) words,  $Y^{k+1}$  is obtained by taking every available action  $u^k$ , calculating with it a possible next state  $x^{k+1} = f(x^k, e^k, u^k)$  and adding to the set the value of the output function  $g$  for this state. The same is done for  $Y'^{k+1}$ .

We then need to compare these sets  $Y^{k+1}$  and  $Y'^{k+1}$  with values, because we have the initial outputs  $y^k$  and  $y'^k$ . We can take  $Y^k$  and  $Y'^k$  as being the single element sets  $\{y^k\}$  and  $\{y'^k\}$ ,

respectively. Assuming that all sets in question have minimal and maximal elements, we obtain the following. Taking as a preorder

$$Y_1 \preceq Y_2 \quad \text{iff} \quad \min Y_1 \preceq \min Y_2$$

(a maximin strategy), we have that the motorcyclist is more vulnerable in one state than in another one if the worst manoeuvre he can make in the first state leads to a worse outcome than the worst he can make in the other state. A more optimistic preorder would be the maximax strategy

$$Y_1 \preceq Y_2 \quad \text{iff} \quad \max Y_1 \preceq \max Y_2,$$

that is, he is more vulnerable in a state if the best possible manoeuvre leads to a worse outcome than the best possible manoeuvre in the other state.

We can now tackle the definitions of hazards and potential impacts.

**Definition 4.**

An exogenous input  $e^k$  is a hazard for a system  $(f, g)$  in state  $x^k$  with respect to  $\preceq$  iff there exists an action  $u^k \in U^k$  such that  $y^{k+1} \prec y^k$ . In this case,  $y^{k+1}$  is called a potential impact.

**Example 6.** The oil spill is a hazard for the motorcyclist if there is a possibility of the motorcyclist making a manoeuvre that results in him falling down the cliff.

**Definition 5.**

An exogenous input  $e^k$  is an unavoidable hazard for a system  $(f, g)$  in state  $x^k$  with respect to  $\preceq$  iff  $\forall u_k \in U^k : y^{k+1} \prec y^k$ .

**Example 7.** The oil spill is an unavoidable hazard for the motorcyclist if, no matter what manoeuvre the motorcyclist attempts, he would still end up falling down the cliff.

As a final remark, “risk” is usually defined as a measure of the set of potential impacts. This measure could be, for example, the sum of damages associated with the potential impacts weighted by their respective probabilities. In the case of the first motorcyclist, the risk could be taken as the probability of falling down the cliff. If, however, we consider the possible outcomes to be injuries and damage instead of alive or dead, we could take the risk as being the expected value of the injuries and damage (the damage weighted by the probabilities).

### 3.3 Adaptive capacity

Given a system in state  $x^k$ , subjected to an exogenous input  $e^k$ , we can define a number of problems:

a) **Optimisation**

choose an action  $u^k \in U^k$  such that  $y^{k+1}$  is optimal, that is,  $\forall u'^k : u'^k \neq u^k$  we have *not*  $(y^{k+1} \prec y'^{k+1})$ .

The optimisation problem as stated here does not necessarily have a unique solution (it may have several or none). In addition, in realistic situations we will not have complete knowledge of  $f$ , and therefore we will at most be able to solve approximate versions of the problem. A more useful question is therefore:

b) **Adaptation**

choose an action  $u^k \in U^k$  such that *not*  $(y^{k+1} \prec y^k)$ .

Such an action avoids a potential impact. We call it effective. For many practical purposes, “effectiveness” is not a clear-cut notion. For example, an action might avoid part of the hazard. Future refinements of the framework will consider this aspect.

**Definition 6.**

An action  $u^k$  is effective for a system  $(f, g)$  in state  $x^k$  subjected to an exogenous input  $e^k$  iff *not*  $(y^{k+1} \prec y^k)$ .

If there are no effective actions against  $e^k$ , then  $e^k$  is an unavoidable hazard. If  $e^k$  is not unavoidable, then problem b) has at least one solution. The set of effective actions available to the system can be used to interpret the notion of adaptive capacity:

**Definition 7.**

The adaptive capacity of a system  $(f, g)$  in state  $x^k$  subjected to an exogenous input  $e^k$  is represented by the set of its effective actions.

**Example 8.** The adaptive capacity of the motorcyclist in the first example is the set of all actions that do not result in the motorcyclist falling down the cliff due to the oil spill. It can be thought of as a measure of, amongst other things, his skill set and the technical specifications of the motorcycle.

### 3.4 Co-evolution of system and environment

One aspect not yet captured by our framework is that vulnerability to climate change is the result of a long-term interaction between the system and the environment. To take this interaction into account, we introduce a model of the environment as a dynamical system:

$$e^{k+1} = h(e^k, u^k, x^k). \quad (5)$$

In most cases, however, the transition function  $h$  is not known and the information we have is of the form

$$e^{k+1} \in E^k = \mathcal{E}(e^k, u^k, x^k), \quad (6)$$

that is, the environment is modelled by a non-deterministic dynamical system. The function  $\mathcal{E}$  returns not just one value, as the function  $h$  does, but an entire set of values.

As in the previous, static case, given  $x^0$  and  $e^0$  we can define a number of problems. In the static case the problems involved finding an action with some property (*e.g.*, optimality). In the dynamic case we need to find a policy

$$\phi : X \times E \rightarrow U, \quad \phi(x^k, e^k) = u^k. \quad (7)$$

The policy  $\phi$  is a function that specifies which actions are to be taken, depending on the state of the system and the exogenous input with which it is faced.

A natural condition on such a function is that the actions it returns should be effective where possible. In addition, we may require:

a) **Optimisation**

$\phi$  should be such that the actions taken drive the system along an optimal trajectory. This assumes that we have a preorder along trajectories. Assuming that  $\preceq^{seq}$  is such a preorder, that is, a preorder on sequences of outputs  $y = (y^1, y^2, \dots, y^K)$ , we require it to be compatible with the original preorder  $\preceq$  in the following way:

$$y \preceq^{seq} y' \Rightarrow \exists k : y^k \preceq y'^k .$$

If a sequence  $y$  is worse than a sequence  $y'$ , then it must be worse for at least one time step  $k$ . An example of such a preorder is:

$$y \preceq^{seq} y' \quad \text{iff} \quad \forall k : y^k \preceq y'^k .$$

As in the static case, the problem will in most cases have several or no solutions, and for realistic examples only approximate versions of the problem will be solvable.

b) **Mitigation** (by controlling  $E^k$ )

For all  $k \in \{1, \dots, K\}$  we require that  $E^k$  contains no unavoidable hazards.

c) **Maintaining adaptive capacity**

For all  $k \in \{1, \dots, K - 1\}$  the set  $U^k$  has effective elements.

Both problems b) and c) can have more than one solution, even when a) has no solution. Moreover, they are dually related in that a policy respects b) iff it respects c).

For a stochastic system, problem b) might translate to the reduction of the probability of all unavoidable hazards below a certain threshold. Similarly, for a less abstract notion of effectiveness, problem c) might require, for example, the improvement of the effectiveness of actions in  $U^k$  and therefore of adaptive capacity. These and other refinements to the framework are in progress and will be presented separately.

As a final remark, once a policy  $\phi$  has been chosen, the relative vulnerabilities of systems in different states can still be assessed using Definition 2.

**Example 9.** Consider the third and fourth motorcyclists described in Section 2.2. They differ from the first two in that they are involved in a sequence of interactions with the environment. If they follow fixed policies, then their evolution is fully determined by their initial states and the respective states of their environments. In this case, Definition 2 applies if we can give a preorder on the trajectories of their output. For instance, we could take as preorder on the trajectories the preorder given on their end-points (as in Example 1, on the road or down the cliff). However, if they do not follow fixed policies, then we need to express the preference criteria by choosing a preorder on the sets of possible trajectories, which we could do by choosing a preorder on the sets of possible end-points of trajectories. If we choose a maximin preorder, then both motorcyclists are going to come out as equally vulnerable, since the worst sequence of action is the same for both of them. On a maximax preorder, however, we would expect the fourth motorcyclist to be more vulnerable than the third, who was able to invest in improving his adaptive capacity.

### 3.5 Multiple agents

It might seem, owing to the insistence of ascribing vulnerability to an entity, that our framework cannot represent multiple agents. This is not the case: in this section we show two possible ways of dealing with interacting systems.

For simplicity we consider two systems:

$$\begin{aligned} f_1 &: X_1 \times E \times X_2 \times U \rightarrow X_1 \\ g_1 &: X_1 \rightarrow Y_1 \\ f_2 &: X_2 \times E \times X_1 \times U \rightarrow X_2 \\ g_2 &: X_2 \rightarrow Y_2. \end{aligned}$$

The systems interact with the environment and with each other:

$$x_1^{k+1} = f_1(x_1^k, e^k, x_2^k, u_1^k) \quad (8)$$

$$x_2^{k+1} = f_2(x_2^k, e^k, x_1^k, u_2^k). \quad (9)$$

Let us assume we have preorders  $\preceq^1$  and  $\preceq^2$  on  $Y_1$  and  $Y_2$ , respectively.

A first problem would be an assessment of the vulnerability of the combined system:

$$\begin{aligned} f_{1,2} &: X_{1,2} \times E \times U_{1,2} \rightarrow X_{1,2} \\ g_{1,2} &: X_{1,2} \rightarrow Y_{1,2} \end{aligned}$$

where

$$X_{1,2} = X_1 \times X_2 \quad (10)$$

$$U_{1,2} = U_1 \times U_2 \quad (11)$$

$$Y_{1,2} = Y_1 \times Y_2 \quad (12)$$

$$f_{1,2}((x_1, x_2), e, (u_1, u_2)) = (f_1(x_1, e, x_2, u_1), f_2(x_2, e, x_1, u_2)) \quad (13)$$

$$g_{1,2}(x_1, x_2) = (g_1(x_1), g_2(x_2)). \quad (14)$$

This assessment requires choosing a preorder on the set  $Y_{1,2}$ , which would “integrate” the two preorders,  $\preceq^1$  and  $\preceq^2$ . For example, we can choose

$$(y_1, y_2) \preceq^{1,2} (y'_1, y'_2) \text{ iff } y_1 \preceq^1 y'_1 \text{ and } y_2 \preceq^2 y'_2.$$

In this case, the roles of the two systems are symmetrical.

We can give more weight to one of the systems by combining the preorders in a lexicographical way:

$$(y_1, y_2) \preceq^{1,2} (y'_1, y'_2) \text{ iff } y_1 \prec^1 y'_1 \text{ or } (y_1 = y'_1 \text{ and } y_2 \preceq^2 y'_2).$$

Here the first system is given more importance, because if its output grows worse, the combined system is considered to be worse off, whereas the output of the second system is only relevant if the first one remains unchanged.

A second possible problem is to assess the vulnerability of each system independently. Taking the case of the first system, we would simply consider the environment as including the second system:

$$\begin{aligned} f'_1 &: X_1 \times E' \times U_1 \rightarrow X_1 \\ g'_1 &: X_1 \rightarrow Y_1 \end{aligned}$$

where

$$E' = E \times X_2 \quad (15)$$

$$x_1^{k+1} = f'_1(x_1^k, (e^k, x_2^k), u_1^k) \quad (16)$$

$$g'_1 = g_1. \quad (17)$$

The problems of optimisation, mitigation and maintaining adaptive capacity can now be raised with respect to the extended environment. Multi-scale analysis becomes important in this case, because the environment will contain a part  $f_2$ , which operates at the same scale as the system  $f_1$ , and another part, given by the evolution of  $e$ , which in general takes place at a much slower pace.

## 4 Preliminary applications

The objective of this section is to relate the framework developed in Section 3 to the IPCC conceptualisation of vulnerability (see Figure 1) and to two recent vulnerability assessments: ATEAM and DINAS-COAST. It is not the objective to evaluate the IPCC conceptualisation and the two assessments, but rather to test the practical applicability of the framework using real examples. The choice of these examples is motivated chiefly by the fact that we have first-hand knowledge of both the IPCC conceptualisation and the two assessments. Future work will apply the framework to vulnerability assessments that are more qualitative in nature, do not follow the IPCC conceptualisation and do not focus primarily on climate change.

As mentioned earlier and discussed in detail by Füssel and Klein [1], the meaning of vulnerability within the context of climate change has evolved over time. This is reflected in the respective assessment reports of the IPCC. In our application we use the definition of vulnerability as provided in the glossary of the Working Group II contribution to the Third Assessment Report [2]. The approaches of both ATEAM and DINAS-COAST were developed to be consistent with this definition. An important difference between the two projects is that DINAS-COAST explicitly considered feedback from human action on the natural system.

### 4.1 IPCC

As mentioned in Section 2.3, the IPCC Third Assessment Report described vulnerability as “a function of the character, magnitude and rate of climate variation to which a system is exposed, its sensitivity, and its adaptive capacity”. It defined vulnerability as “the degree to which a system is susceptible to, or unable to cope with, adverse effects of climate change, including climate variability and extremes” [2, p. 995]. The extent to which this definition can be made operational for assessing vulnerability is limited because the defining concepts are either very general or their meaning is unclear. In addition, vulnerability is said to be a function of exposure, adaptive capacity and sensitivity, but nothing is said about the form of this function. As a result of this unclarity we can only verify whether all elements of the IPCC definition are contained in our framework and whether there are any obvious contradictions between the two.

There are four defining elements in the IPCC definition, two of which can be mapped directly to primitives used in our definition. The first element, the “degree to which a system is susceptible to, or unable to cope with”, is represented in our definition by the preorder  $\preceq$ . These preference criteria on the set of outputs  $Y$  make it possible to assert that the system may end up in an undesirable state, thus being unable to cope with some stimulus. The second element, the “character, magnitude and rate of climate variation to which a system is exposed” describes the (climate) stimulus to which the system is exposed. In our definition this element is the exogenous input  $e$ . Since we want to be able to consider non-climatic input as well, we do not limit  $e$  to climate stimuli.

The other two defining elements in the IPCC definition, sensitivity and adaptive capacity, cannot be mapped directly to our primitives. We consider both concepts to be more complex properties of a system, which are therefore not suitable as starting points for a definition. However, both concepts can be defined using our primitives. “Sensitivity” is a well-established concept in system theory, characterising how much a system’s output is affected by a change in its input. It requires the differentiability of the functions  $f$  and  $g$ . If this requirement is met, it can be shown that a system cannot be vulnerable to an exogenous input if it is not sensitive to that input, which agrees with the IPCC definition. However, in our framework this requirement and the notion of sensitivity are not necessary to define vulnerability.

The fourth element, adaptive capacity, is defined by us as the set of effective actions available to the system. It is a more complex notion than vulnerability in that its definition relies on four primitives, not three. In addition to a dynamical system, exogenous input and a preorder, endogenous input is required to define adaptive capacity (see Definition 7). In contrast to the IPCC conceptualisation, knowledge of adaptive capacity is not required for assessing vulnerability, as is illustrated by the case of simple systems (as in Example 1). However, adaptive capacity will influence the vulnerability of the more complex systems typically considered by the IPCC. As shown in Section 3.4 (problem c) and Example 9), assessments of vulnerability and adaptive capacity are interrelated: their influence on one another depends on the preference criteria chosen.

## 4.2 ATEAM

The project ATEAM (Advanced Terrestrial Ecosystem Analysis and Modelling; <http://www.pik-potsdam.de/ateam/>) was funded by the Research Directorate-General of the European Union from 2001 to 2004. It was concerned with the risks that global change poses to the interests of people and organisations in Europe relying on the services provided by ecosystems. It involved thirteen partners and six subcontractors, whose joint activities resulted in the development of a vulnerability mapping tool [17]. Focusing on agriculture, forestry, carbon storage and energy, water, biodiversity and mountains, an explicit objective of ATEAM was to assess vulnerability rather than merely potential impacts. The project adopted the IPCC conceptualisation of vulnerability, which required combining information on potential impacts with information on adaptive capacity (see Figure 1). Socio-economic data was used to assess adaptive capacity on a sub-national scale, in a way that allowed it to be projected into the future using the same set of scenarios as for the assessment of potential impacts [18]. The information on potential impacts and adaptive capacity was then combined in a series of vulnerability maps.

When taking a closer look at ATEAM using the formal framework of Section 3, we first need to identify the framework’s three primitives. ATEAM aimed “to assess where in Europe people may be vulnerable to the loss of particular ecosystem services, associated with the combined effects of climate change, land use change and atmospheric pollution” [19, p. 3]. Thus, the entity are people in Europe who rely on ecosystem services. The assessment was carried out on a coupled human-ecological system, a system that receives both exogenous input (the stimuli) and endogenous input (the human actions). The evolution of such a system can be represented by Equation 3, where  $U^k$  represents the management actions people can apply to adapt to potential impacts and thus maintain the ecosystem services on which they rely. These actions are usually specific to the ecosystem service considered. For example, a management action for ensuring the ecosystem service “food supply” could be to irrigate agricultural land.

The second primitive is the stimulus or exogenous input  $e \in E$ , to which the system’s vulnerability was assessed. This input was given by the scenarios of climate, land use and nitrogen deposition, which represent the possible evolutions of the environment. The scenarios were based on the IPCC SRES storylines (for details see [19]).

The third primitive notion concerns the preference criteria, which relate to the loss of ecosystem services. We will discuss the preference criteria in more detail below.

Given these three primitive notions, it is now possible to interpret ATEAM as assessing the vulnerability of a region (more accurately: people in a region) in state  $x^k$  to an exogenous input  $e^k$  with respect to a preorder  $\preceq$ . For clarity of presentation, we will consider only one transition of the system. First, the set of possible outputs  $Y^{k+1}$  needs to be determined, that is, the system’s output for all actions in the set of endogenous inputs  $U^k$  needs to be calculated. Second, this set of outputs needs to be compared to the previous output  $y^k$ , using an appropriate preorder (see Example 5).

However, in the case of ATEAM it was not possible to determine the set of potential impacts, because the transition function  $f$  (Equation 3), which represents the dynamics of the coupled human-ecological system, was not known. The available knowledge, in the form of ecological and hydrological models, did not consider the feedback from human action to ecosystems. Therefore, the non-deterministic system was simplified into a deterministic one by choosing some average action  $\tilde{u}$ , which is independent of the exogenous input, and assessing the evolution of the system only for this one element of the set of possible actions. This average action represents “management as usual”. The evolution of the deterministic system was then provided by the ecological and hydrological models:

$$x^{k+1} = f_{\tilde{u}}(x^k, e^k). \quad (18)$$

Multi-dimensional output was produced for every scenario and indicators were used to compare the outputs. Indicator values were normalised for each ecosystem service, producing values between 0 and 1. The preorder on the normalised values was not made explicit in ATEAM. Rather, for each output the respective indicators were displayed on a map.

The multi-dimensional output could be seen as the combined outputs of several systems of regions (here “combined” is taken as in Section 3.5). A benefit of the indicator-based approach is that comparisons could be made between these component systems (*cf.* Definition 3). To enable such comparisons was one of the major objectives of ATEAM.

Up to this point, the approach was that of a traditional assessment of potential impacts. However, ATEAM also assessed the third element of the IPCC definition, adaptive capacity. Adaptive capacity was captured by an index representing society’s ability to adapt. The adaptive capacity index is a real number between 0 and 1. It was developed by building a statistical model from observed socio-economic data, which was then applied to the IPCC SRES scenarios to produce future projections. The approach will be described in detail in [18].

The adaptive capacity index representing the socio-economic situation was an estimate of the size of the set of actions  $U^k$  that are available. The size of this set was then assumed to be an indication of the size of the set of effective actions, since the latter is a subset of the former. The more actions are available, the more likely it is that some of these would be effective. Thus, the adaptive capacity index could be seen as an estimate of the size of the set of effective actions.

### 4.3 DINAS-COAST

The project DINAS-COAST (Dynamic and Interactive Assessment of National, Regional and Global Vulnerability of Coastal Zones to Climate Change and Sea-Level Rise; <http://www.dinas-coast.net>) was also funded by the Research Directorate-General of the European Union from 2001 to 2004. Five partners and two subcontractors worked together to develop the dynamic, interactive and flexible CD-ROM-based tool DIVA (Dynamic and Interactive Vulnerability Assessment) [20]. DIVA enables its users to assess coastal vulnerability to sea-level rise and explore possible adaptation policies. Whilst also following the IPCC conceptualisation, DINAS-COAST took a somewhat different approach to assessing vulnerability compared to ATEAM in that it included in the representation of the vulnerable system feedback from human action to the environment.

At the core of DIVA is an integrated model, which itself is composed of modules representing different natural and social coastal subsystems [21]. The model is driven by sea-level and socio-economic scenarios and computes the geodynamic effects of sea-level rise on coastal systems, including direct coastal erosion, erosion within tidal basins, changes in wetlands and the increase of the backwater effect in rivers. Furthermore, it computes socio-economic impacts, either directly due to sea-level rise or indirectly via the geodynamic effects. In contrast to traditional impact assessments, DINAS-COAST also considers the effects of adaptation actions.

More formally, given an initial state  $x^0$ , DIVA takes a sequence of exogenous inputs  $(e^1, e^2, \dots, e^K)$  representing the evolution of the environment from time 1 to  $K$ , with which it produces a sequence of outputs  $(y^1, y^2, \dots, y^K)$  representing the evolution of the coastal system:

$$(y^1, y^2, \dots, y^K) = h((e^1, e^2, \dots, e^K), x^0, \phi) \quad (19)$$

where

$$h : X \times E^K \times \Phi \rightarrow Y^K,$$

$X$  is the set of states of the system,  
 $E$  is the set of exogenous inputs,  
 $\Phi$  is the set of adaptation policies, and  
 $Y$  is the set of outputs of the system.

Let us now analyse this model in terms of the three primitives of our framework. The first primitive, the vulnerable entity, is the coastal system. It is represented as a discrete dynamical system  $(f, g)$  that receives exogenous input (the stimuli) and endogenous input (the adaptation actions). The evolution of the system is given by Equation 3 and its output function by Equation 2.

The state  $x$  of the system comprises all variables that determine the state of the natural and social subsystems. This large set includes such diverse variables as wetland area, tourist arrivals, dike heights and tidal basin sediments. The exogenous input  $e$  contains the following socio-economic and climate variables: sea level, temperature, land use, population growth and gross domestic product per capita. The output  $y$  includes state variables and diagnostic variables of the system, such as people at risk to flooding, land loss, economic losses and costs of protecting the coast. The actions contained in the set of endogenous inputs  $U$  are (i) do nothing, (ii) build dikes, (iii) move away and (iv) nourish the beach or tidal basins.

Given  $f$  and  $g$ , the vulnerability of the system could be assessed by calculating the transition of the system for every adaptation action  $u \in U$  and comparing the resulting set of possible

outcomes  $Y^{k+1}$  with the previous output  $y^k$  (see Example 5). However, instead of doing this, DIVA includes a third function, the adaptation policy. The adaptation policy  $\phi$  returns an adaptation action  $u$  for every state of the coastal system, given the input it receives from the environment (Equation 7).

The model's computation is performed by the application of  $f$ ,  $g$  and  $\phi$ . The composition of the adaptation policy  $\phi$  with the state transition function  $f$  transforms the non-deterministic system  $(f, g)$  into a deterministic one  $(f', g)$ :

$$x^{k+1} = f(x^k, e^k, u^k) = f(x^k, e^k, \phi(x^k, e^k)) = f'(x^k, e^k) \quad (20)$$

Whilst the state transition and the output function are unique, the adaptation policy can be selected from:

- No adaptation: the model only computes potential impacts.
- Full protection: raise dikes or nourish beaches as much as is necessary to preserve the status quo (*i.e.*,  $x^0$ ).
- Optimal protection: optimisation based on the comparison of the monetary costs and benefits of adaptation actions and potential impacts.

The stimulus or exogenous input ( $e \in E$ ) to which the system's vulnerability is assessed was provided by sea-level, land-use and socio-economic scenarios. Similarly to ATEAM these were developed on the basis of the IPCC SRES storylines.

DIVA does not produce a unique measure or indicator of vulnerability. The model's output has many components and no preorder is given on the set of outputs  $Y$ . However, since the output is quantitative, a total order exists on each component of the output. The monetary components of the output can be summed up and compared, which is the basis for the optimal protection policy. DIVA does not specify a preorder to compare the non-monetary components of the output. Rather, it is left to the user to explore and compare the outputs that are produced by choosing different adaptation policies and scenarios. For this purpose, the model is provided with a graphical user interface that allows for the visual comparison of the outputs for different regions, time steps, scenarios and adaptation policies in form of graphs, tables and maps.

## 5 Conclusions and future work

In this paper we presented the contours of a formal framework of vulnerability to climate change. The formal framework is based on a grammatical investigation that led from the everyday meaning of vulnerability to the technical usage in the context of climate change. The most important result of this investigation is that the definition of vulnerability requires the specification of three primitives: the entity that is vulnerable, the stimulus to which it is vulnerable and a notion of "worse" and "better" with respect to the outcome of the interaction between the entity and the stimulus. Section 3 presented a mathematical translation of this result, grounded in systems theory. In addition, it introduced refinements that capture the informal concepts of adaptive capacity and mitigation. Section 4 served as a first test of the framework by assessing whether it can represent concepts used in recent work of which the authors have first-hand knowledge.

Preliminary findings of this test include that the three determinants of vulnerability as identified by the IPCC correspond only in part with the three primitives of our formal framework and that ATEAM and DINAS-COAST have chosen not to specify the preorder on their models' respective outputs. Instead, they specified several preorders on components of their outputs, leaving room for interpretation by the user. However, it has not been the purpose of this paper to evaluate these projects, in particular because the current version of the framework is too rudimentary for such a task. A more important finding is that the preparation of this paper has allowed and motivated scientists from very different disciplines (the authors, workshop participants and informal reviewers) to communicate clearly about an issue of common interest, thereby enriching each other's understanding of the issue. At the same time, the paper has shown that there is scope for many refinements, specialisations and applications of the framework, which means that much work remains to be done to develop it into a useful tool.

The definitions in this paper aimed at showing that a certain type of mathematical theory can account for a simplified grammar of vulnerability rather than at being of immediate use to researchers in the field. A major part of the work to be done will concern structural refinements: formulating stronger, more precise definitions for more complex systems, in a way that makes it easy to deal with continuous time, stochasticity, fuzziness, multiple scales, *etc.* The problems of optimisation, mitigation and maintaining adaptive capacity must be formulated for these systems in ways that relate them to questions asked in vulnerability assessments. To do so will enable us to incorporate results from the fields of control theory, game theory and decision theory, which was, after all, one of the motivations for developing our framework.

These theoretical developments should be accompanied by practical applications that elaborate on those in Section 4. The analytical framework must be informed by the large body of results available from past case studies and by the needs of ongoing vulnerability assessments. For example, we will interact with the case-study teams involved in the project NeWater so as to develop a systematic way of including indicator-based approaches in our framework.

As mentioned in Section 1, it is increasingly argued that the climate change community could benefit from experiences gained in food security and natural hazards studies. These communities have their own well-developed fields of research on vulnerability, although there are important differences with vulnerability assessment carried out in the context of climate change [5, 22]. The framework proposed in this paper could be used to analyse approaches to vulnerability assessment in these communities, as well as in the climate change community. This could make more explicit and thus lead to a better understanding of the perceived and real differences between the respective models of vulnerability in use. Moreover, it will serve to test the framework proposed here. It will be a challenge to see whether the framework can capture in mathematical terms the complexity and richness of individual communities, sectors and regions, as well as of the factors leading to their vulnerability. In addition, the value of the framework for qualitative approaches to vulnerability assessment needs to be demonstrated, especially in those places where data are scarce.

On a final note, we realise that some may perceive a formal framework as limiting the flexibility required to capture the breadth and diversity of issues relevant to vulnerability assessment. Others may consider the mathematical approach to developing the framework as an impediment to discussion and application. As stated before, the framework is not intended to be prescriptive, nor is it meant to exclude non-mathematical viewpoints on vulnerability. The least we hope to achieve is that our framework makes researchers aware of the potential confusion that can arise from not being precise about fundamental concepts underpinning their research. At the most,

we hope they will recognise the potential benefits of testing, applying and further developing the framework proposed here. Every attempt has been made to make the formal description of the framework as accessible as possible to the mathematically challenged, a group that includes the second author of this paper.

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