ELSEVIER

Economic Modelling

Volume 24, Issue 2
March 2007

ISSN 0264-9993

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ELSEVIER
Economic Modelling 24 (2007) 272–294
Economic Modelling
www.elsevier.com/locate/econbase

Technological spillovers within multi-region models: Intertemporal optimization beyond the Negishi approach

Marian Leimbach [a,*], Ottmar Edenhofer

[a] PIK - Potsdam Institute for Climate Impact Research, P.O. Box 60 12 03, D-14412 Potsdam, Germany

Accepted 28 July 2006

Abstract

An alternative approach to multi-region modeling will be presented. This approach (called modular approach) is able to reproduce results obtained with the traditional as well as a modified Negishi approach. We redesigned the Negishi approach to make it applicable in modeling technological spillovers induced by foreign direct investments. While the modular approach is able to completely internalize the external effects from technological spillovers, which result in higher welfare gains, the modified Negishi approach cannot completely internalize them. The latter is due to the fact that shadow prices from sectoral production functions are used in order to feed the process of iteratively adjusting the Negishi welfare weights. Under the modular approach, the way of finding an equilibrium solution in a dynamic and multi-regional framework is allocation-based and differs from existing price-oriented methods. We discuss the characteristics of the underlying adjustment algorithm which, in contrast to the joint maximization of the Negishi approach, is embedded in a decentralized optimization process. Results from numerical model experiments will demonstrate the advantage of this novel approach.

JEL classification: O41; C61; F21; F17

Keywords: Multi-region modeling; Technological spillovers; Decentralized optimization; Foreign direct investments

1. Introduction

This paper focuses on multi-region modeling including external effects like technological spillovers. Within economic theory, the concept of externalities has long been investigated. Externalities arise, for instance, due to market imperfections, public goods and spillover effects.

* Corresponding author. Tel.: +49 331 288 2566.
E-mail address: leimbach@pik-potsdam.de (M. Leimbach).

0264-9993/$ - see front matter
doi:10.1016/j.econmod.2006.07.005

The classical instrument to internalize external effects is a Pigouvian tax, which can be derived from the shadow prices of a counterfactual model experiment. However, if the externality causes a non-convex model structure, pareto (global) optimality of the solution cannot be guaranteed and shadow prices are misleading in the case of a local optimum. In modeling regional linkages (e.g. capital and trade flows) in a dynamic context, dealing with external effects like technological spillovers is additionally complicated, because of the feedbacks between regional linkages and external effects. Recent literature (Blomström et al., 1999; Hejazi and Safarian, 1999) identified a strong link between foreign direct investments and spillovers. This paper aims to provide a model framework that allows analysis of the impacts of foreign investments and spillovers on regional welfare and economic growth.

Our starting point in multi-region modeling is the Negishi approach. Some well-known models in climate economics, e.g. MERGE (Manne et al., 1995; Manne and Richels, 1995) and RICE (Nordhaus and Yang, 1996), applied the Negishi algorithm in order to find a general equilibrium in an intertemporal optimizing framework. The traditional Negishi approach, however, has to be modified in order to deal with foreign direct investments and spillovers. This will be explained in more detail later on. Yet this modified Negishi approach is limited in capturing the external effect induced by technological spillovers. We present an alternative algorithm of coupling regions, which is able to capture the external effect completely.

The paper is structured as follows: challenges of multi-region modeling will be discussed in Section 2. Methods and solution techniques of multi-region modeling are summarized; problems that arise in the presence of interregional spillovers are highlighted. Our alternative approach is based on the concept of modularity. The model structure distinguishes between region modules and a trade module. The mathematical structure of the model is presented in Section 3. We demonstrate the iterative algorithm that searches for an equilibrium solution within the modular approach. In Section 4, we discuss the characteristics of the underlying adjustment process and contrast them to the characteristics and capabilities of the Negishi approach. Preliminary results from numerical experiments, depicting welfare and terms of trade implications, are presented in Section 5. The advantage of the modular approach is demonstrated for the case of the presence of spillovers. We end with some conclusions in Section 6.

2. Multi-region modeling methods

Multi-region modeling becomes a challenging task when different regional interactions are considered. Several flows (e.g. goods trade, capital flows, tradeable permits, knowledge spillovers) between heterogenous regions form a complex pattern of interaction. In classical economics and trade theory, prices are the major tool for coordinating regional interactions (cf. Samuelson, 1952; Negishi, 1972). In an intertemporal problem, as it is the subject of investigation here, entire time paths of prices have to be determined so as to equilibrate imports and exports, supplies and demands during each period simultaneously. Since each commodity is differentiated between regions and time periods, the number of prices and quantities rises rapidly. This curse of dimensionality persists in spite of the most recent progress in computational performance. That is also why first steps in applied economic analysis of dynamic multi-regional problems were taken in the more aggregated framework of economic growth models (e.g. Manne and Richels, 1992).

Early work on algorithms that help to find equilibrium prices numerically was summarized by Scarf (1984). More recently, e.g. Kumar and Shubik (2004) and Luenberger and Maxfield (1995) presented advanced algorithms for the computation of competitive equilibria. These algorithms employ a standard fixed-point method as a fundamental component, as other general purpose algorithms for determining equilibria also do. In the literature different termini for the categorization of adjustment algorithms are mentioned, e.g. Walrasian vs. Marshallian, tatonnement vs. non-tatonnement. Like many others, Luenberger and Maxfield (1995) distinguish between price-based and allocation-based algorithms. The former (including especially the standard Walrasian excess demand algorithm) is the most applied one. Manne and Richels (1992) applied such an iterative algorithm based on the Danzig-Wolfe decomposition method.

The Negishi approach, as a well-known solution technique for multi-region modeling (Manne and Rutherford, 1994; Leimbach and Toth, 2003), can be classified as an allocation-based algorithm. In particular, the Negishi approach is distinguished by a welfare adjustment process that guarantees that the intertemporal budget constraints are satisfied. Technically, the Negishi approach integrates the regions in a single model. It therefore differs from approaches with decentralized decision-making processes. Essentially, it internalizes the coordination function which in decentralized models is played by the market or a virtual auctioneer. By means of adjustable welfare weights, the regions' utility functions are combined in a single global welfare function. Negishi (1972) demonstrated the correspondence between a competitive equilibrium and a maximum point of a social welfare function which is a linear combination of utility functions of individual consumers, with the weights in the combination in inverse proportion to the marginal utilities of income. Numerically, the welfare optimum can be computed based on the method of sequential joint maximization (Dixon, 1975; Rutherford, 1992).

As part of the most recent discussion in endogenous growth literature, Farmer and Lahiri (2005) studied the problem of finding an equilibrium in a two-country model setting with externalities. They could prove the existence of an equilibrium solution (unique equilibrium) under strict assumptions on the initial state only. Farmer and Lahiri also demonstrated that the presence of externalities prevents factor price equalization. Greiner and Semmler (2002), too, investigated a multi-region problem characterized by externalities, which in this case are due to knowledge spillovers. They found two equilibria (balanced growth paths). Again, initial values predetermine the long-run growth path. General results from analyses of regional interactions based on analytical models can only be derived for simplified models and under restrictive assumptions. In all other cases, numerical algorithms are needed to obtain model solutions, i.e. to find a general equilibrium and/or a pareto optimum.[1,2]

We started with the Negishi approach in modeling regional interactions within a framework of economic growth models. However, as it turns out (see below) the capability of the Negishi approach in dealing with technological spillovers is limited. Based on an alternative allocation-based adjustment algorithm, we developed a model that bears the challenge of taking spillovers and foreign investments into account and which is suited for numerical experiments. This model is composed of single optimization problems of decentralized actors (i.e. regions). On a meta-level, a control entity or auctioneer coordinates the local solutions in an iterative fashion. The virtual coordinator has to compute an optimal allocation of the common resources. The present approach considers all traded goods as common resources. Based on the allocation of the resources, the agents determine their economic activities. There is an exchange of information between the coordinator and the agents until the global situation cannot be improved anymore.

The equilibrium reached in this way, however, is a conditional one, based on the weights the coordinator assigns to each agent's improvement in evaluating the global situation. In applying a

[1] As for the relation between equilibria and optima, see for instance Ginsburgh and Waelbroeck (1981).

[2] An interesting technique of finding optimal solutions in markets with non-convexities is presented by O'Neill et al. (2005). They expanded the set of commodities by one extra good for each externality.

different approach to multi-region modeling, Bahn et al. (1998) assume economies of the same state of development which implies equal weights. Such an assumption, obviously, restricts the application of that multi-regional model. The present model will follow the rationale of the Negishi approach. A distinguished equilibrium can be obtained by equalizing intertemporal trade balances.

Multi-regional models, pursuing the Negishi approach (e.g. Nordhaus and Yang, 1996; Leimbach and Toth, 2003), are usually implemented in the primal nonlinear programming format. In contrast, state-of-the-art multi-regional computable general equilibrium models are implemented in a mixed complementarity format (cf. Rutherford, 1995). Duality properties of optimization models and complementary slackness conditions are employed by this format. Either type of multi-region modeling faces problems in the presence of spillover effects. Spillovers represent a kind of externality and increasing return to scale effect, respectively, that may introduce non-convexities into the model structure.

However, in dealing with a problem setting characterized by externalities, where the common convexity assumptions of the general equilibrium theory do not hold, we favor the more open structure of the nonlinear programming format over the stringent mixed complementary programming (MCP) format. The application of the MCP format is limited due to the occurrence of non-convexities. In particular, as we will see later, technological spillovers cause deviations between import and export prices and between regional rates of return on capital. These deviations distort the complementarity conditions utilized by the MCP format.

In order to discriminate the present approach from the Negishi approach and alternative decentralized approaches we will call it the modular approach (in reference to its modular structure—see below).

3. The basic model

In this section, we present our model. The model is composed of region modules and a trade module. The region modules encapsulate multisectoral Ramsey-type economic growth models. We shall give a rather compact and general description of that model part. In Section 5, we will provide specifications of the implemented computational model.

We investigate a decision problem without market imperfections, i.e. the spillover externality is internalized by the regional actors. This implies an equivalence of the model either formulated as a market model or a social planner model. Hence, a representative agent can be assumed to summarize households' consumption decisions and firms' investment and trade decisions. The trade module represents a meta-optimizing algorithm that can be conceived as an auctioneer who balances interactions of decentralized agents. Finding a solution for the coupled model is an iterative process.

Throughout the model presentation, we use the following indices:

t	time periods
i, k	regions
j	goods
r	iterations.

With $J=\{G, F\}$ and $j \in J$, the following types of trading goods are distinguished:

G	consumption good
F	investment good.

Each good is produced in a different sector. Hence, j also represents a sectoral index. We denote the sectoral index by a superscript throughout the model presentation. Although the model and the equations, respectively, are time-discrete in principle, we use the continuous form of representing time. Time derivatives are represented as usual. Each variable actually bears the time, region and iteration index. For transparency reasons, we will suppress them as often as possible.

3.1. Region module

Each region module is represented by an economic growth model that includes a welfare maximizing objective function max U! with

$$U = \int_{t=1}^{\infty} f[C(t)] \cdot \mathrm{e}^{-\rho t} \mathrm{d}t. \tag{1}$$

This welfare function measures the utility of the region's representative household. Utility is a function f of the consumption path $C(t)$ subject to discounting by the pure rate of time preference ρ. For f it holds that

$$f'[C]>0; f''[C]<0.$$

Production functions g with capital K and some exogenously given other production factors M generate sectoral output Y:

$$Y^j = g^j[A^j, K^j, M^j]. \tag{2}$$

Variable A denotes the productivity level which represents total factor productivity here. Capital is allocated from a common pool. Thus, perfect cross-sectoral mobility of capital is implicitly assumed (we neglect the real-world vintage and putty-clay structure of the capital stock):

$$K^G + K^F = K. \tag{3}$$

Capital accumulation follows the standard capital stock equation of motion (I and δ represent domestic investments and the depreciation rate, respectively) extended by investment goods imports (X^F):

$$\dot{K}_i = I_i + \sum_k X_{ki}^F - \delta_i \cdot K_i. \tag{4}$$

For simplicity reasons, we assume perfect substitutability between domestic and imported capital goods.[3] The same assumption applies to consumption goods. The output of the consumption goods sector represents the regional gross product net of investments. It is used to meet demands on consumption and exports, while being incremented by imports:

$$Y_i^G = C_i + \sum_k (X_{ik}^G - X_{ki}^G). \tag{5}$$

X_{ik} denotes the export from region i to region k. It simultaneously denotes the import of region k from region i which, however, is part of the optimization of another region. Particular

[3] Lee (1995) analyzed an endogenous growth model assuming imperfect substitutability between domestic and imported capital goods.

constraints ensure equivalence of both (see below). Note that the trade variables represent net export and net import values. The usage of separate export and import variables, which for net values actually could be omitted, is due to the subsequent modeling of technological spillovers.

The investment goods sector provides domestic investments and meets foreign demands on investment goods:

$$Y_i^F = I_i + \sum_k X_{ik}^F. \tag{6}$$

In order to establish a link to foreign direct investments, while keeping the model simple, we assume that all investment goods export is accompanied by foreign direct investments. Foreign direct investments, which include physical investments, are considered simultaneously as part of the current account, i.e. exports/imports, and the financial account, i.e. capital transfers (cf. IMF, 1993). Since the current account and the financial account are complements, no further equations are needed to close the model with respect to foreign direct investments and other capital transfers (with the latter as a counterpart of consumption goods export/import), if we neglect reserve changes and net incomes.[4] An intertemporal trade balance introduced below handles the associated accounting issues representatively.

The range of regional interactions usually modeled is extended by technological spillovers. Technological spillovers that increase productivity may be due to foreign direct investments. Empirical research, reported by Takii (2004), demonstrated for several countries that foreign firms (resulting from foreign direct investments) tend to have higher productivity than domestic ones, hence improving the host country's aggregated productivity. Likewise, empirical results presented by Lee (1995) imply that imported capital goods have a much higher productivity than domestically-produced capital goods. Within our model an additional change of the total factor productivity in a region i is a function of foreign direct investments of region k in region i and of productivity differences between both regions:

$$\dot{A}_i = h(X_{ik}^F, \Delta A) \text{ with } \Delta A = A_k - A_i. \tag{7}$$

The productivity differences are computed based on data from the previous iteration.

The present modular approach towards multi-region modeling is distinguished by the use of trade flow boundaries $\bar{X}$. These boundaries are computed by the trade module in the previous iteration. If i denotes the region under consideration, for exports it holds that:

$$X_{ik}^j \geq \bar{X}_{ik}^j. \tag{8}$$

Analogously the following import constraint holds:

$$X_{ki}^j \leq \bar{X}_{ki}^j. \tag{9}$$

Both constraints are binding, since

$$\frac{\partial U_i}{\partial \bar{X}_{ik}^j} < 0$$

[4] Denoting the export of goods and services with X, the import with IM, net incomes with NI, capital transfers and foreign investments with CT and the change of reserve assets with RC, we get the following basic balance of payment equating current and financial account (cf. IMF, 1993):

$$X - \text{IM} + \text{NI} = \text{CT} - \text{RC}.$$

and

$$\frac{\partial U_i}{\partial \bar{X}^j_{ik}} > 0.$$

This guarantees that interregional trade flows are balanced. The region module is completed by several initial conditions:

$$K(0) = k \tag{10}$$

$$A(0) = a \tag{11}$$

and non-negativity conditions:

$$C, A, I, K, K^j, Y^j, X^j \geq 0. \tag{12}$$

The set of control variables Q of each region can be denoted by:

$$Q = \{I, K^j, X^j\}.$$

3.2. Trade module

The purpose of the trade module is to mediate between the region modules (i.e. to clear markets and ensure intertemporal balancing) and to determine trade flow boundaries that correspond to a competitive equilibrium. Since all regions can only be made better off if and only if a new trade activity makes a positive profit at the current equilibrium prices (cf. Scarf, 1990, p. 379), the objective function O of the trade module maximizes the total gains z from a shift in the trade structure. These are obtained from the difference between the importers' marginal utility (based on import price pi) of additional import units $(\bar{X} - X)$ and the exporters' marginal utility loss (based on export price pe):

$$O = \max \int_{t=1}^{\infty} z[pi(t), pe(t), X(t), \bar{X}(t)]\mathrm{d}t \tag{13}$$

with

$$z = \sum_{i=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{1} [(pi^j_{ik} - pe^j_{ik}) \cdot (\bar{X}^j_{ik} - X^j_{ik})]. \tag{14}$$

Prices are endogenous in the integrated system. However, they are exogenous for the region modules as well as for the trade module. They are based on the shadow prices of constraints (8) and (9), which can be described in the form of partial derivatives (with U^* as maximum welfare in iteration r):

$$pi^j_{ki} = \frac{\partial U^*_i}{\partial \bar{X}^j_{ki}} \tag{15}$$

$$pe^j_{ik} = \frac{\partial U^*_i}{\partial \bar{X}^j_{ik}}. \tag{16}$$

The potential trade flows $\bar{X}$, which represent the export/import boundaries in the region modules, serve as control variables in the trade module. The price differential within the objective function is formed by the import price of region k and the export price of region i (with $i \neq k$). These prices may not completely converge. This is due to the effect of the intertemporal budget constraint (see below) which for regions with a current account deficit requests to export tradeables. The designated exporter may have higher marginal utility (shadow prices) than the importing region with respect to the trading good. Each export of such a good, however, increases its shadow price and hence the difference to the respective price within the importing region.

Since the shadow prices cannot be expected to converge offhand (even in the equilibrium), world market prices (which are needed to compute the intertemporal trade balance) are determined on the basis of average values (with n representing the number of regions):

$$\tilde{p}^j = \left[\frac{\sum_{i=1}^{n} \sum_{k=1}^{n} pi_{ik}^j}{(n-1) \cdot n} + \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} pe_{ik}^j}{(n-1) \cdot n} \right] / 2 \quad i \neq k. \tag{17}$$

While suppressing the time index, we emphasize that these prices are time-variant.

An intertemporal trade balance (intertemporal budget constraint) has to be met by each region:

$$\int_{t=1}^{\infty} B_i(t) \mathrm{d}t = 0 \tag{18}$$

with

$$B_i = \sum_{j=1}^{l} \left(\tilde{p}^j \cdot \sum_{k=1}^{n} [\bar{X}_{ik}^j - \bar{X}_{ki}^j] \right). \tag{19}$$

This equation serves to level off the trade deficits of each region in the long run and prevents in a similar way as within the Negishi approach implausible redistribution effects. Due to the complementary relation between trade and capital transfers, this equation represents also a substitute of balancing final net foreign assets which accumulate capital transfers and return rates from foreign investments.

Finally, each region may either be an exporter or an importer of a particular good over a given time period τ:

$$\int_{t=1}^{\tau} \sum_{k=1}^{n} \bar{X}_{ik}^j(t) \mathrm{d}t \cdot \int_{t=1}^{\tau} \sum_{k=1}^{n} \bar{X}_{ki}^j(t) \mathrm{d}t = 0. \tag{20}$$

One could increase flexibility by taking this constraint into account for each point in time separately. This, however, could lead to artificial investment goods exports in anticipation of spillover gains from re-exports in subsequent periods.

3.3. Iteration process

Capturing interactions between the regions, i.e. balancing trade and investment flows, is an iterative process. We run this process numerically. Within each iteration, first the region modules, confronted with new export/import boundaries, are solved and then the trade module. Within the first iteration, an arbitrary set of export/import boundaries has to be selected.

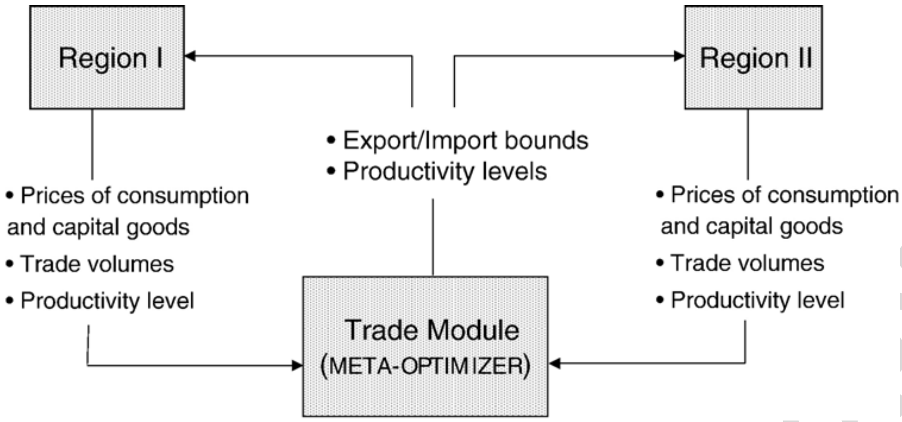

Fig. 1. Data flow between modules.

The data flow between the modules (see Fig. 1 for a two-region example) and the adjustment process work as follows: the region modules send trade volumes, export prices and import prices to the trade module. The trade module uses this information in order to adjust the export/import boundaries, which it sends back.

Furthermore, the current productivity levels of each region are made available for each other region's optimization within the next iteration. This iterative adjustment process ends when the return rates on capital converge to a stationary point and the trade structure does not change anymore, i.e. for all export variables it holds that:

$$|\bar{X}^j - X^j| \leq \varepsilon \tag{21}$$

or

$$|\bar{X}^j_r - \bar{X}^j_{r-1}| \leq \varepsilon. \tag{22}$$

In order to stabilize the iterative algorithm and support iteration progress, we first change Eq. (18) to

$$\int_{t=1}^{\infty} B_i(t)\mathrm{d}t \leq \varepsilon. \tag{23}$$

The value of parameter ϵ has to be chosen close to, but significantly different from zero. Second, the potential change of trade flows, represented by the difference $\bar{X} - X$, is restricted to a fraction γ of the respective output levels:

$$|\bar{X}^j_{ki} - \bar{X}^j_{ik}| \leq \gamma \cdot Y^j_i. \tag{24}$$

The convergence behavior and iteration progress can be influenced by parameter γ.

The iteration process does not reflect the real adjustment process of markets, and the export/import boundaries determined by the trade module are purely algorithmic devices. The result of the iteration process, however, resembles the outcome of market processes. The present iteration process is an allocation-based algorithm (cf. Section 2). It differs from the Walrasian auctioneer and other excess demand algorithms that, by means of price adjustments, iteratively clear markets and/or balance trade flows. Above all, it is an adjustment algorithm that operates in an intertemporal model setting which may include transitional (i.e. off-the-steady-state) dynamics.

4. Modular approach vs. Negishi approach

4.1. Traditional Negishi approach

In this section, we will contrast the modular approach with two versions of the Negishi approach. The traditional Negishi approach plays the counterpart in the first step. In order to investigate problems with spillover externalities we developed a modified version of the Negishi model. This is the subject of the second part of this section.

In Table 1, we want to compare the adjustment processes of the classical Negishi approach and the present modular approach for a conventional model setting without spillover effects. This comparison is based on the following equilibrium conditions:

- balanced intertemporal budget constraint
- clearance of trade markets
- equalized rates of return on capital.

In contrast to standard approaches based on a Walrasian tatonnement process, here, the market clearance condition will be met by means of appropriate constraints in both approaches from the outset. This applies after some initial iterations also to the intertemporal budget constraint within the modular approach and to the return rates within the Negishi approach. While the Negishi approach iterates towards an evened intertemporal budget constraint, the modular approach iterates towards equalized return rates on capital. The next section will show whether differences in the adjustment processes will yield different results.

Return rates represent the rental price of capital. In the standard Heckscher-Ohlin model, trade results in an equalization of factor prices. Meeting the equilibrium condition of equalized return rates, however, depends first on a neoclassic type of production function, where

$$g'[K] \geq 0; g''[K] \leq 0$$

holds in particular, and second on certain initial conditions. Cunat and Maffezzoli (2004) demonstrated that significant differences in the capital-labor ratios of different countries make this equalization impossible. This is associated with a specialization on either capital-intensive or labor-intensive goods. Furthermore, in the presence of externalities induced by foreign direct investments, regional return rates on capital may differ as shown by Farmer and Lahiri (2005). Convergence has to be tested as the case arises. For the general case, we only require a stationary point of the return rates to represent an equilibrium condition.

The Negishi approach is challenged when spillover effects have to be taken into account. This is first due to the need to model trade flows in a bilateral form. In its common application (cf. Leimbach

Table 1
Adjustment process

Negishi approach	Modular approach
Iterative adjustment of welfare weights	Iterative adjustment of flow boundaries
Equal rates of return in each iteration	Intertemporal budget constraint is balanced in each iteration
Iteratively obtaining balanced intertemporal budget constraint	Iteratively obtaining equalized rates of return on capital

and Toth, 2003), the Negishi approach is based on the shadow prices of tradeables as derived from the trade balance equation (X_i represents net export of region i):

$$\sum_{i=1}^{n} X_i = 0.$$

Within the bilateral model formulation (with k as another region index), this trade balance changes to

$$\sum_{i=1}^{n} \left(\sum_{k=1}^{n} X_{ik} - \sum_{k=1}^{n} X_{ki}\right) = 0.$$

This can be transformed to

$$\sum_{i=1}^{n} \sum_{k=1}^{n} (X_{ik} - X_{ik}) = 0.$$

This equation is met by any arbitrary value for X_{ik} and results in an infeasibility of any change of the right-hand side. Hence, this trade balance has no meaningful shadow price and prevents the Negishi approach from being operable.

4.2. Modified Negishi approach

Using alternative shadow prices could be a way out. Therefore, we reformulated the model described above to make it applicable under the Negishi approach. This reformulation includes all constraints from the region module (except for Eqs. (8) and (9)) supplemented by a global welfare objective function (with global welfare maximum W and Negishi weights w):

$$W = \max \sum_{i=1}^{n} w_i \cdot U_i \tag{25}$$

and a trade restriction analogous to Eq. (20).

The shadow prices of the instances of the production functions (Eq. (2)) can then be used to obtain consumption goods prices and investment goods prices. These prices allow us to compute the intertemporal budget constraint (analogously to the left-hand side of Eq. (23)) and to adjust the Negishi weights.

The modified Negishi approach shares the same adjustment process as the traditional Negishi approach. It only differs in the origin of the prices that underlie the iterative process of obtaining a balanced intertemporal budget constraint. The equivalence of the used production prices with market prices, however, can only be guaranteed if there is an unrestricted flow of goods and capital. The modified Negishi approach may even then fail in the case of existing externalities. Spillovers exhibit price-relevant external effects that are bound to interregional linkages. They cannot be grasped by production prices. In contrast, the modular approach provides a straightforward way to derive market-relevant prices by means of Eqs. (15) and (16). Global optimality in the case of externalities, however, cannot be guaranteed for the modular approach either.

5. Model experiments

The primary aim of numerical experiments is to validate the model presented in Section 3. This is done, first, by testing the convergence behavior, second, by contrasting the results with those of the modified Negishi approach, and third, by means of sensitivity analyses. A particular focus will be on the comparison of the results of the modular and the Negishi approach in the spillover case.

As an instance of the function f (Eq. (1)), we apply a common logarithmic or Bernoullian utility function (cf. Nordhaus, 1994, p. 12):

$$f(C) = L \cdot \ln \frac{C}{L}. \tag{26}$$

L represents the regions' population. It simultaneously forms the exogenously given production factor labor. The production function for the consumption goods sector is specified as a Cobb-Douglas function (with the production factors capital K, labor L, energy E and the elasticity parameters α and η):

$$Y^G = \mathrm{e}^{\kappa t} A \cdot [(1-\theta)K]^{\alpha} \cdot L^{\eta} \cdot E^{1-\alpha-\eta}. \tag{27}$$

In addition to productivity changes induced by spillover effects, A is assumed to change exogenously according to growth rate κ. Variable θ denotes the share of total capital stock which is allocated to the investment goods sector. Investment goods production is assumed to be a function of capital only:

$$Y^F = \psi \cdot (\theta K)^{\phi}. \tag{28}$$

The elasticity parameter ϕ is assumed to be constant over time for simplicity reasons. With ϕ equal to 1, this equation becomes a Leontief-type production function and parameter ψ could be interpreted as technological coefficient (investment goods output per unit capital stock). Following the neoclassical assumption of diminishing marginal productivity, we chose a value for ϕ that is close to, but significantly lower than 1. This production function then exhibits decreasing returns to scale, and the conventional rewarding of factor prices in accordance with its marginal productivities is incomplete. Implicitly, there is a kind of monopoly rent that restricts further entries of firms. However, this does not limit the model's capability in finding an interregional competitive equilibrium, which is the focus here.

The production factor energy results from transforming primary energy carriers, either domestic (Y^R) or foreign (X^R) ones:

$$E_i = \nu_i \cdot (Y_i^R - X_{ik}^R + X_{ki}^R). \tag{29}$$

Primary energy carriers are assumed to represent a third tradable good (with $R \in J$, while goods set J is extended to $J = \{G, F, R\}$). While this intentionally provides additional variety in regional interactions, we simultaneously restrict intraregional complexity by assuming the resource extraction function to be modeled as a time trend (with ω as growth rate):

$$\dot{Y}^R = \omega Y^R. \tag{30}$$

Next, we specify the technological spillover function h:

$$h = \dot{A}_{ir} = \sum_{k=1}^{n} \left[\left(\zeta_1 \cdot \frac{X_{kir}^F}{K_{ir}} \right)^{\zeta 2} \cdot \beta \cdot \max\left(0, A_{k,r-1} - A_{i,r-1}\right) \right]. \tag{31}$$

Note that the foreign direct investments variable is divided by the capital stock in order to avoid scaling effects (otherwise larger regions would get higher productivity gains). While ζ_1

represents an additional scaling parameter, ζ_2 ($0<\zeta_2<1$) depicts the elasticity of productivity changes on foreign direct investments. β represents a spillover coefficient, i.e. the intensity of technological spillovers.

Finally, we introduce for diagnostic purposes only a current account (CA) equation:

$$\mathrm{CA}_{ik} = \sum_j (\tilde{p}^j / \tilde{p}^G \cdot [X_{ik}^j - X_{ki}^j]) \tag{32}$$

and a net foreign assets (FA) equation (with CT=CA, following a simplified balance of payment concept):

$$\dot{\mathrm{FA}} = \mu \cdot \mathrm{FA} + \mathrm{CT}. \tag{33}$$

The return rates on capital (μ) are computed as negative growth rates of the marginal utilities of capital represented by the shadow prices λ of the capital stock Eq. (4):[5]

$$\mu = -\frac{\dot{\lambda}}{\lambda}. \tag{34}$$

The time horizon of model simulations is from 1990 to 2050, including twelve 5-year time steps. We omit terminal and transversality conditions beyond the intertemporal budget constraint. Thus, terminal effects may occur in numerical results. This, however, only affects intraregional dynamics. Due to the presence of the intertemporal budget constraint (Eqs. (19) and (23)) there is no limitation for the search for the competitive interregional equilibrium.

We started with a model including the following generic regions:

- IR—developed world region;
- DR—developing world region.

The developed region is characterized by a higher productivity level and a higher initial capital stock (per capita), the developing region by higher energy resources endowment and population growth. While data are used that are in an order of those provided by international databases, the model is not at all calibrated. Hence, results should only be interpreted in a qualitative sense. All modules are programmed in GAMS (Brooke et al., 1992) and numerically solved with the nonlinear programming solver CONOPT3. As part of the results of a single optimization run, GAMS is capable to provide shadow price data (called marginal values). The programs are available from the authors upon request. For the selected parameters and initial values see Appendix A.

5.1. Non-spillover case

Within a first set of model experiments based on the modular approach, we assume that technological spillovers do not exist and technical progress is completely exogenous. In analyzing the results, we first take a look at the convergence behavior. The upper graphics of Fig. 2

[5] The negative growth rate of the marginal utility of capital corresponds to the marginal productivity of capital.

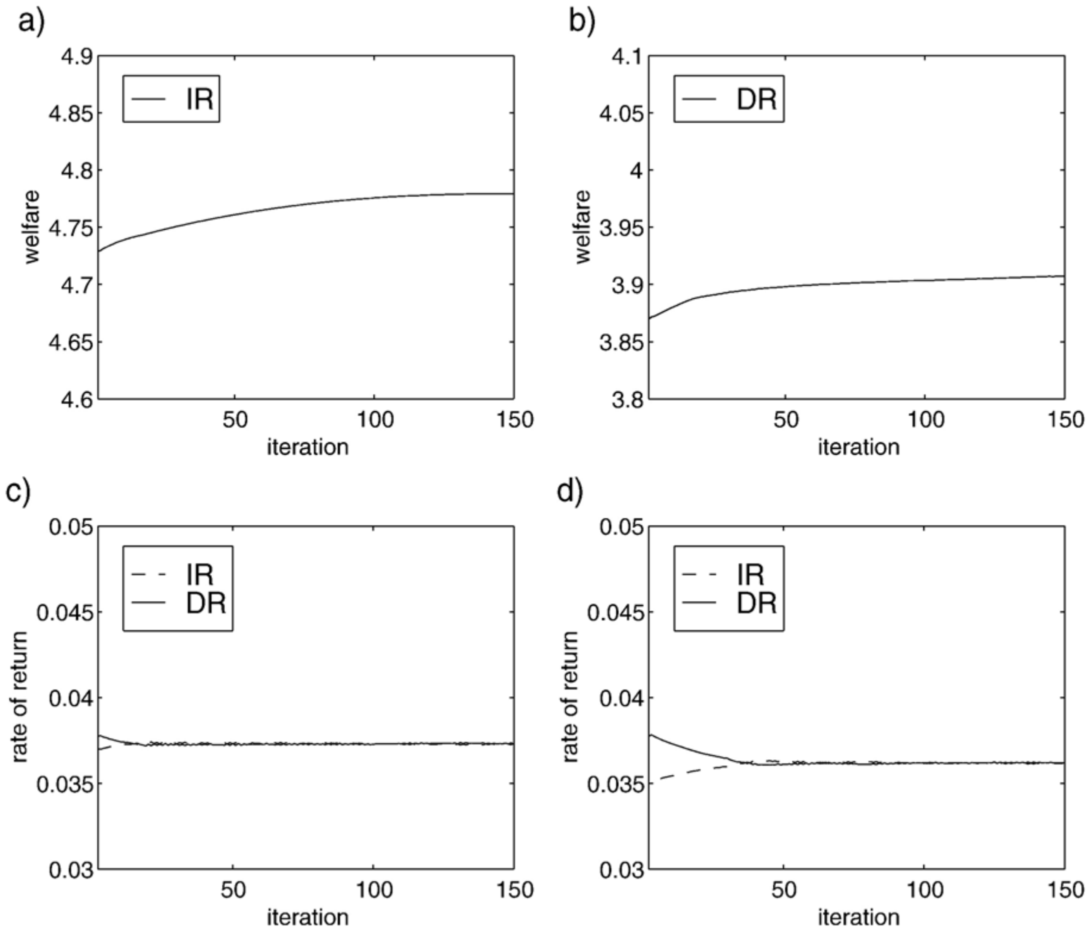

Fig. 2. Convergence of welfare (a and b) and return rates on capital in 2015 (c and d) (IR—developed world region, DR—developing world region).

demonstrate the convergence of welfare measures. Around 100 iterations are needed in this example. Convergence can be sped up by increasing parameter γ. This, however, will simultaneously increase the risk of failing convergence.

The lower graphics show the convergence of the return rates on capital of the year 2015. The convergence process is almost completed after 50 iterations. This, however, is not a uniform result. The return rates in other years exhibit a different speed of convergence. Moreover, the required number of iterations also depends on the initial flow boundaries and, in particular, on the initial ratio between capital stock and output. Fig. 2d resulted from a model run where the initial capital stock in IR was increased from 6 to 8 trillion US dollars. The implied divergence of initial return rates delays convergence. Nevertheless, the convergence behavior is quite robust.

As a next quality check of the modular approach, its results are compared to those from the Negishi approach. First, it turns out that the return rates presented in Fig. 2 converge towards the respective values that result from the Negishi approach. Second, the Negishi solution in terms of main variables can be reproduced completely with a sufficient numerical precision. Given the methodological differences, this correspondence is remarkable. Correspondence appears after around 50 iterations for per capita consumption, gross product, and welfare figures. More iterations are needed to obtain export trajectories that fit to each other. Nevertheless, even the trade pattern could be reproduced quite well. Fig. 3 shows the convergence of the current account

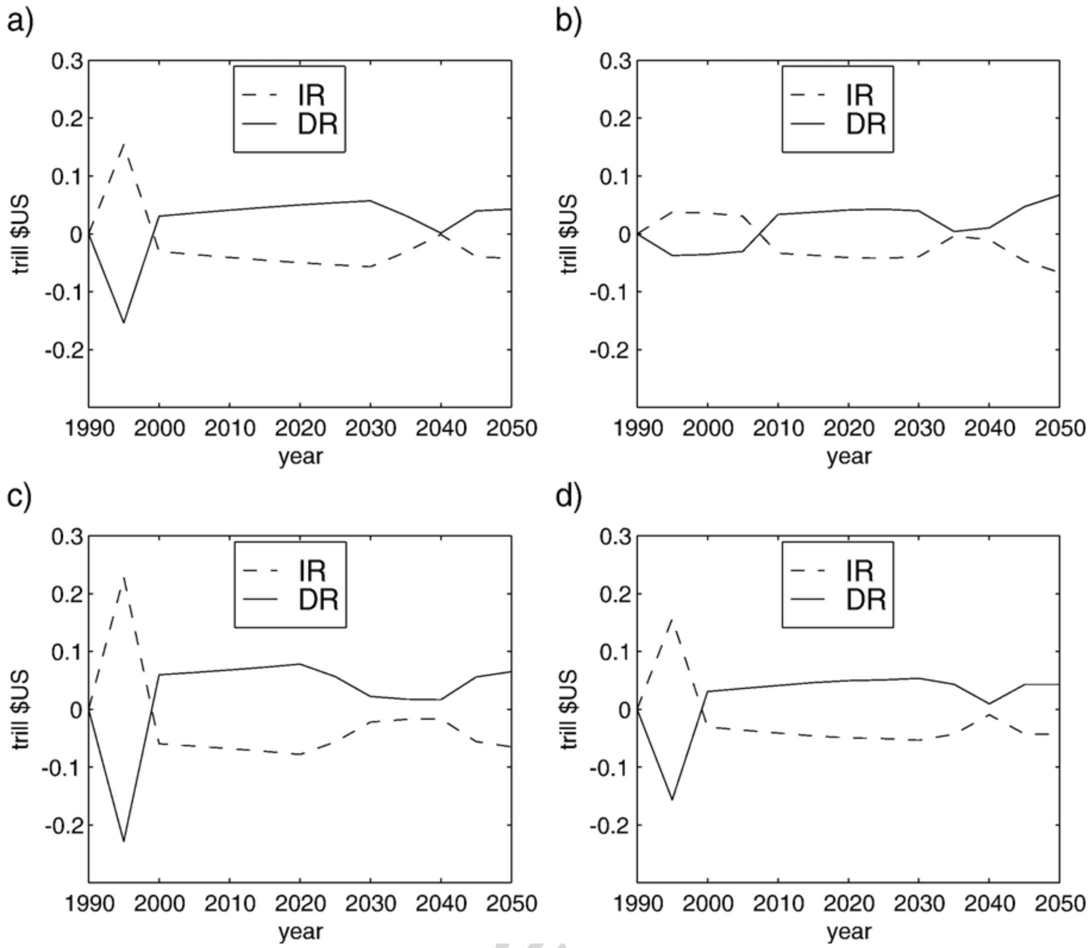

Fig. 3. Current account resulting from the Negishi approach (a) and from the modular approach after (b) 10 iterations, (c) 40 iterations, (d) 300 iterations (IR—developed world region, DR—developing world region).

obtained from the modular approach as well as the correspondence with the current account obtained from the Negishi approach. In the following we will analyze the trade and foreign investment structure indicated by Fig. 3.

Due to differences in resources endowments and productivity, the regions' trade profits vary. The developing region, endowed with affluent resources, benefits most from trade and international capital mobility (2.2% in relative welfare units compared to 1.2% for the developed region). A major merit of the model is its capability to provide insights into the dynamics of regional interactions. Fig. 4 illustrates some details of the intertemporal and bilateral trade structure. Besides the expected result that the developing region exports primary energy, we see that there is a sustained export of final goods and temporary export of investment goods from IR to DR. Primary energy exports will lead to leveling off the current account deficit of DR and reducing the foreign assets of IR, which arise due to substantial initial foreign direct investments of IR in DR.[6] This is a typical result within an intertemporal optimizing framework. There is a bias to immediately adjust the capital stock to a level from which it is easier to approach the steady state. The level of these "induced" foreign direct investments depends on the initial capital stock

[6] Note that in Figs. 4 and 7 net exports are represented by their physical equivalents, partly measured in $ units. No price-relevant information (discounting, changes in relative prices) is included.

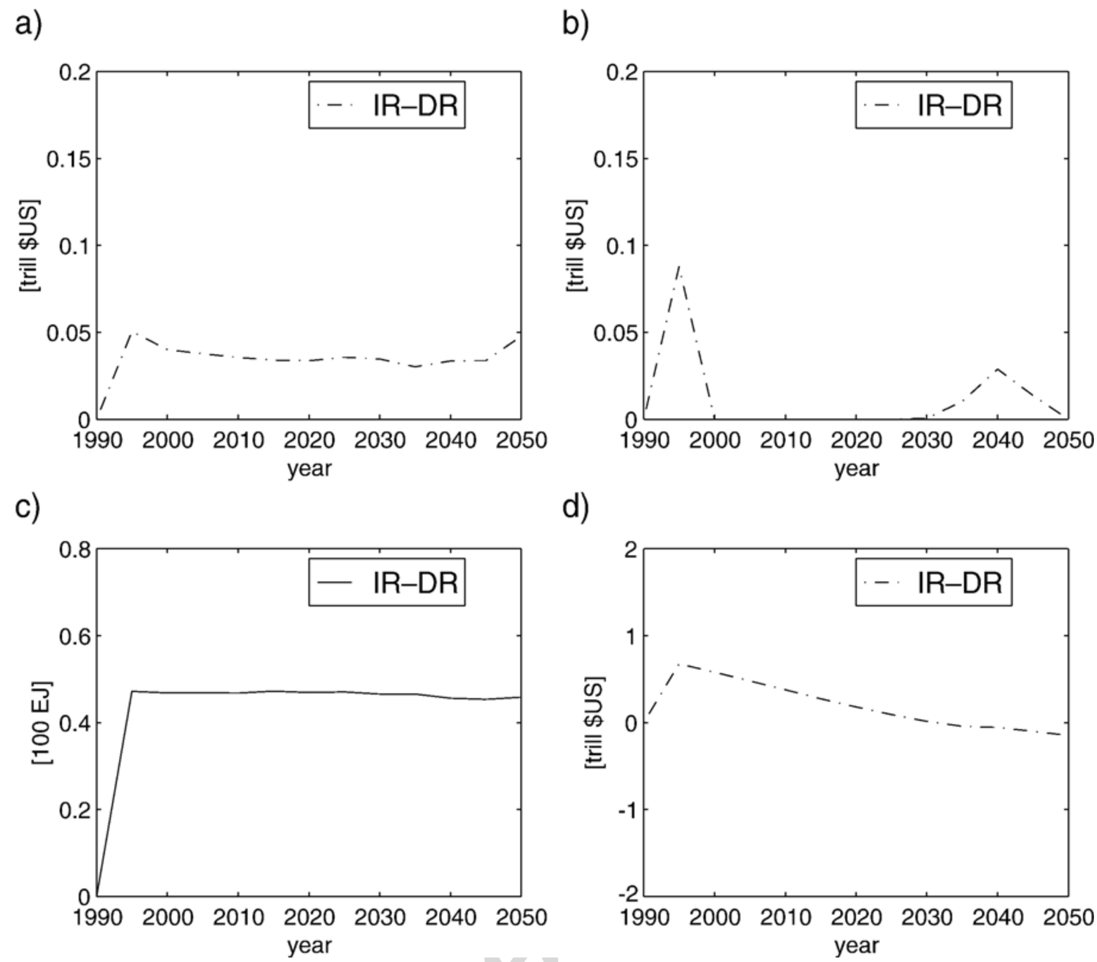

Fig. 4. Net export of (a) consumption goods, (b) investment goods, (c) primary energy (first mentioned region in the legend always denotes the exporter), and (d) net foreign assets—discounted (IR—developed world region, DR—developing world region).

levels. Increasing the initial capital stock in IR (from 6 to 8 trillion US dollars) leads to a reversed trade pattern with respect to consumption goods. IR's exports of investment goods increase further. DR's resulting current account deficit is equalized by primary energy and consumption goods exports.

5.2. Spillover case

Within the second set of model experiments, we take technological spillovers into account. Consequently, technological progress is partly endogenized. From an economic point of view, the question arises of whether there are welfare gains for both regions and what changes result in the trade and capital flow structure. From a methodological point of view, the more important question relates to the differences between the Negishi approach and the modular approach. Is there any result that would demonstrate the value of the modular approach beyond that of a computationally expensive substitute of the Negishi approach?

In Fig. 5, we contrast the welfare results of the Negishi and the modular approach. More precisely, welfare differences between several spillover cases (which differ by spillover intensity) and the non-spillover case are shown for both approaches. In order to get a broad picture, we varied the spillover intensity over a wide range. Considering the combined results for IR and DR, it turns out that in almost

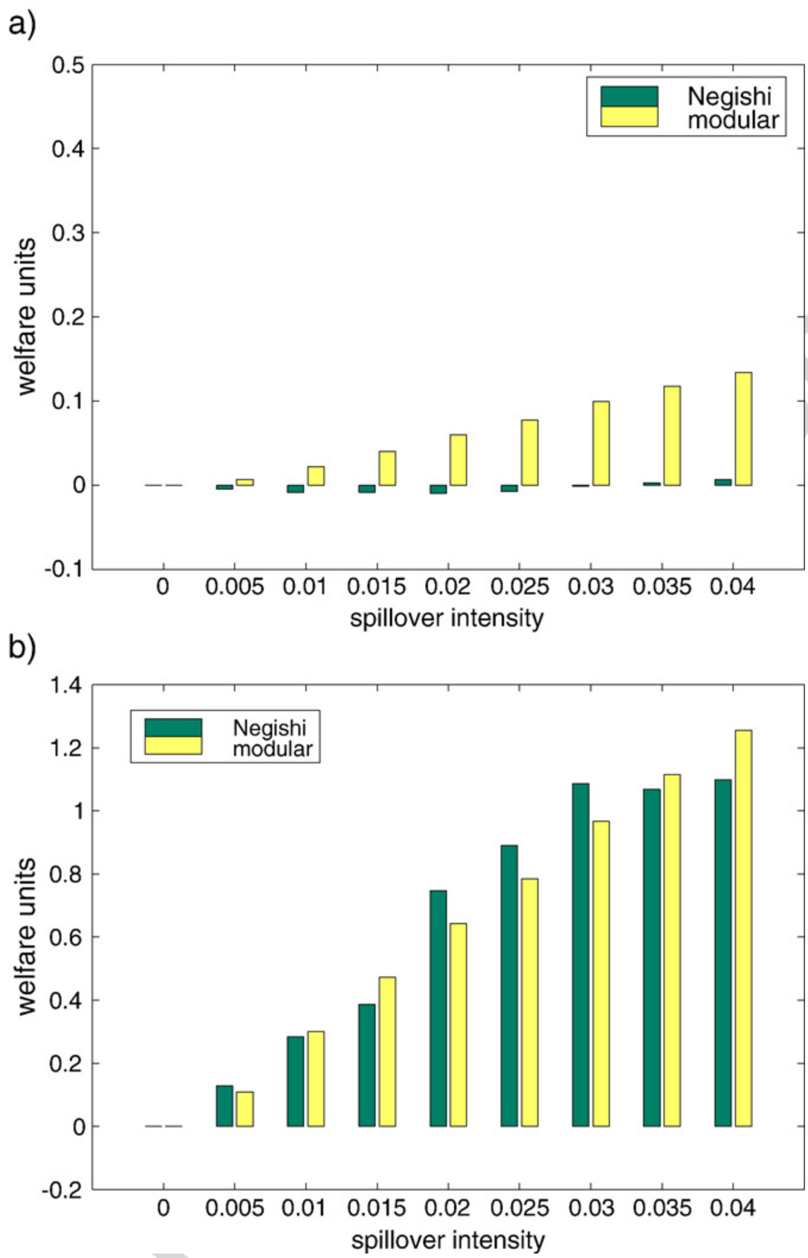

Fig. 5. Welfare differences between spillover and non-spillover cases in (a) IR (developed world region) and (b) DR (developing world region).

half of the single variants the modular approach dominates the Negishi approach in terms of the objective value. In particular, with the modular approach we obtain higher welfare values for IR in each case and for DR in some cases.

The reason for the differences between the Negishi and the modular approach can be found in the usage of shadow price information. Externalities caused by the spillover effect lead to a divergence of the marginal values of exports and imports of the same good. This, precisely, applies to the investment goods prices of the host country of spillovers. The modified Negishi approach computes goods prices on the basis of the shadow prices of the sectoral production functions (Eqs. (27), (28) and (30)). But based on these production prices, external effects that result from regional interactions cannot be captured completely. The modular approach, in contrast, is able to completely internalize the spillover effect. This is due to an adjustment process

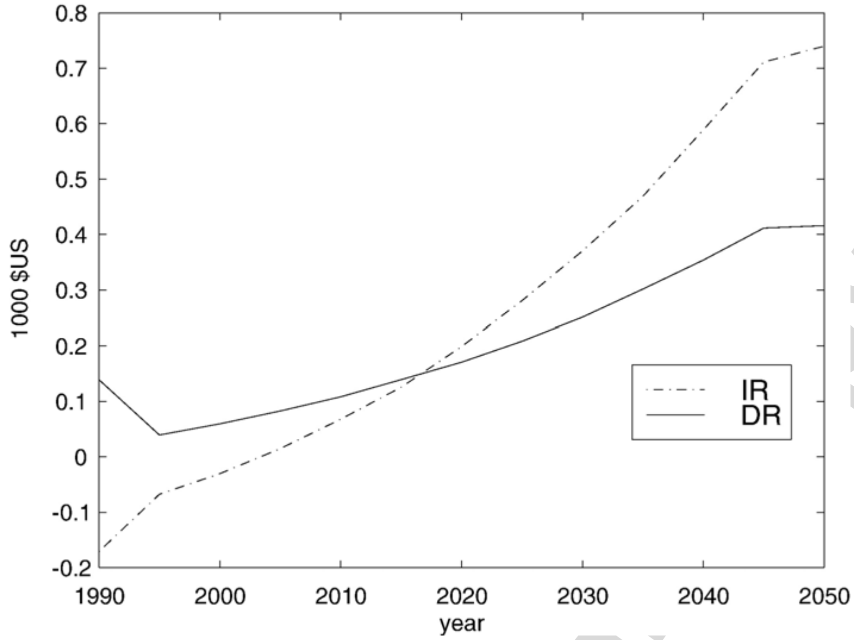

Fig. 6. Per capita consumption gains from spillover effects (IR—developed world region, DR—developing world region).

which is based on shadow prices of export and import bounds as introduced in the region modules (Section 3.1). In the presence of spillover effects, the modular approach is clearly preferable. This statement holds unless we would argue that the related positive external effects cannot be anticipated.

Taking a look at Fig. 5 from the perspective of the modular approach only, we observe two expected results: first, the welfare gains are higher, the higher the spillover intensity is, and second, the welfare gains are higher for the developing region—the recipient of spillovers. This pattern is more differentiated when looking at the dynamics. Assuming a default spillover intensity of β=0.015, Fig. 6 illustrates the consumption gains from the spillover effect. While gains in both regions increase with time, there are yet significant differences in the patterns of gains. The developing region gains in all periods. The increase is moderate. The developed region, in contrast, loses in initial periods, but gains more later. Due to the discounting effect, consumption gains of the developed region do not become manifest in an equal increase in welfare. The developing region increases welfare by 13.2% in relative terms, whereas the developed region increases welfare by 1.9% only. The level of gains, in general, depends on the specification of parameter β. There is no empirical foundation for β so far. Hence, we shall stress again that this result can only be interpreted in a qualitative sense.

With spillover effects being taken into account, trade in investment goods claims significant shares in total trade—see Fig. 7.[7] DR receives a sustained flow of foreign direct investments. Fig. 7, furthermore, shows an overall intensification of trade and capital mobility in the spillover case (compared to Fig. 4). Consumption and welfare gains of the developing region are directly linked to productivity increases caused by technological spillovers. Positive feedbacks to the developed

[7] Fig. 7 illustrates results from the modular approach as well as from the Negishi run. The qualitative pattern is quite similar. The lower total amount of foreign investments with the Negishi results (in a number of periods there is no foreign investment at all), however, indicates a lower level of technological spillovers to become realized.

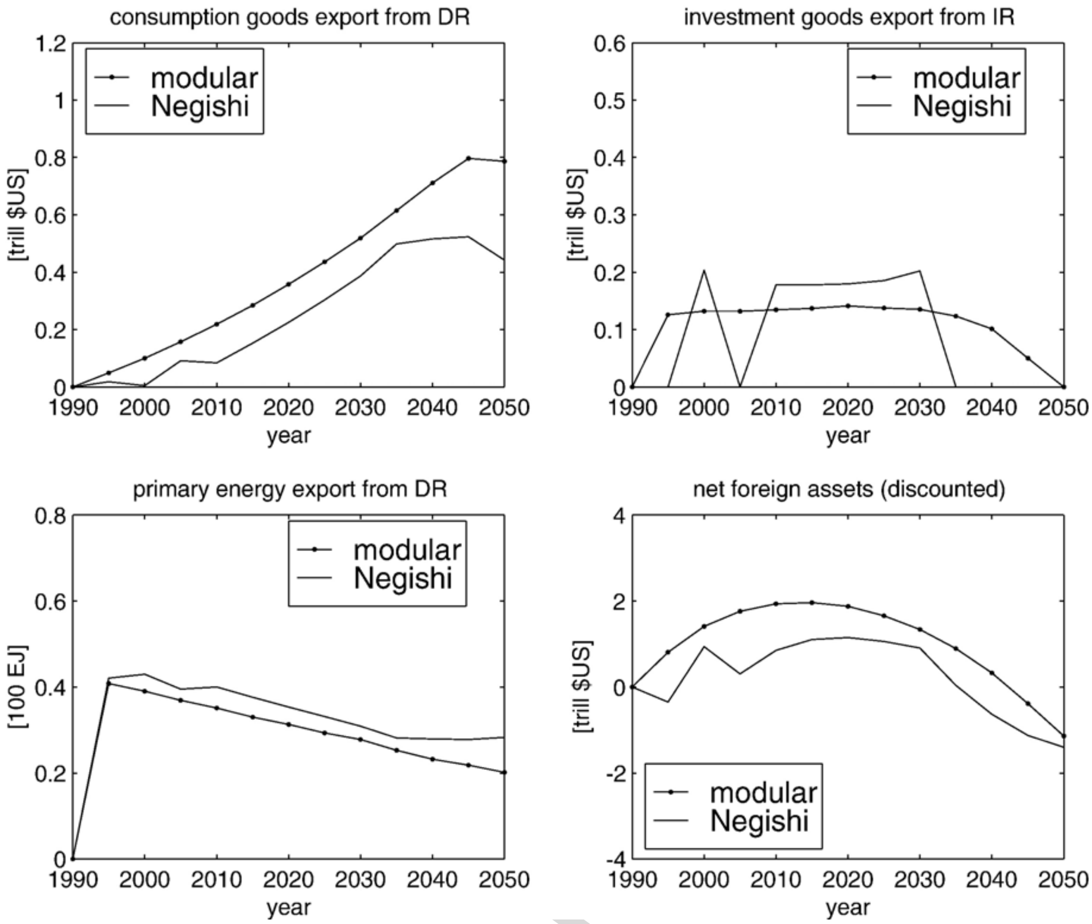

Fig. 7. Trade structure and net foreign assets in the spillover case.

region, while on a moderate level only, are mainly due to higher prices of investment goods in comparison to prices of consumption goods. The developed region being the exporter of investment goods benefits from this. In order to meet the intertemporal trade balance and leveling off the net foreign assets, respectively, the developing region compensates the expansion of investment goods imports (compared to the non-spillover case) increasingly by consumption goods exports.

By introducing spillover effects the mathematical model structure becomes non-convex and multiple optima may exist. Within the modular approach, this does not apply to the single models (region modules and trade module). Within the region modules, the representative agents have no direct control over this external effect. However, the spillover effect influences the shadow prices in the region modules and due to the exchange of shadow price information, the spillover effect gets into consideration within the trade module. Consequently, the modular approach provides a solution which corresponds to a situation where the representative agents anticipate technological spillovers. The above algorithm of finding the optimal solution to the multi-region optimization problem does not guarantee that a global optimum is found. Nevertheless, the convergence process in finding the local optimum, although not as smooth as the convergence in the non-spillover case (see Fig. 2), is quite robust (see Fig. 8).

We ran sensitivity analyses in order to study the robustness of the modular approach. The spillover coefficient β was varied within the interval [0, 0.04]. Smooth changes of welfare

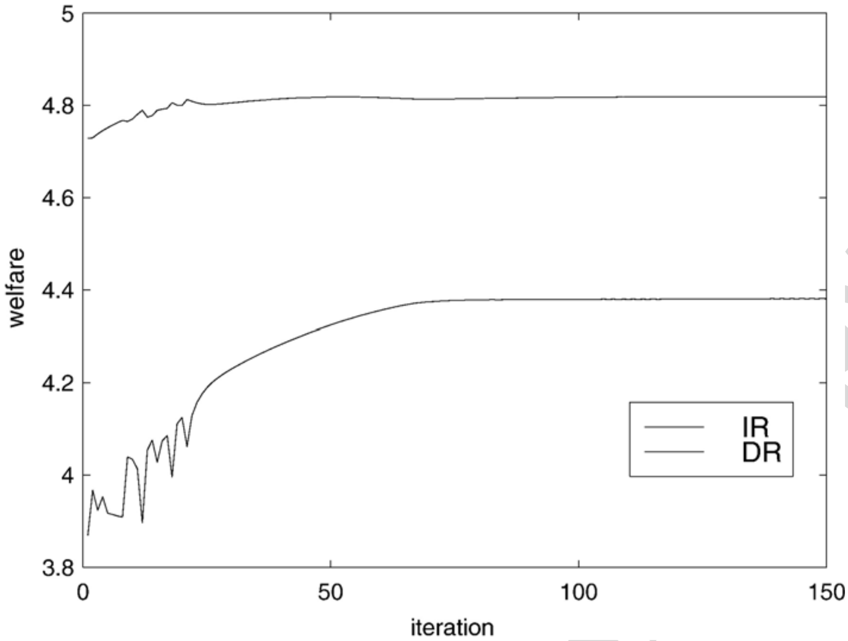

Fig. 8. Convergence of welfare measure (IR—developed world region, DR—developing world region).

and per capita consumption occurred over a wide range (see Figs. 5 and 9). This demonstrates robustness. However, even with moderate spillover intensity, the regional return rates on capital do not converge to a common level. This is due to the external effect and corresponds with the findings of Farmer and Lahiri (2005). Whereas with moderate spillover intensity, the return rates converge to only slightly different levels (Fig. 10 a and b), increasing the spillover coefficient further will result in a significant deviation of the return rates (Fig. 10 c

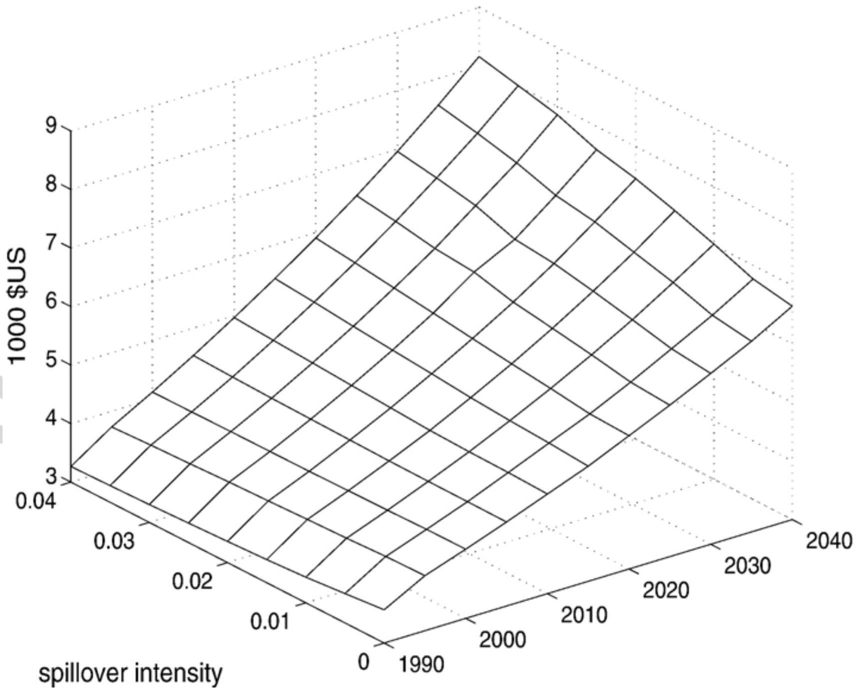

Fig. 9. Per capita consumption sensitivity on spillover intensity in IR (developed world region).

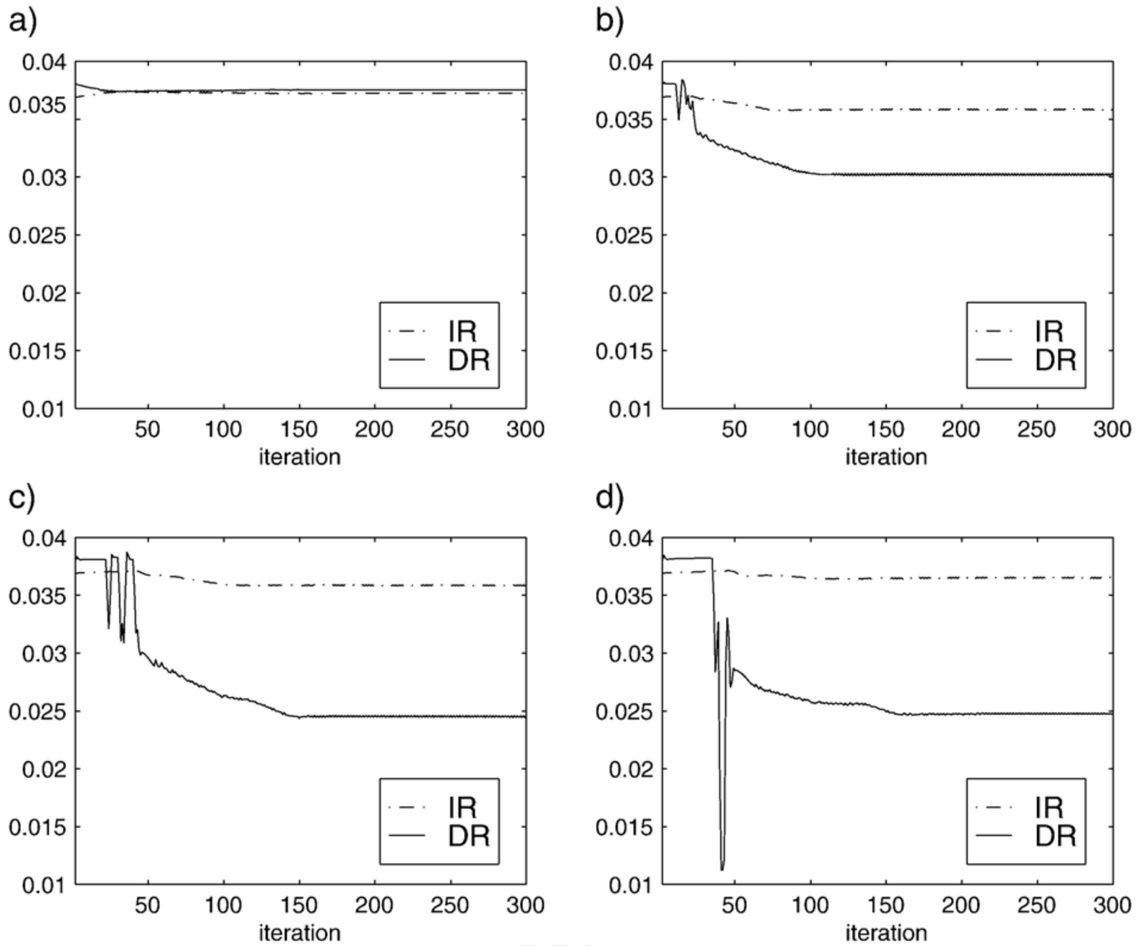

Fig. 10. Convergence of return rates in 2010 with varying spillover intensity: (a) β=0.0, (b) β=0.02, (c) β=0.04, (d) β=0.06 (IR—developed world region, DR—developing world region).

and d).[8] The latter brings the model into a state where robustness disappears and the algorithm fails to find a stationary point for the return rates. Thus, one crucial point of further research is to empirically find the reasonable range of the spillover intensity. Empirical foundation and sensitivity analyses are also needed with respect to the other parameters of the technological spillover function ζ_1 and ζ_2.

6. Conclusions

We presented a novel approach to multi-region modeling which is applicable in an intertemporal optimization framework. We compared the results from this modular approach with those from the well-known Negishi approach. In a first step, we showed the correspondence between the solutions of both approaches for a problem setting without externalities. This is remarkable in face of the technical differences of the underlying algorithms. Hence, welfare optimality, which is proven for the Negishi approach under common convexity assumptions (Negishi, 1972), is indicated for the modular approach. Welfare and pareto optimality, respectively, cannot be guaranteed when spillover effects are taken into account. The modular

[8] With the modified Negishi approach, convergence fails for a spillover intensity beyond 0.05.

approach has an important advantage in this case since the Negishi approach has limited capabilities to model them. We demonstrated the advantage of the modular approach. Supplemented by robustness properties, the modular approach turns out to be a superior alternative in modeling interregional linkages including externalities like technological spillovers.

However, the non-convex model structure implied by integrating spillover effects challenges the modular approach too. With the possibility of multiple local optima, a general problem of trade simulations in an optimal control framework becomes apparent: in a multi-regional setting, quite different trade patterns can produce similar welfare. Careful empirical calibration is important to bound optimal trade flows to plausible ranges. Moreover, modeling spillovers from foreign investments on a net base can be criticized. On the other hand, despite deficiencies in the model structure, the fact that there is only a small positive feedback from technology spillovers to the technologically leading world region (which in essence loses in relative terms) might give a new argument to the Lucas Paradox (cf. Lucas, 1990, 'Why Doesn't Capital Flow from Rich to Poor Countries?'). It partially explains why real world capital transfers towards developing regions are not as high as could be expected from return rate differentials.

Appendix A

Default parameters and initial values

ρ	0.03
δ	0.08
ν	1.0
β	0.015
ζ_1	$0.01 \cdot \frac{\tilde{p}^F}{\tilde{p}^G}$
ζ_2	0.4
γ	0.002
ϵ	0.0000001
κ	0.008
α_{IR}	0.28
α_{DR}	0.3
η_{IR}	0.67
η_{DR}	0.6
ψ_{IR}	0.16
ψ_{DR}	0.16
ϕ	0.9
ω	0.01
k_{IR}	6 trillion $US
k_{DR}	4 trillion $US
a_{IR}	2.5
a_{DR}	1.2
$L_{\mathrm{IR}}(0)$	0.5 billion (constant)
$L_{\mathrm{DR}}(0)$	0.8 billion (grows by 1%)
$Y^R_{\mathrm{IR}}(0)$	20 EJ
$Y^R_{\mathrm{DR}}(0)$	120 EJ
FA(0)	0.0
$\bar{X}^j_l$	0.0

References

Bahn, O., Haurie, A., Kypreos, S., Vial, J.-P., 1998. Advanced mathematical programming modeling to assess the benefits from international CO_2 abatement cooperation. Environmental Modeling and Assessment 3, 107–115.

Blomström, M., Globerman, S., Kokko, A., 1999. The determinants of host country spillovers from foreign direct investments: review and synthesis of the literature. SSE/EFI Working Paper Series in Economics and Finance, vol. 339. School of Economics, Stockholm.

Brooke, A., Kendrick, D., Meeraus, A., 1992. GAMS—A User's Guide, Release 2.25. The Scientific Press, San Francisco.

Cunat, A., Maffezzoli, M., 2004. Neoclassical growth and commodity trade. Review of Economic Dynamics 7, 707–736.

Dixon, P.B., 1975. The Theory of Joint Maximization. North-Holland, Amsterdam.

Farmer, R., Lahiri, A., 2005. A two-country model of endogenous growth. Review of Economic Dynamics 8, 68–88.

Ginsburgh, V.A., Waelbroeck, J.L., 1981. Activity Analysis and General Equilibrium Modelling. North-Holland, Amsterdam.

Greiner, A., Semmler, W., 2002. Externalities of investment, education and economic growth. Economic Modelling 19, 709–724.

Hejazi, W., Safarian, A.E., 1999. Trade, foreign direct investment, and R&D spillovers. Journal of International Business Studies 30, 491–511.

IMF, 1993. Balance of Payment Manual. International Monetary Fund, Washington.

Kumar, A., Shubik, M., 2004. Variations on the theme of Scarf's counter-example. Computational Economics 24, 1–19.

Lee, J.-W., 1995. Capital goods import and long-run growth. Journal of Development Economics 48, 91–110.

Leimbach, M., Toth, F.L., 2003. Economic development and emission control over the long term: the ICLIPS aggregated economic model. Climatic Change 56, 139–165.

Lucas, R.E., 1990. Why doesn't capital flow from rich to poor countries? American Economic Review 80, 92–96.

Luenberger, D.G., Maxfield, R.R., 1995. Computing economic equilibria using benefit and surplus functions. Computational Economics 8, 47–64.

Manne, A.S., Richels, R., 1992. Buying Greenhouse Insurance: the Economic Cost of Carbon Dioxide Emission Limits. The MIT Press, Cambridge.

Manne, A.S., Richels, R., 1995. The greenhouse debate: economic efficiency, burden sharing and hedging strategies. The Energy Journal 16, 1–37.

Manne, A.S., Rutherford, T.F., 1994. International trade, capital flows and sectoral analysis: formulation and solution of intertemporal equilibrium models. In: Cooper, W.W., Whinston, A.B. (Eds.), New Directions in Computational Economics. Kluwer, Dordrecht, pp. 191–205.

Manne, A.S., Mendelsohn, M., Richels, R., 1995. MERGE—a model for evaluating regional and global effects of GHG reduction policies. Energy Policy 23, 17–34.

Negishi, T., 1972. General Equilibrium Theory and International Trade. North-Holland, Amsterdam.

Nordhaus, W.D., 1994. Managing the Clobal Commons. The MIT Press, Cambridge, MA.

Nordhaus, W.D., Yang, Z., 1996. A regional dynamic general-equilibrium model of alternative climate-change strategies, American Economic Review 86, 741–765. Climatic Change 34, 327–336.

O'Neill, R.P., Sotkiewicz, P.M., Hobbs, B.F., Rothkopf, M.H., Stewart, W.R., 2005. Efficient market-clearing prices in markets with nonconvexities. European Journal of Operational Research 269–285.

Rutherford, T.F., 1992. A sequential joint maximization algorithm for applied general equilibrium problems. Working Paper, University of Western Ontario.

Rutherford, T.F., 1995. Extension of GAMS for complementarity problems arising in applied economic analysis. Journal of Economic Dynamics and Control 19, 1299–1324.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review 42, 283–303.

Scarf, H., 1984. The computation of equilibrium prices. In: Scarf, H., Shoven, J. (Eds.), Applied General Equilibrium Analysis. Cambridge University Press, Cambridge.

Scarf, H., 1990. Mathematical programming and economic theory. Operations Research 38, 377–385.

Takii, S., 2004. Productivity differentials between local and foreign plants in Indonesian manufacturing, 1995. World Development 32, 1957–1969.