Indicating the proximity to a critical threshold: The example of a bifurcation in a stochastic Ocean model

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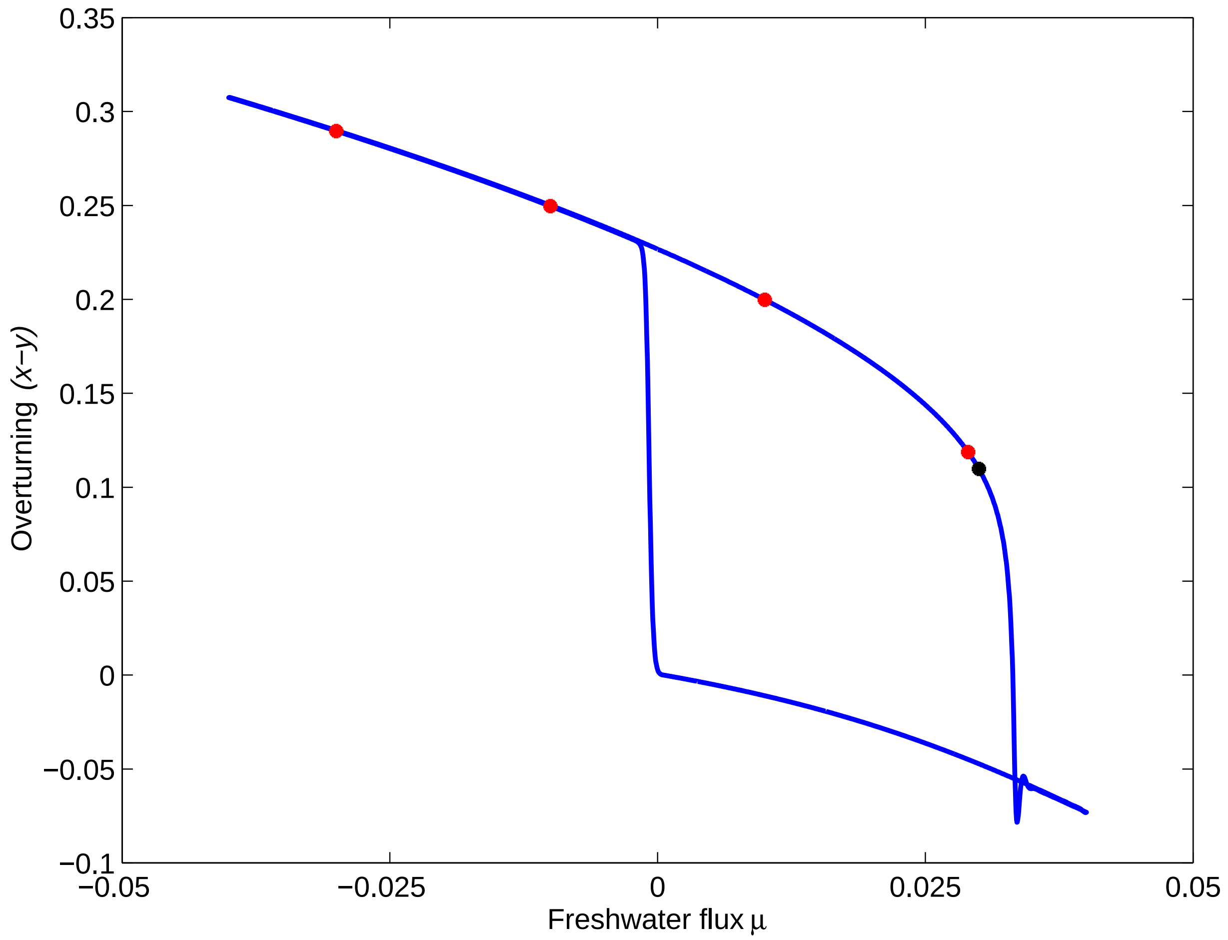
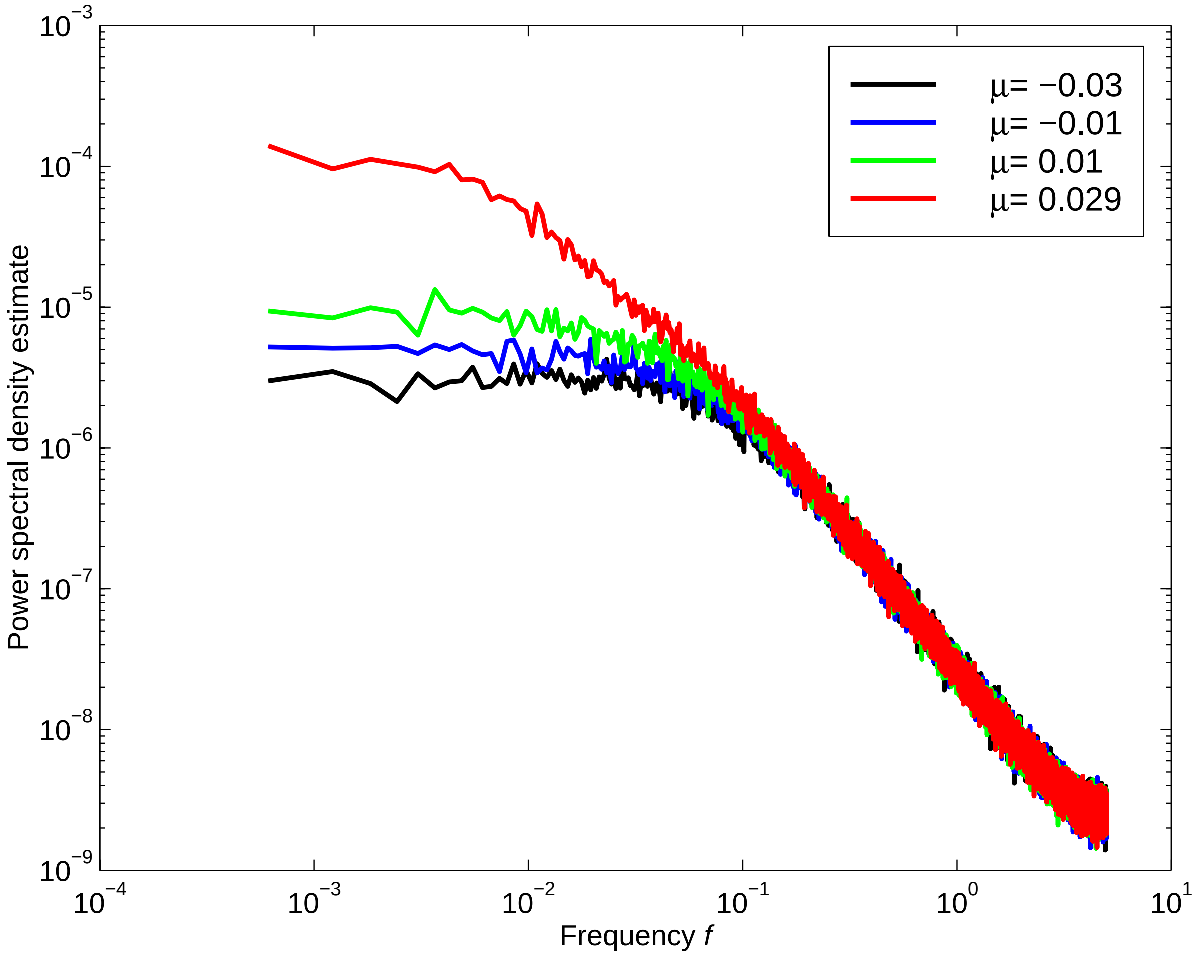

0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
Overturning (x−y)
−0.05
−0.025
0
0.025
0.05
Freshwater flux μ


10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
Power spectral density estimate
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency f
μ= −0.03
μ= −0.01
μ= 0.01
μ= 0.029

Reduced Stommel model

- Assumption: Temperature relaxation much faster than salinity fluctuations

- Stommel model can be reduced to 1-D model describing salinity difference

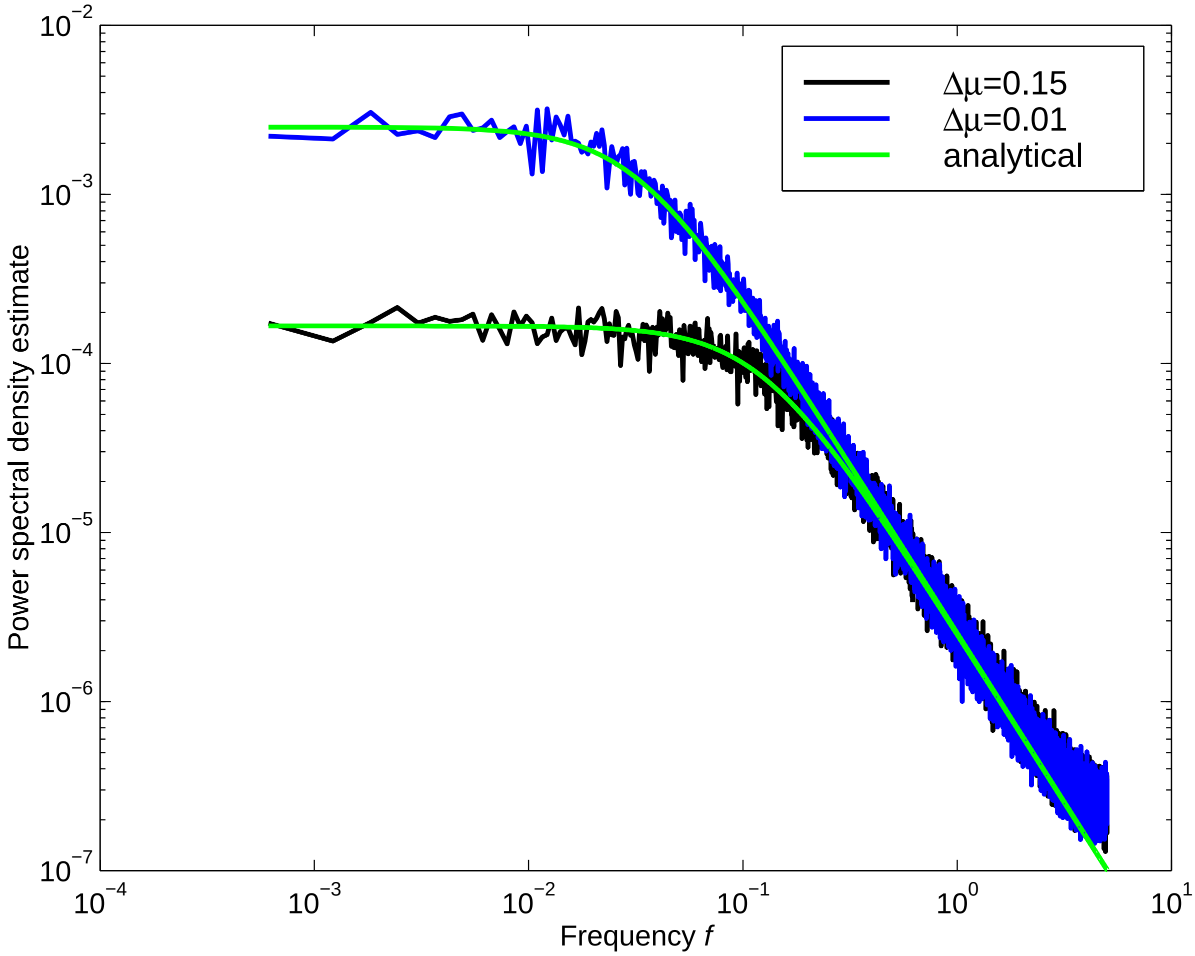
$$\dot{y} = -|1-y|y + \mu + \sigma\xi$$

- y nondimensional salinity difference, μ mean freshwater flux, bifurcation parameter, $\sigma\xi$ stochastic freshwater flux, σ standard deviation, ξ white noise process

- Reduced model retains relevant properties of full model: saddle-node bifurcation, hysteresis behavior

- Allows analytical treatment of stochastic differential equations

Power spectral density



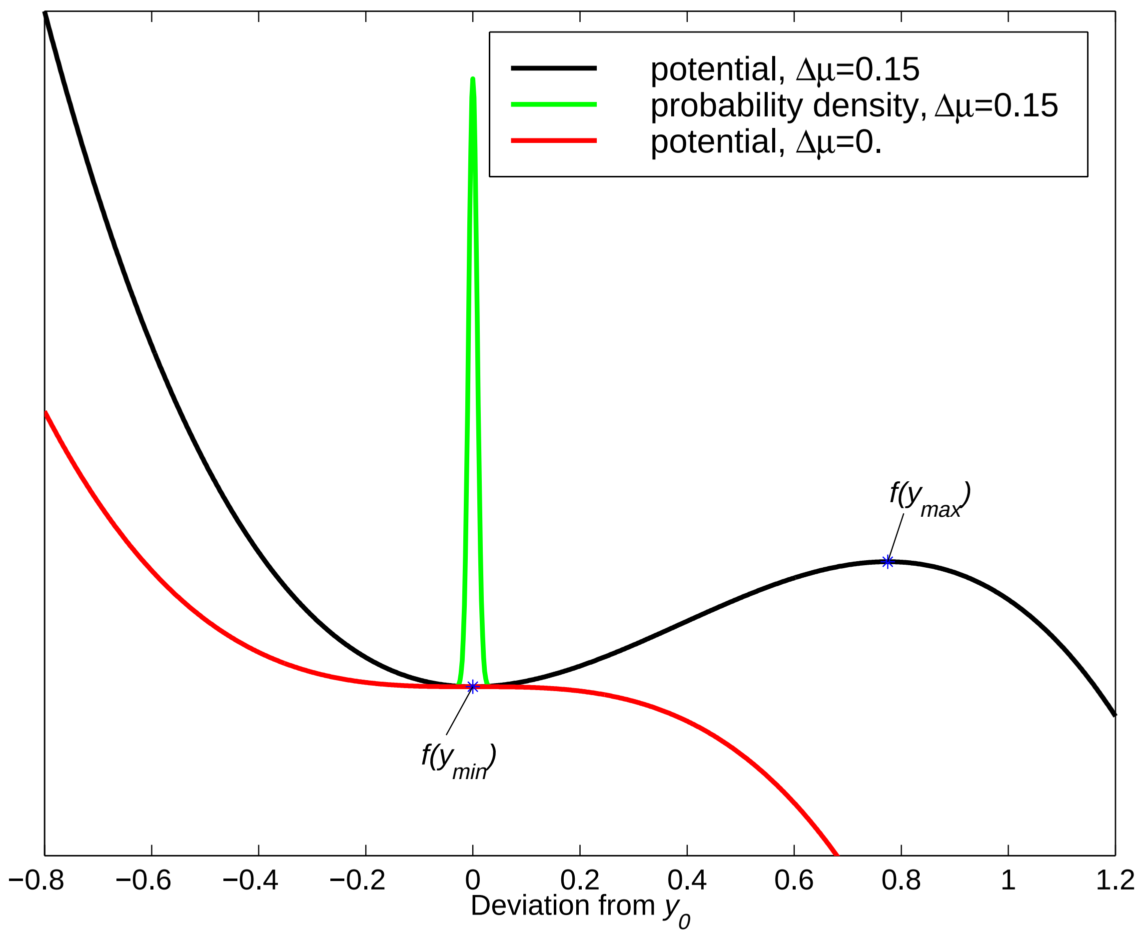
- Analytical expression for power spectral density from linearization

$$S(\omega, \Delta\mu) = \frac{\sigma^2}{4\Delta\mu + \omega^2}$$

- Diffusion coefficient inversely proportional to distance from bifurcation point $\Delta\mu$

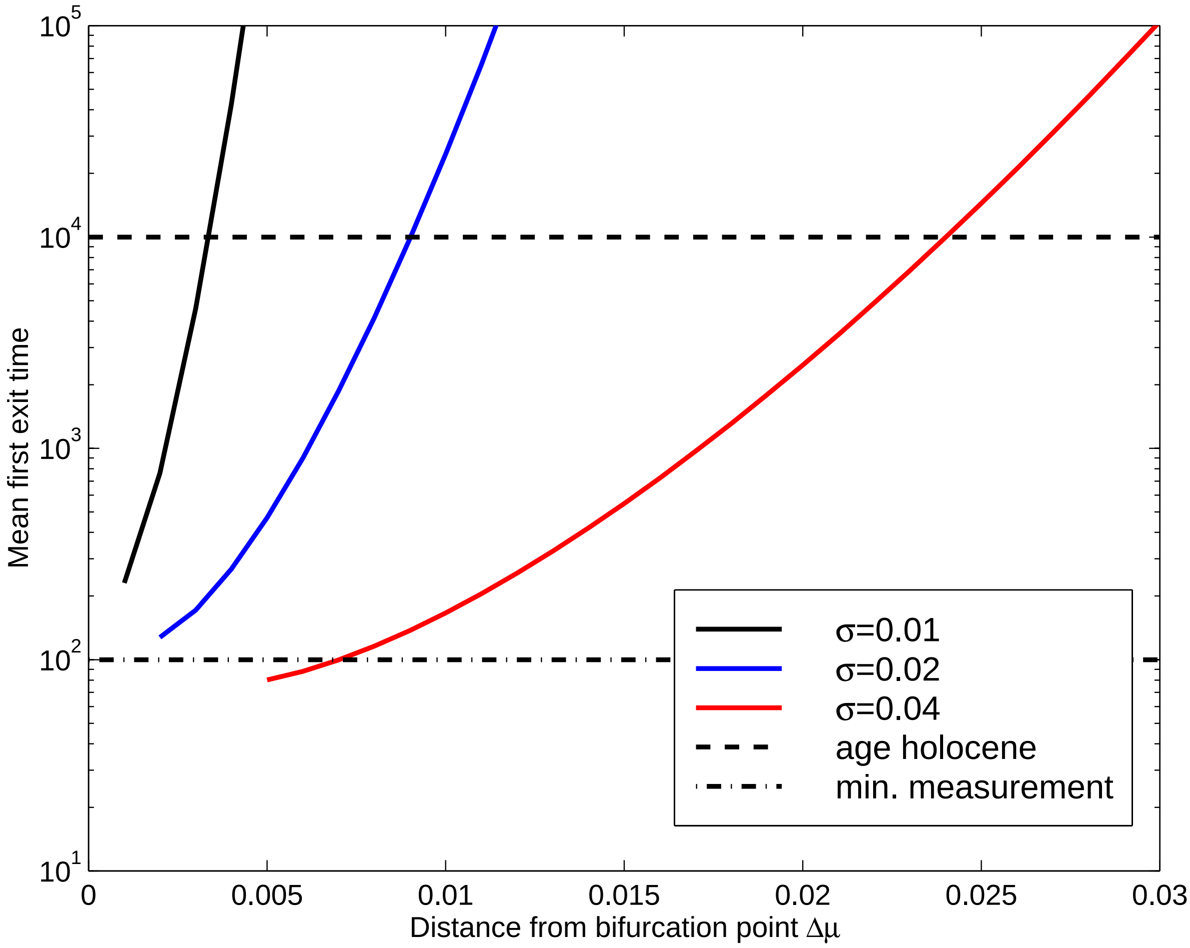
- Cutoff frequency proportional to square root

Probability density at bifurcation



- Steady state propability density: Obtained from Fokker-Planck equation, “potential solution”
- Cubic potential
- System is within potential well
- Depth of potential well decreases as system is moved to bifurcation
- At bifurcation point potential well vanishes

Stability of the THC



- Stability of the THC can be expressed as mean first exit time from potential well

$$\tau(\sigma, \Delta\mu) = 2\pi(4\Delta\mu)^{-\frac{1}{2}} \exp\left(\frac{1}{3\sigma^2}(4\Delta\mu)^{\frac{3}{2}}\right)$$

- Interesting timescales:

  10^4 years: age of holocene

  10^2 years: rough estimate of minimal time needed for measurement and possibly correction of freshwater input

Summary and Conclusions

- Spectral properties allow us to draw conclusions on the distance from the bifurcation point
- Generic property of saddle-node bifurcation, might also be valid for more comprehensive models
- Stability of THC sufficient for estimating distance from bifurcation point
- Stochastic properties may give valuable insights into quantities not easily available from averaged measurements