Indicating the proximity to a critical threshold: The example of a bifurcation in a stochastic Ocean model

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Introduction

- Critical Thresholds in the Climate System may have drastic consequences for humankind if exceeded
- Exact location of thresholds often unknown or current position with respect to threshold difficult to determine
- Indicator that can be used to determine distance from threshold possibly useful
- Use of simple stochastic box model of North Atlantic Thermohaline Circulation (THC): Stommel model with stochastic freshwater flux
- Spectral properties possible indicator for distance from bifurcation point

Reduced Stommel model

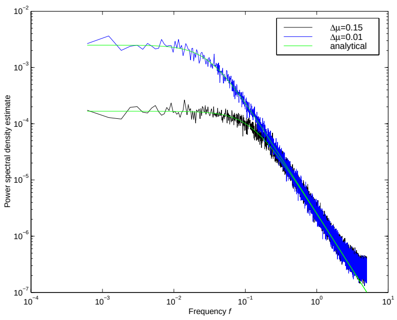
- If temperature relaxation is assumed to be much faster than salinity fluctuations, Stommel model can be reduced to 1D system in y only:

$$\dot{y} = -|1-y|y + \mu + \sigma\xi$$

- y nondimensional salinity difference, μ mean freshwater flux, bifurcation parameter, $\sigma\xi$ stochastic freshwater flux, σ standard deviation, ξ white noise process
- Reduced model retains relevant properties of full model: saddle-node bifurcation, hysteresis behavior

Analytical solution for power spectral density

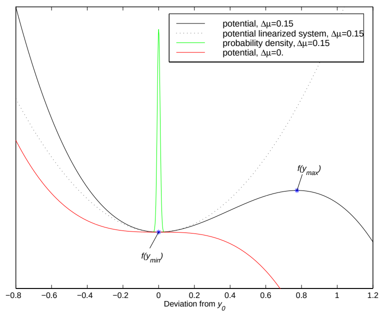
- Analytical expression for power spectral density from linearization around steady state solution
- Power spectral density dependent on distance from bifurcation point $\Delta\mu$: close to bifurcation increase in diffusion coefficient and decorrelation time



$$S(\omega, \Delta\mu) = \frac{\sigma^2}{4\Delta\mu + \omega^2}$$

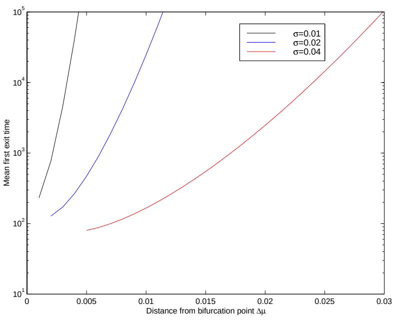
Probability density at bifurcation

- Probability density function calculated from Fokker-Planck equation
- Stationary solution: calculated from potential
- Cubic potential, potential well becomes shallower close to bifurcation
- potential well vanishes at bifurcation



Stability of the THC

- Stochastic system: Stability THC = Mean first exit time from potential well
- Analytical expression mean first exit time from Kramer's formula
- Stronger than exponential increase with distance from bifurcation point
- Strong decrease with rising noise amplitude σ



$$\tau(\sigma, \Delta\mu) = 2\pi(4\Delta\mu)^{-\frac{1}{2}}\exp\left(\frac{1}{3\sigma^2}(4\Delta\mu)^{\frac{3}{2}}\right)$$

Conclusions

- Stochastic description provides more information than deterministic description
- Spectral properties possible indicator for distance from bifurcation point
- Generic property of stochastic saddle-node bifurcation => results should also appear in more complex models, but still to be investigated