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A Dynamic Memory of Fracture Processes in Ice Shelves

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Abstract

Future sea-level rise has the potential to affect a large fraction of the world's urbanized areas and many generations to come. In order to design suitable mitigation and adaptation measures, accurate sea-level projections are required. Ice dynamics within the Antarctic Ice Sheet constitutes the major uncertainty in state-of-the-art model approaches. Floating ice shelves around the Antarctic coastline play an indirect but essential role for ice-discharge estimates.

In order to gain a better understanding of the Antarctic ice-sheet-shelf system and to provide physically sound estimates for future sea-level rise we have developed the Potsdam Parallel Ice Sheet Model (PISM-PIK). Many numerical issues had to be solved in order to generate a realistic representation of ice-shelf dynamics. In a dynamic equilibrium simulation of the entire Antarctic ice sheet under present day conditions, we could reproduce the characteristic pattern of ice flow, with a smooth transition between the different flow regime, in a way that is consistent with observations. PISM-PIK code was merged back into PISM and is now used by many groups around the world.

The position of the grounding line, which separates the grounded and floating ice, strongly influences the stress balance between ice shelves and ice sheet. In a model intercomparison I could show that, by the use of a combined shallow approximation of the stress balance, PISM features the same quality of steady-state grounding-line migration and reversibility as models using higher-order approximations. With the help of sub-grid techniques we improved PISM's performance for coarse resolutions as well.

Ice loss from the Antarctic ice shelves mainly occurs via calving of icebergs which is a highly complex process involving different spatial and temporal scales. I proposed a first-order kinematic calving parameterization which is based on the local spreading rates along the calving front which is associated with the propagation of preexisting fractures. In this simple formulation, in most cases modeled ice shelves remain within the basins where they are presently observed, mainly controlled by the confining boundary geometry via the non-local stress balances.

Fractures in ice shelves often form for certain conditions at the inlets or at shear margins when ice flows along prominent geographical features. Hence the calving at the ice front is connected with fracture formation far upstream. Therefore, I introduced a field state variable which remembers the damage history along the flow towards the calving front. It appears that active fracture zones strongly affect the flow dynamics especially in the small buttressing ice shelves. My *fracture density* approach nicely captures this interaction of fracturing and ice-shelf dynamics and provides the tools for more sophisticated calving parameterizations in large-scale land-ice models.

Zusammenfassung

Der bevorstehende Anstieg des Meeresspiegels wird sich weltweit auf die urbanisierten Küstengebiete und auf viele künftige Generationen auswirken. Um passenden Anpassungs- und Vermeidungsmaßnahmen planen zu können, bedarf es möglichst genauer Meeresspiegelprojektionen. Besonders die Modellierung von eisdynamischen Prozessen im Antarktischen Eisschild birgt dabei noch große Unsicherheiten. Vor allem die schwimmenden Eisschelfe entlang der antarktischen Küste spielen dabei eine zwar indirekte, aber ganze besondere Rolle.

Um das System von Landeis, Eisströmen und Eisschelfen in der Antarktis besser zu verstehen und somit physikalisch begründete Abschätzungen des zu erwartenden Meeresspiegelanstieges zu erhalten, haben wir das *Potsdam Parallel Ice Sheet Model* (PISM-PIK) entwickelt. Damit gelingt nun eine realistische Darstellung der Eisschelfdynamik. In einer dynamischen Gleichgewichtssimulation des Antarktischen Eisschildes unter heutigen Klimabedingungen, konnten wir das charakteristische Eisflussmuster reproduzieren, bei dem die unterschiedlichen dynamischen Regime fließend ineinander übergehen. PISM-PIK wurde mit PISM vereint und wird als solches von unterschiedliche Arbeitsgruppen weltweit genutzt.

Die Position der Aufsetzlinie, welche das Landeis vom Schelfeis trennt, beeinflusst maßgeblich das Spannungsgleichgewicht zwischen den beiden Teilen. Ich konnte im Rahmen eines Modellvergleichs zeigen, dass PISM trotz näherungsweiser Lösung des Spannungsgleichgewichts, in der Lage ist, qualitativ das Verhalten der Aufsetzlinie nach einer Störung richtig zu beschreiben. Dabei steht PISM selbst Modellen ohne Näherungen in nichts nach. Besonders mit Hilfe von Methoden, die die Bewegung der Aufsetzlinie linear zwischen den Gitterpunkten interpolieren, konnten wir das Modellverhalten von PISM wesentlich verbessern, insbesondere für grobe Auflösungen.

Der Antarktische Eisverlust geschieht vor allem durch das Abkalben von Eisbergen. Dieser Prozess basiert auf einer Reihe von komplexen Mechanismen, welche auf verschiedenen räumlichen und zeitlichen Skalen interagieren. Ich habe eine einfache kinematische Beschreibung für das Kalben gefunden, welche dieses Phänomen in erster Ordnung be-

schreibt, und zwar basierend auf den lokalen Dehnungsraten. Dabei wird angenommen, dass sich unter starker Dehnung bereits existierenden Risse weiter ausbreiten. Auf diese Weise kann gewährleistet werden, dass Eisschelfe im Modell stabil innerhalb von Buchten existieren können, welche durch Berge oder Inseln geschaffen werden. Diese Eigenschaft lässt sich letztendlich auf nichtlokale Eigenschaften des Spannungsgleichgewichtes zurückführen.

Das Kalben entlang der Eisschelfe wird durch Risse beeinflusst, die sich schon lange vorher und meist weiter im Inland gebildet haben. Dies geschieht beispielsweise an den Eiszufüssen oder dort, wo das Eis an den Rändern entlang schert. Deshalb habe ich eine Feldvariable eingeführt, die diese Verbindung berücksichtigt, indem sie die Rissgeschichte entlang des Eisflusses bis zur Front speichert. Dabei wird deutlich, dass dynamisch aktive Risszonen die Flussdynamik beeinflussen können und das ganz besonders in den kleineren Eisschelfen, welche im Allgemeinen eine starke rückgerichtete Kraft auf die Eisschilde ausüben. Meine Beschreibung als Rissdichte berücksichtigt genau diese Interaktion von Rissbildung und Eisschelfdynamik und öffnet damit den Weg für realistischere Parametrisierungen von Kalbungsprozessen in großskaligen Eisschildmodellen.

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Introduction

1.1. Ice Shelves and Climate Change

Sea-level rise constitutes one of the main impacts of anthropogenic climate change on society. The understanding of observed sea-level rise has significantly improved in the last years (Cazenave & Llovel, 2010, Church *et al.*, 2011, Gregory *et al.*, 2012). While thermal expansion of the warming ocean and melting mountain glaciers made the largest contribution in the past, the role of the polar ice sheets in Greenland and Antarctica has increased over the last decades (Van den Broeke *et al.*, 2011, Rignot *et al.*, 2011, Shepherd *et al.*, 2012, Hanna *et al.*, 2013). The large ice sheets bear a potential raise of sea level by around 70 m (Alley *et al.*, 2005). Accurate process-based models are required to understand the involved complex processes and interactions as sound basis for confident projections of future sea-level rise (Bamber & Aspinall, 2013, Levermann *et al.*, 2013).

In the Greenland Ice Sheet, surface melt is the predominant response to the warming climate that is captured well by contemporary surface mass-balance models. In Antarctica, however, ice loss into the surrounding ocean mainly occurs by the release of icebergs from the floating ice shelves (van der Veen, 1999). Most of the Antarctic coastline is, in fact, fringed by ice shelves (Rignot *et al.*, 2008, Bindshadler *et al.*, 2011). This exposed location at the interface of ice, atmosphere and ocean involves diverse interacting processes and a high sensitivity to moderate climate change. Both, sub-surface melting and calving of icebergs, are consequently essential components in the evaluation of climate-induced changes regarding the dynamics of the Antarctic ice sheet.

Most uncertainty in the projections of future solid-ice discharge results from a potential dynamic instability of the West Antarctic Ice Sheet, where the ice rests on an inward-sloping bed under the sea surface (Joughin & Alley, 2011, see Fig. 1.1). A

self-amplifying retreat of the ice sheet would result in a 3.3 m rise of the global sea level (Bamber *et al.*, 2009). Similar settings can be found in East Antarctica. The stability of these regions is highly connected with the stability of the protecting ice shelves (Dupont & Alley, 2005, 2006): Ice shelves provide a retentive force onto the inland ice flow, acting as buffers. Climate-induced processes can decrease this buttressing effect, resulting in an acceleration of the ice discharge from inland ice and contributing to a rising sea-level (Rignot, 2006, Goldberg *et al.*, 2009, Gudmundsson, 2013). Additionally, the discharge from the Antarctic continent could be enhanced by increased snowfall in the future climate (Winkelmann *et al.*, 2012).

State-of-the-art ice-flow models provide a comparably well representation of the different flow regimes and of grounding-line migration as sensitive indicator of changes in the boundary conditions (Bindshadler *et al.*, 2013, Pattyn *et al.*, 2013). However, none of these models has a representation of the underlying fracture processes, which are typical for polychrystalline materials such as ice. Fracture formation in ice shelves results from strong shear or tensile flow and can be enhanced by surface melt or basal melt (Pritchard *et al.*, 2012, Rignot *et al.*, 2013, MacAyeal & Sergienko, 2013). In turn, the existence of fractures potentially alters the macroscopic mechanical integrity of the ice body and preconditions the calving of icebergs. This feedback influences the flow pattern within the ice shelf and hence its stability. Under certain conditions ice shelves can disintegrate as observed in several cases along the Antarctic Peninsula in the last decades (Cook & Vaughan, 2010). This thesis describes the basic steps towards an ice-sheet model that accounts for all relevant physical processes needed for a comprehensive assessment of interactive responses to a changing climate and hence for a better estimate of sea-level contributions.

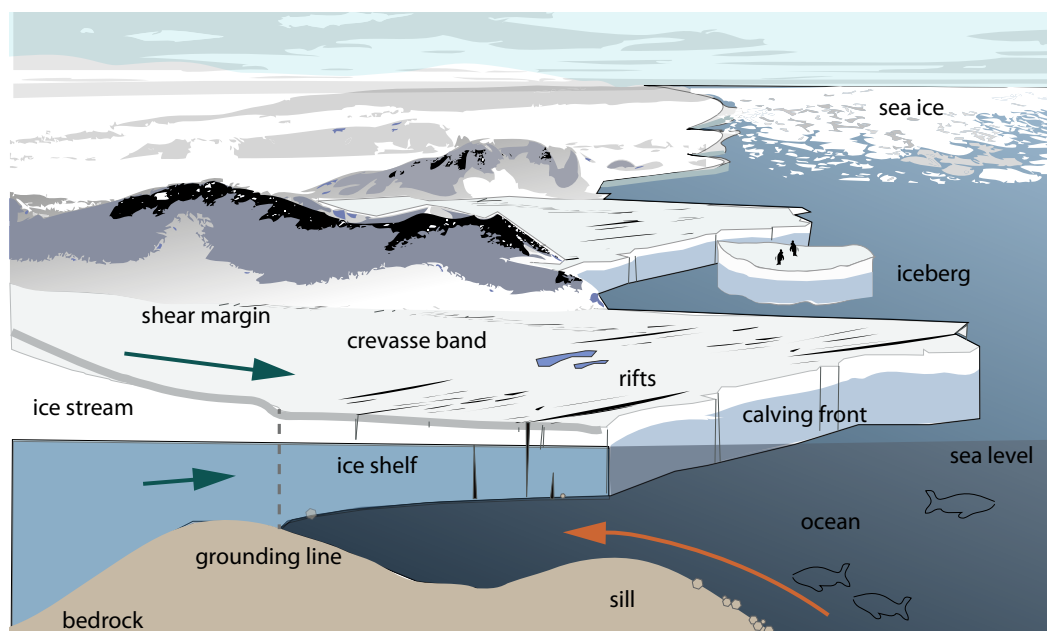


Figure 1.1 Schematic of an ice-stream/ice-shelf system draining into the ocean, flow direction indicated by green arrows. Ocean melt occurs when warm waters enter the cavity beneath the ice shelf indicated by orange arrow. Shear along mountain coast initiates fracture formation.

1.2. Scope and contents of the thesis

Ice shelves are the floating extensions of the large ice sheets and are mainly present in Antarctica. Due to their prominent location between grounded ice sheet, ocean and atmosphere the complex ice-shelf dynamics cannot be discussed in isolation from these influential drivers. In order to study the response of the Antarctic ice sheet to changing climate conditions the appropriate representation of the ice shelves in a continental-scale numerical model is of high priority.

The first article of my thesis (Sect. 2.1, [Winkelmann *et al.* \(2011\)](#)) introduces the Potsdam Parallel Ice Sheet Model (PISM-PIK) especially designed for the application to the entire Antarctic Ice Sheet system. The model code is based on the three-dimensional, thermomechanically-coupled Parallel Ice Sheet Model (PISM), which was separately developed by our colleagues in Alaska, and which has been primarily applied to the Greenland Ice Sheet. The article summarizes various novel model components, which together enable realistic simulations of ice streams and of ice-shelf flow. In a first important step, the advection scheme for the transport of shelf ice was replaced with a formulation that ensures mass conservation. This transport scheme was modified at the ice-shelf front such that the motion of the front can be continuously described, irrespective of the grid resolution (see Appendix, [Albrecht *et al.* \(2011\)](#)). Ice-shelf fronts appear in nature as steep ice walls due to the calving of icebergs. This shape is preserved in our treatment of the ice fronts, allowing for a proper definition of the force imbalance at the ocean-facing boundary. The calving-front boundary condition (CFBC) is of great importance in PISM, since it affects the flow in the entire ice domain. Mathematically the stress balance within the ice shelves is commonly approximated in terms of the Shallow Shelf Approximation (SSA, [Morland, 1987](#), [Weis *et al.*, 1999](#)). This defines an elliptic boundary-value problem in two dimensions where the velocities within the confined domain emerge from an optimization algorithm. Ice domains that are purely floating, such as detached icebergs, cannot be properly described in SSA: the problem is ill-posed. Hence, these patches need to be ignored in PISM-PIK simulations.

Ice streams are another dynamic feature of interest. Ice streams are fast-flowing conduits of ice that couple the slow-moving interior of the ice sheet to ice shelves at the margins. Along flowlines starting in the interior of the ice sheet, creep of flow, that is dominated by basal-shear stress, gives way to a “shelfy”, membrane-stress dominated flow regime before the ice stream becomes afloat as ice shelf. These regions are often located in narrow fjords between jagged mountains and typical flow speeds can range over several orders of magnitude. PISM-PIK provides a novel way of capturing this transition based on the hybrid approach by [Bueler & Brown \(2009\)](#). The Stokes equation provides a full description of all stress regimes involved in ice flow but simulations based on the

Stokes equation are very computationally expensive. Shallow approximations provide simpler and faster alternatives that are applicable to continental-scale simulations. Scale analysis reveals the dominant (zeroth order) terms of the stress balance based on the fact that the horizontal scale is much larger than the vertical for considered applications. For the flow regime in the interior, where the ice is mainly frozen to the bed, the Shallow Ice Approximation (SIA) applies. In contrast, for the floating ice shelves which experience no friction at their base, the SSA represents an appropriate simplification. Ice streams comprise aspects of both limiting flow regimes. Instead of taking into account higher-order terms of the stress balance, PISM-PIK simply superposes SIA and SSA velocity contributions throughout the entire ice domain. This approach is supported by theoretical considerations (Schoof, 2006, 2007) and is tested in flowline experiments within the Marine Ice Sheet Model Intercomparison Project (MISMIP, Pattyn *et al.*, 2012).

The second paper of my thesis (Sect. 2.2, Martin *et al.* (2011)) describes the application of PISM-PIK in an equilibrium simulation of the Antarctic ice sheet including the ice shelves. The climatic boundary conditions were parameterized according to present-day observations. Relevant parameters are constrained in an ensemble study such that observed large-scale characteristics are best reproduced, e.g., ice thickness distribution, ice volume and velocity pattern (Lythe *et al.*, 2001, Joughin *et al.*, 2002, Jezek *et al.*, 2003). A special focus of the work was on the representation of sliding-dominated flow within the ice streams and the steady-state position of the grounding line.

The grounding line is the boundary between the ice shelf and the ice stream where ice transitions from grounded to floating. Acceleration or thinning of the ice in this vicinity result in a retreat of the grounding line, which, in turn, feeds back to the dynamics both upstream and downstream. The dynamics in the vicinity of the grounding line are complicated. The validity of ice-sheet models depends very much on an adequate representation of grounding line motion. Together with our colleagues at the University of Alaska, Fairbanks, I merged PISM-PIK into the PISM code and implemented a routine which determines the grounding line position within the fixed mesh grid based on linear interpolation (Gladstone *et al.*, 2010). In two dimensions, the fraction of a grid cell's area covered with grounded ice hence determines a corresponding reduction in basal friction in the cell. The unified and updated PISM joined the second version of the Marine Ice Sheet Model Intercomparison Project (MISMIP3d, Sect. 2.3, Pattyn *et al.*, 2013). MISMIP3d defined a plan-view setup where basal friction in a spot close to the steady-state grounding-line is temporarily perturbed, causing a partial advance of the grounding line. Theoretically, the perturbed grounding line should then retreat to its initial position after the perturbation is switched off. The performance of PISM is compared with other state-of-the-art models, some of which take into account higher-

order terms or even the full set of stresses from the Stokes equation.

A special feature of the ice flow at the grounding line is the abrupt shift in the dynamic regime, which often finds its expression as a sharp bend in observations of the surface elevation. The gradient in surface elevation determines the driving stress which translates gravitational forces into a lateral drag towards thinner ice. In a finite-difference model using shallow approximation, as in PISM, horizontal gradients are typically computed using centered differences with respect to the neighboring grid cells. In this way the driving stress at the grounding line tends to be overestimated on the nearly flat floating side and underestimated at the steeply sloped grounded side. Making use of one-sided gradients to both side of the grounding line significantly improves the approximation of the driving stress in this dynamically important region. MISIP3d experiments have been repeated with the new PISM configuration to confirm a reasonable quality of grounding line motion, especially for coarse resolutions as used in continental-scale simulations (Sect. 2.4, [Feldmann *et al.* \(2014\)](#)).

A equilibrium simulation of the entire Antarctic ice sheet and ice shelf system cannot be simulated without a realistic representation of the ice losses. The calving of ice bergs contributes the largest portions of these losses in Antarctica and occurs in diverse patterns ([Benn *et al.*, 2007](#)). Various factors can stimulate calving, but it is the internal stresses determined by the geometry that controls the different calving regimes ([Alley *et al.*, 2008](#), [Bassis *et al.*, 2008](#), [Bassis & Jacobs, 2013](#)). Rifts act as precursor of calving icebergs and form in floating ice shelves mainly as a result of tensile stresses. Observation suggest that rifting occurs particularly in regions where the ice-shelf flow can spread more easily after passing beyond confining geometric features. This two-dimensional spreading can be quantified by the strain rates along the two principal flow axes, the eigenvalues of the strain-rate tensor. In the fifth paper of my thesis (Sect. 2.5, [Levermann *et al.* \(2012\)](#)), we propose a simple functional relationship between calving rate and the product of the two principal spreading rates that is associated with the average speed with which ice is shed from the terminus. The proportionality constant of this *eigencalving* formula comprises all relevant ice properties and can be estimated from observed velocity fields. The inferred range of values is applied in the model to individual Antarctic ice shelves and to the entire Antarctic ice sheet (Sect. 2.2, [Martin *et al.* \(2011\)](#)).

Eigencalving cannot resolve individual calving events but it provides an average rate of iceberg removal and hence a continuous description. Steady-state positions of calving fronts are possible, where the speed of the ice shelf at the terminus is equal and opposite of the calving rate. This calving speed depends on the spreading rate, and the spreading rate is calculated form the spatial velocity gradient. In this way *eigencalving* uses the non-locality of the stress balance in SSA, and the confining geometry can indirectly

control the steady-state calving front position. These long-range effects can reveal interesting reactions in mechanically coupled ice shelves when one ice shelf is strongly perturbed. This is motivated by the observation of the response of the calving front of the Larsen B Ice Shelf in the years after the collapse of Larsen A in 1995. After that event a gradual retreat of the Larsen B calving front has been registered before the final break-up occurred in 2002. In a simplified setup of two ice shelves connected by an ice bridge, the robustness of this mechanism is tested for varying parameters, such as the connective width (Sect. 2.6, Albrecht & Levermann (2014b)). I modified the calculation of the adaptive time step in the model such that the maximum calving speed can at most erode one grid cell length per time step. This allows for a comparison of time scales of observed and calculated calving-front retreat.

The formation of rifts and hence the release of icebergs is preconditioned by fractures (Hulbe *et al.*, 2010). These fractures mostly form in dynamically active regions farther upstream, e.g., in the vicinity of the grounding line or in shear margins. From these formation areas the fractured ice is carried with ice flow, forming elongated fracture bands, which are observed in satellite images. Inverse modeling indicates that these active fracture bands appear to be extremely low in viscosity and affect the flow dynamics in the entire ice shelf (Khazendar *et al.*, 2007, Borstad *et al.*, 2012). Especially in the small buttressing ice shelves, horizontal stresses cannot be transferred effectively across the fracture-weakened zones. This results in large velocity gradients across these narrow sections and comparably high flow speeds in the interior of the ice shelf. Furthermore, fracture dynamics and interaction provide a plausible explanation for the abrupt disintegration of ice shelves that occurred several times along the Antarctic Peninsula (e.g., Scambos *et al.*, 2003, MacAyeal *et al.*, 2003, Glasser & Scambos, 2008, Scambos *et al.*, 2009, MacAyeal & Sergienko, 2013).

These studies emphasize that fracture processes cannot be considered just as boundary effects of the ice flow. In fact they control the flow dynamics and the stability of ice shelves (Doake, 2001) and hence the ice discharge of ice sheets into the surrounding oceans. However, fractures are discrete discontinuities and cannot be individually resolved in a fluid-dynamic framework. In the seventh paper, presented in this thesis (Sect. 2.7, Albrecht & Levermann (2012)), I propose a model approach for the parameterization of fracture dynamics in commonly used continuum-mechanics ice models, here applied to PISM. In this approach, fracture formation areas can be identified by a material-strength criterion (Vaughan, 1993), which expresses that ice can support only a limited effective stress before failure. In these regions fractures are assumed to grow with a rate that is proportional to the prevailing maximum spreading rate. The average spacing and size of fractures is expressed in terms of a *fracture density* field. A high *fracture density* limits the growth of further fractures accounting for local

fracture interaction. As a scalar tracer *fracture density* can be simply transported with the ice flow by advection. If the fractured ice passes through flow regimes where shearing and spreading are comparably small, fractures can close. This healing effect is approximated analogous to fracture growth. With this model framework based on first-order assumptions of continuous-damage modeling we are able to explain many observed features in plan-view fracture patterns. However, in this formulation *fracture density* describes the evolution of fractures in only one effective direction.

The eighth article of my thesis (Sect. 2.8, [Albrecht & Levermann \(2014a\)](#)) considers the feedback of fractures onto the flow dynamics in ice shelves and ice streams. The coupling is introduced by reducing the effective viscosity according to the density of fractures as supported by previous studies (e.g., [Pralong & Funk, 2005](#), [Humbert et al., 2009](#), [Duddu & Waisman, 2012](#), [Borstad et al., 2013](#)). With this simple coupling, self-amplifying effects can be studied in order to explain various aspects of observed flow patterns and to evaluate the buttressing effect of certain ice shelves. Furthermore, enhanced fracture formation by atmospheric or oceanic forcing can be considered. These effects may promote calving and even ice-shelf disintegration. The *fracture density* approach focuses on the qualitative interactions of fracturing and dynamic ice flow and acts as a link in the causal reasoning from observed phenomena to future risks. In the article we present improvements of the numerical schemes, compare different failure criteria and give an outlook to future development steps and applications.

1.3. Overview

This thesis is organized around eight scientific articles, which are either published or under review. In the following I give an overview of the contents of each individual article which is presented in Section 2. Based on the work for my diploma thesis in 2009 I have worked on a more technical paper, which is attached in the Appendix.

Article 1

The Potsdam Parallel Ice Sheet Model (PISM-PIK) - Part 1: Model Description

R. Winkelmann, M.A. Martin, M. Haseloff, T. Albrecht, E. Bueler, C. Khroulev, A. Levermann

This article describes the Potsdam Parallel Ice Sheet Model (PISM-PIK) with various novel modules for a better representation of the ice streams and the floating ice shelves. PISM-PIK is based on the Parallel Ice Sheet Model (PISM, [Bueler & Brown, 2009](#)), version stable 0.2. Torsten Albrecht improved the transport scheme and the representation of ice-shelf-front motion therein (see separate publication in Appendix). This was the basis for the implementation of an appropriate boundary condition at the front, which Torsten Albrecht and Marianne Haseloff implemented together. A main contribution by Torsten Albrecht was the development and implementation of a physically-based calving law, which is presented in more detail in Article 5. Torsten Albrecht and Constantine Khroulev have merged PISM-PIK modifications into PISM version stable 0.4.

Article 2

The Potsdam Parallel Ice Sheet Model (PISM-PIK) - Part 2: Dynamic equilibrium simulation of the Antarctic ice sheet

M. A. Martin, R. Winkelmann, M. Haseloff, T. Albrecht, E. Bueler, C. Khroulev, A. Levermann

The performance of PISM-PIK in a dynamic equilibrium simulation of the Antarctic ice-sheet system is analyzed and compared against observations. All coauthors participated in the development of the model code (see Article 1) and the discussion of the results as well as in the improvement of the manuscript.

Article 3

Grounding-line migration in plan-view marine ice-sheet models: results of the ice2sea MISMIP3d intercomparison

F. Pattyn, L. Perichon, G. Durand, L. Favier, O. Gagliardini, R.C.A. Hindmarsh, T. Zwinger, T. Albrecht, S. Cornford, D. Docquier, J.J. Fürst, D. Goldberg, G.H. Gudmundsson, A. Humbert, M. Hütten, P. Huybrechts, G. Jouvet, T. Kleiner, E. Larour, D. Martin, M. Morlighem, A.J. Payne, D. Pollard, M. Rückamp, O. Rybak, H. Seroussi, M. Thoma and N. Wilkens

The merged and improved PISM version 0.5 participated in the model intercomparison MISMIP3d to investigate grounding-line migration in a marine ice-sheet-shelf system. Torsten Albrecht and Moritz Hütten ran the PISM simulations and the analysis of the results. Frank Pattyn coordinated the intercomparison, while the other coauthors represent different ice models.

Article 4

Resolution-dependent performance of grounding line motion in a shallow model compared to a full-Stokes model according to the MISMIP3d intercomparison

J. Feldmann, T. Albrecht, C. Khroulev, F. Pattyn, A. Levermann

This article indicates the main improvements of the latest model development by Torsten Albrecht and Constantine Khroulev using the computational setup of the model intercomparison MISMIP3d that was coordinated by Frank Pattyn (see Article 3). Johannes Feldmann ran the simulations and evaluated the results supervised by Torsten Albrecht. The manuscript was written by Johannes Feldmann supported by Torsten Albrecht and Anders Levermann.

Article 5

Kinematic first-order calving law implies potential for abrupt ice-shelf retreat

A. Levermann, T. Albrecht, R. Winkelmann, M. A. Martin, M. Haseloff, I. Joughin

A first-order kinematic calving parameterization, called *eigencalving*, is proposed as representation of large-scale iceberg formation at ice-shelf fronts. Stabilizing effects of the confining geometry can be accounted for via the non-local character of the stress-balance formulation which results in steady-state ice-shelf fronts in consistency with satellite

observations. Anders Levermann and Torsten Albrecht developed the physical basis. Torsten Albrecht implemented the parameterization into PISM, performed the model simulations and validation of the model results against observational data provided by Ian Joughin. Together with Ricarda Winkelmann, Maria Martin and Marianne Haseloff they discussed the results and the manuscript.

Article 6

Spontaneous ice-front retreat caused by disintegration of adjacent ice shelf in Antarctica

T. Albrecht and A. Levermann

Application of the *eigencalving* parameterization (Article 5) to a setting of adjacent ice shelves that are dynamically connected, can reveal a spontaneous dynamic response of the one ice shelf if the other disintegrates. Focused on the abrupt collapse events in the Larsen area in the last decades this phenomenon is studied using PISM-PIK. Torsten Albrecht performed the experiments and wrote the article, supported by Anders Levermann.

Article 7

Fracture field for large-scale ice dynamics

T. Albrecht and A. Levermann

A macroscopic *fracture-density* field variable is introduced into the prognostic continuum ice-flow model PISM accounting for fracture initiation, fracture growth and transport of fractures with the ice flow. Torsten Albrecht designed the concept based on first-order assumptions and performed the simulations for the largest Antarctic ice shelves. Torsten Albrecht wrote the manuscript, supported by Anders Levermann.

Article 8

Fracture-induced softening for large-scale ice dynamics

T. Albrecht and A. Levermann

This study is based on the previous Article 7 and considers the full feedback of fracture processes and ice-flow dynamics. This formulation enables simulations of more realistic velocity distributions within ice shelves and ice streams and contributes to a more

confident estimate of the stability of the small buttressing ice shelves fringing Antarctica. Torsten Albrecht refined the concepts of the continuous fracture-density formulation, improved the two-dimensional transport scheme and created a physically sound basis for a fracture-enhanced calving parameterization. Torsten Albrecht wrote the manuscript supported by Anders Levermann.

Original Manuscripts

2.1. The Potsdam Parallel Ice Sheet Model (PISM-PIK) - Part 1: Model description

Abstract: We present the Potsdam Parallel Ice Sheet Model (PISM-PIK), developed at the Potsdam Institute for Climate Impact Research to be used for simulations of large-scale ice sheet-shelf systems. It is derived from the Parallel Ice Sheet Model (Bueler and Brown, 2009). Velocities are calculated by superposition of two shallow stress balance approximations within the entire ice covered region: the shallow ice approximation (SIA) is dominant in grounded regions and accounts for shear deformation parallel to the geoid. The plugflow type shallow shelf approximation (SSA) dominates the velocity field in ice shelf regions and serves as a basal sliding velocity in grounded regions. Ice streams can be identified diagnostically as regions with a significant contribution of membrane stresses to the local momentum balance. All lateral boundaries in PISM-PIK are free to evolve, including the grounding line and ice fronts. Ice shelf margins in particular are modeled using Neumann boundary conditions for the SSA equations, reflecting a hydrostatic stress imbalance along the vertical calving face. The ice front position is modeled using a subgrid-scale representation of calving front motion (Albrecht et al., 2011) and a physically-motivated calving law based on horizontal spreading rates. The model is tested in experiments from the Marine Ice Sheet Model Intercomparison Project (MISMIP). A dynamic equilibrium simulation of Antarctica under present-day conditions is presented in Martin et al. (2011).

2.2. The Potsdam Parallel Ice Sheet Model (PISM-PIK) - Part 2: Dynamic equilibrium simulation of the Antarctic ice sheet

Abstract: We present a dynamic equilibrium simulation of the ice sheet-shelf system on Antarctica with the Potsdam Parallel Ice Sheet Model (PISM-PIK). The simulation is initialized with present-day conditions for bed topography and ice thickness and then run to steady state with constant present-day surface mass balance. Surface temperature and sub-shelf basal melt distribution are parameterized. Grounding lines and calving fronts are free to evolve, and their modeled equilibrium state is compared to observational data. A physically-motivated calving law based on horizontal spreading rates allows for realistic calving fronts for various types of shelves. Steady-state dynamics including surface velocity and ice flux are analyzed for whole Antarctica and the Ronne-Filchner and Ross ice shelf areas in particular. The results show that the different flow regimes in sheet and shelves, and the transition zone between them, are captured reasonably well, supporting the approach of superposition of SIA and SSA for the representation of fast motion of grounded ice. This approach also leads to a natural emergence of sliding-dominated flow in stream-like features in this new 3-D marine ice sheet model.

2.3. Grounding-line migration in plan-view marine ice-sheet models: results of the ice2sea MISMIP3d intercomparison

Abstract: Predictions of marine ice-sheet behaviour require models able to simulate grounding-line migration. We present results of an intercomparison experiment for plan-view marine ice-sheet models. Verification is effected by comparison with approximate analytical solutions for flux across the grounding line using simplified geometrical configurations (no lateral variations, no buttressing effects from lateral drag). Perturbation experiments specifying spatial variation in basal sliding parameters permitted the evolution of curved grounding lines, generating buttressing effects. The experiments showed regions of compression and extensional flow across the grounding line, thereby invalidating the boundary layer theory. Steady-state grounding-line positions were found to be dependent on the level of physical model approximation. Resolving grounding lines requires inclusion of membrane stresses, a sufficiently small grid size (<500 m), or subgrid interpolation of the grounding line. The latter still requires nominal grid sizes of <5 km. For larger grid spacings, appropriate parameterizations for ice flux may be imposed at the grounding line, but the short-time transient behaviour is then incorrect and different from models that do not incorporate grounding-line parameterizations. The numerical error associated with predicting grounding-line motion can be reduced significantly below the errors associated with parameter ignorance and uncertainties in future scenarios.

Please find the full manuscript, published in *Journal of Glaciology* Vol. 59, No. 215, 410-422 (2013), [here](#).

2.4. Resolution-dependent performance of grounding line motion in a shallow model compared to a full-Stokes model according to the MISMIP3d intercomparison

Abstract: Making confident statements about the evolution of an ice sheet-shelf-system with a numerical model requires its capability of reproducing the migration of the grounding line. Here we show that the SIA/SSA hybrid-type Parallel Ice Sheet Model (PISM), with its recent improvements, is capable of modeling the grounding line motion in a perturbed ice sheet-shelf-system. The model is set up according to the three-dimensional Marine Ice Sheet Intercomparison Project (MISMIP3d) and simulations are carried out across a broad range of spatial resolutions. Using 1) a linear interpolation of the grounding line with locally interpolated basal friction and 2) an improved driving stress computation across the grounding line, the reversibility of the grounding line (i.e. its retreat after an advance forced by a local perturbation of basal resistance) is captured by the model even on medium and low resolutions ($\Delta x > 10$ km). The transient model response is qualitatively similar to that of higher-order models but reveals a higher initial sensitivity to perturbation on very short time scales. Our findings support the application of PISM to the Antarctic Ice Sheet from a regional up to the continental scale also on relatively low spatial resolutions.

Please find the full manuscript, published in *Journal of Glaciology* Vol. 60, No. 220, 353-360 (2014), [here](#).

2.5. Kinematic first-order calving law implies potential for abrupt ice-shelf retreat

Abstract: Recently observed large-scale disintegration of Antarctic ice shelves has moved their fronts closer towards grounded ice. In response, ice-sheet discharge into the ocean has accelerated, contributing to global sea-level rise and emphasizing the importance of calving-front dynamics. The position of the ice front strongly influences the stress field within the entire sheet-shelf-system and thereby the mass flow across the grounding line. While theories for an advance of the ice-front are readily available, no general rule exists for its retreat, making it difficult to incorporate the retreat in predictive models. Here we extract the first-order large-scale kinematic contribution to calving which is consistent with large-scale observation. We emphasize that the proposed equation does not constitute a comprehensive calving law but represents the first-order kinematic contribution which can and should be complemented by higher order contributions as well as the influence of potentially heterogeneous material properties of the ice. When applied as a calving law, the equation naturally incorporates the stabilizing effect of pinning points and inhibits ice shelf growth outside of embayments. It depends only on local ice properties which are, however, determined by the full topography of the ice shelf. In numerical simulations the parameterization reproduces multiple stable fronts as observed for the Larsen A and B Ice Shelves including abrupt transitions between them which may be caused by localized ice weaknesses. We also find multiple stable states of the Ross Ice Shelf at the gateway of the West Antarctic Ice Sheet with back stresses onto the sheet reduced by up to 90% compared to the present state.

2.6. Spontaneous ice-front retreat caused by disintegration of adjacent ice shelf in Antarctica

Abstract: Antarctic ice-discharge constitutes the largest uncertainty in future sea-level projections. Floating ice shelves, fringing most of Antarctica, exert retentive forces onto the ice flow. While spontaneous ice-shelf retreat has been observed, it is generally considered a localized phenomenon. Here we show that the disintegration of an ice shelf may induce the spontaneous retreat of its neighbor. As an example, we reproduce the spontaneous but gradual retreat of the Larsen-B ice-front as observed after the disintegration of the adjacent Larsen-A ice-shelf. We show that the Larsen-A collapse yields a change in spreading rate in Larsen-B via their connecting ice-channels and thereby causes a retreat of the ice-front to its observed position of the year 2000. This mechanism might be particularly relevant for the role of East Antarctica and the Antarctic Peninsula in future sea-level.

Please find the full manuscript, published in *Earth and Planetary Science Letters* 393, 26-30, (2014), [here](#).

2.7. Fracture field for large-scale ice dynamics

Abstract: Recent observations and modeling studies emphasize the crucial role of fracture mechanics for the stability of ice shelves and thereby the evolution of ice sheets. Here we introduce a macroscopic fracture-density field into a prognostic continuum ice-flow model and compute its evolution incorporating the initiation and growth of fractures as well as their advection with two-dimensional ice flow. To a first approximation, fracture growth is assumed to depend on the spreading rate only, while fracture initiation is defined in terms of principal stresses. The inferred fracture-density fields compare well with observed elongate surface structures. Since crevasses and other deep-reaching fracture structures have been shown to influence the overall ice-shelf dynamics, we propose the fracture density field introduced here be used as a measure for ice softening and decoupling of the ice flow in fracture-weakened zones. This may yield more accurate and realistic velocity patterns in prognostic simulations. Additionally, the memory of past fracture events links the calving front to the upstream dynamics. Thus the fracture-density field proposed here may be employed in fracture-based calving parameterizations. The aim of this study is to introduce the field and investigate which of the observed surface structures can be reproduced by the simplest physically motivated fracture source terms.

Please find the full manuscript, published in *Journal of Glaciology* Vol. 58, No. 207, 165-176 (2012), [here](#).

2.8. Fracture-induced softening for large-scale ice dynamics

Abstract: Floating ice shelves can exert a retentive and hence stabilizing force onto the inland ice sheet of Antarctica. However, this effect has been observed to diminish by fracture-coupled dynamic processes within the protective ice shelves leading to accelerated ice flow and hence to a sea-level contribution. In order to better understand the role of fractures in ice dynamics we apply a large-scale continuum representation of fractures and related fracture growth into the prognostic Parallel Ice Sheet Model (PISM). To this end we introduce a higher-order accuracy advection scheme for the transport of the two-dimensional fracture density across the regular computational grid. Dynamic coupling of fractures and ice flow is attained by a reduction of effective ice viscosity proportional to the inferred fracture density. This formulation implies the possibility of a non-linear threshold behavior due to self-amplified fracturing in shear regions triggered by small variations in damage threshold. As a result of prognostic flow simulations flow patterns with realistically large across-flow velocity gradients in fracture-weakened regions as seen in observations are reproduced. This model framework is expandable to grounded ice streams and accounts for climate-induced effects on fracturing and hence on the ice flow dynamics. It further allows for an enhanced fracture-based calving parameterization.

Discussion and Conclusions

The Antarctic ice shelves at the interface of ice sheet, atmosphere and ocean play a major role in the discussion on global sea-level rise. This connection appears to be indirect but diverse important processes occur and interact within the floating ice shelves. Especially the formation of fractures and the related release of calving icebergs has been widely neglected in past consideration. In my thesis I focussed on the questions (1) how discontinuous fracture-related processes can be represented within the framework of a continuous large-scale ice-flow model and (2) how the coupling to the ice flow can be quantified such that observed phenomena can be reproduced and understood. My thesis delineates the successive steps of the development of an ice-sheet model which applies to these questions and therewith to some central questions of the scientific community. A profound understanding of ice-shelf interaction requires interdisciplinary approaches including material science. As a team we designed various model components to improve the representation of the ice shelves. This work is the basis for realistic simulations of the entire Antarctic Ice sheet.

Based on the open-source model PISM (see documentation: www.pism-docs.org), we developed novel strategies in order to capture the ice-shelf flow in an appropriate way. First, I modified the transport scheme of shelf ice to ensure mass conservation. For the realistic representation of the motion of the ice-shelf calving-front, I introduced a sub-grid scheme for the ice transport. This made it possible to apply correct stress-boundary condition at this ice-shelf front. This was necessary in the used mathematical framework in order to produce ice velocities in reasonable agreement with observations. In PISM this affects not only the floating ice shelves. In order to realize a smooth transition between the different flow regimes we use a superposition of the two used shallow approximations (SSA and SIA) over the entire ice domain. Especially within the fast ice streams draining into the ice shelves, the “shelfy” flow character is well represented. The superposition is also supported by variational scale-analysis of the underlying equations (Schoof, 2006). The performance of this model configuration was

tested within the context of the MISMIP model intercomparison, verified against a semi-analytical solution (Pattyn *et al.*, 2012). All these modification entered PISM-PIK, which was then applied in an equilibrium simulation of the entire Antarctic Ice Sheet. Several parameters had be constrained within a comprehensive ensemble study. The inferred overall ice thickness and velocity distribution matches the observations to a high degree and reveals flow features with a range in velocity magnitude of more than three orders.

Together with our colleagues at the University of Fairbanks, Alaska, I was able to merge PISM-PIK into an updated version of PISM. In this model configuration the position of the grounding line between shelf ice and land ice as well as the corresponding basal friction are considered on a linearly interpolated subgrid. This treatment allows for a more accurate representation of grounding-line motion within a finite-difference model even for comparably coarse resolutions. This was tested in a pursuing model intercomparison, MISMIP3d, where PISM was able to reproduce the same quality of grounding-line curvature after perturbation and reversibility as models with higher-order approximations of the Stokes stress balance. The participation in such a model intercomparison generally supports the acceptance of the model within the scientific community. PISM is now widely used by modelers around the world, applied to both the prehistoric Laurentide Ice Sheet, the Greenland Ice Sheet and the Antarctic Ice Sheet (e.g., Golledge *et al.*, 2012, Van Pelt & Oerlemans, 2012, Bindshadler *et al.*, 2013, Aschwanden *et al.*, 2013).

PISM's results in the MISMIP3d experiments even improved by a modification of the driving-stress approximation upstream and downstream of the grounding line. The default centered-difference scheme was replaced by one-sided differences avoiding differencing across the kink in surface elevation at the grounding line. Reversibility, i.e. the return of the grounding line to its initial position after switching off the localized basal perturbation, is obtained also for coarse resolutions of 10 km and lower as currently used in continental-scale simulations with PISM. Even the transient response times agree with those inferred by full-Stokes models. Furthermore a resolution-dependent bias in the steady-state position of the grounding line could be eliminated. Accurate grounding line curvature, reversibility also for coarse resolutions and transient response time close to the results of a full-Stokes model are important properties of a model in order to provide more confident projections of solid-ice discharge across the grounding line in response to changing climatic boundary conditions.

Besides the grounding line also the position of the calving front is a sensitive indicator of changes in the internal ice dynamics as response to changing climatic forcing. However, the complex calving mechanism comprises both the frequent shedding of small

ice slabs as well as the sporadic release of huge tabular icebergs expressed as a calving rate. Even though most of the ice discharge from Antarctica occurs via calving, this process had been widely avoided in models or was estimated with semi-empirical approaches. Based on observations and previous analyses we derived a first-order relationship between the inferred spreading of the ice shelf in the areas close to the front and the average calving rate. The proportionality constant may be associated with geometric and glaciological properties of the ice at the front. In fact, the proportionality constant derived from observed velocities is about an order of magnitude larger for the less confined major ice shelves Ross, Ronne and Larsen C than for smaller ice shelves, such as Amery or Filchner. The *eigen-calving* parameterization cannot explain the complex physics involved but it provides a physically-motivated tool to account for calving in large-scale ice-flow models. Furthermore the spreading rates are calculated from the stress balance in SSA, which is represented by a two-dimensional elliptic boundary value problem. Hence, the geometry of an ice shelf, i.e. the coastline and the position of the calving front, affects the overall flow field in a non-local manner. In this way the calving front is coupled to the geometry of the embayment, enabling realistic steady-state ice-shelf configurations. We used the calving parameterization in an equilibrium simulation under present-day climate conditions where the steady-state calving front of the major Antarctic ice shelves agreed well with observations. We could further show that, based on our assumptions, multiple steady-state ice-shelf configurations are possible, involving significantly reduced backstresses. In fact, PISM permits a quality of stability analysis which is unique so far and has been intensively discussed within the ice-modeler community.

Regarding the non-locality of the underlying stress-balance approximation we used the calving parameterization in a setting of dynamically connected ice shelves. We expanded the concept of adaptive time steps, considering the calculated calving rate as horizontal mass flux on the regular grid. This was necessary in order to evaluate the speed of calving-front retreat after perturbations. In simulations with PISM-PIK we investigated a mechanism where an abrupt disintegration of the one ice shelf induces a spontaneous calving front retreat of the other. Such a response was observed during the years after the breakup of the Larsen A Ice Shelf in 1995, possibly promoting a cascade of effects ultimately causing the disintegration of Larsen B seven years later. Similar settings can be found in several ice shelves around the Antarctic coast. The described mechanism has not been discussed before and emphasizes the strength of PISM for the understanding of complex interactions occurring in ice-sheet/ice-shelf systems.

The *eigen-calving* parameterization is justified by the observations that ice-shelf flow tends to be less confined when approaching the front. This leads to spreading also across the main flow in coincidence with the formation of long rifts from pre-existing fractures in those areas. Long intersecting rifts ultimately cause the release of icebergs into the

ocean. Fracture formation plays a major role in this interplay and needs an appropriate representation in ice-shelf models. This finding motivated the introduction of a scalar field variable, which measures the abundance of fractures in a certain region, called *fracture density*. This continuous tracer can be easily transported with the calculated ice flow within the standard finite-difference model framework of PISM. Depending on the prevailing stress regime, additional fractures can form or existing fractured ice can get healed. Fracture interaction is introduced by a reducing factor in the source term. *Fracture density* simply accounts for the accumulation or deactivation of fractures in the ice on its way down the stream. This conception can be associated with a memory of the stress history that the ice experienced during advection. In order to identify regions, where fractures most likely form, I decided for a stress-threshold formulation: the von-Mises criterion is commonly used for the fracture initiation based on the strength of the material ice. I had to find a simple physical way to quantify the fracture growth in these regions expressed in terms of a *fracture-density* contribution rate. Likewise the deactivation of fractures by healing needed to be quantified, although very little is known about such processes. Assuming a first-order description of the underlying processes, the number of tuning parameter is kept to a minimum of four, namely two rate factors and two thresholds. The parameter values could be constrained in a minor ensemble study applied to the Ronne-Filchner Ice Shelf comparing the evolving *fracture density* pattern with those inferred from observations. In this way I could reproduce observed pattern of fracture bands with origin in certain regions of high shear stresses close to geographical features or along the inlets. The shape of the fracture bands appears even more pronounced when a modified advection scheme is applied reducing the angle-dependence of the flow with respect to the underlying grid axes. Also resolution influences the unwanted numerical dissipation of the fracture bands with distance from the formation area.

However, I designed the *fracture density* to account for the interaction of fracture processes and the ice flow. In this framework the *fracture density* with values bounded between zero (no fractures) and one (totally damaged) applies as enhancement factor reducing the effective viscosity. This formulation is based on the assumption that fractured ice behaves on a macroscopic scale kinematically equivalent as intact ice of lower rigidity. This approach allows for a simple coupling of fracture dynamics and ice-flow dynamics which bears the potential for self-amplifying feedbacks. This has been exemplarily investigated in a shear regime, where the von-Mises threshold was successively decreased, mimicking external changes in climatic conditions. If fracture formation is activated also the shearing intensifies by corresponding softening. This leads to further fracture formation also downstream of the initial formation area, where the previously formed fractures are transported to. In this way the evolving active fracture bands enable intense across-flow gradients in ice-flow velocity as indicated by inverse modeling techniques. Fracture-weakened shear regions can cause a reduction

in buttressing and possibly a destabilization especially of the smaller ice shelves. The discharge from the interior regions of the ice sheet would consequently accelerate, which could lead to a significant contribution to sea-level rise. The idea of *fracture density* provides a simple coupling of fracture phenomena and ice flow and links these processes over time and space. This attribute is reminiscent of a memory, which stores information about the history of experienced stress regimes and undergone fracture formation.

The concepts of *fracture density* and of *eigen-calving* are rather simple but various phenomena can be studied and understood qualitatively in this framework. The destabilization and disintegration of ice shelves is one prominent scientific question which may be investigated in more detail with the current model configuration. Involved first-order functional relationships could be replaced by more sophisticated approaches if more validation data is available. Furthermore, a coupling of both concepts could be used to account for the preconditioning of calving by pre-existing fractures. Rifts form from deep fractures and have been observed to initiate and terminate in fracture bands when they propagate transversally. This phenomenon and many others could be accounted for in such a coupled formulation.

I am convinced that first-order model approaches generally are a reasonable way to gain an elementary understanding of certain observed phenomena. It can furthermore provide a proper basis for model-based projections of future ice discharge and sea-level contributions. The Fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC, <http://ipcc.ch>) will - for the first time - explicitly present such projections for the near future based on different emission scenarios. This is of tremendous societal importance since many densely populated areas and cultural heritage are particularly vulnerable and at risk. In this sense I hope that my work can contribute ultimately to intelligent decisions on the future of humanity.

Appendix

Parameterization for subgrid-scale motion of ice-shelf calving fronts

T. Albrecht, M. A. Martin, M. Haseloff, R. Winkelmann, A. Levermann

Abstract: A parameterization for the motion of ice-shelf fronts on a Cartesian grid in finite-difference land-ice models is presented. The scheme prevents artificial thinning of the ice shelf at its edge, which occurs due to the finite resolution of the model. The intuitive numerical implementation diminishes numerical dispersion at the ice front and enables the application of physical boundary conditions to improve the calculation of stress and velocity fields throughout the ice-sheet-shelf system. Numerical properties of this subgrid modification are assessed in the Potsdam Parallel Ice Sheet Model (PISM-PIK) for different geometries in one and two horizontal dimensions and are verified against an analytical solution in a flow-line setup.

Please find the full manuscript, published in *The Cryosphere* 5, 35-44 (2011), [here](#).

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Erklärung

Diese Arbeit ist bisher an keiner anderen Hochschule eingereicht worden. Sie wurde selbständig und ausschließlich mit den angegebenen Mitteln angefertigt.

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Torsten Albrecht

